Testing Independence of Error Terms:  
**The Durbin-Watson Statistic** (Text Section 12.3)

One assumption of our linear regression model is that the error terms are independent. A common violation of this assumption occurs when each error term is related to its immediate predecessor ($\varepsilon_i$ is related to $\varepsilon_{i-1}$). This is mostly likely to occur when the data points were observed in some sort of meaningful time sequence (weekly sales data, for example). This type of relationship is called *first order autocorrelation*. The parameter $\rho$ is used to represent first order autocorrelation, where $-1 \leq \rho \leq +1$. If error terms exhibit first order autocorrelation then they follow the relation:

$$
\varepsilon_t = \rho \varepsilon_{t-1} + \mu_t
$$

where the $\mu_t$ values are assumed to be independent $\text{N}(0, \sigma^2)$.

The *Durbin-Watson* statistic is typically used to test: $H_0: \rho = 0$ vs. $H_1: \rho > 0$ since when error terms are correlated in business and economic applications, the correlation tends to be positive (Reference: Neter, Kutner, Nachtsheim, and Wasserman, *Applied Linear Statistical Models*, 4th Edition, pg. 497). It does this by measuring the correlation between error terms and their immediate predecessors:

$$
D = \frac{\sum (e_t - e_{t-1})^2}{\sum e_t^2}
$$

The statistic $D$ ranges in value from zero to four. When the error terms are independent we expect $D$ to be close to 2. “Small” values of $D$ suggest that error terms tend to cluster (positive autocorrelation); “large” values of $D$ suggest that error terms tend to alternate (+, -, +, -) (negative autocorrelation). Critical values for the one-sided test for positive autocorrelation can be found in Table B.7 on page 1349 (it’s based on $n$, the sample size, and $p$, the number of independent variables in the model).

The decision rule is a little different: If $D < d_L$ you’d reject $H_0$ and conclude that the error terms exhibit positive autocorrelation; if $D > d_U$ you’d fail to reject $H_0$ and conclude that the error terms do not have positive autocorrelation, and if $d_L \leq D \leq d_U$ the test is inconclusive.

Minitab can calculate this statistic automatically. It’s under ‘Options’ in the regression setup window.

You can also test for negative autocorrelation by using $4 - D$ instead of $D$ for your test statistic.
Example 1:

Sales_A = 50.6 + 2.51 Week

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Coef</th>
<th>StDev</th>
<th>T</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>50.621</td>
<td>2.995</td>
<td>16.90</td>
<td>0.000</td>
</tr>
<tr>
<td>Week</td>
<td>2.5142</td>
<td>0.1687</td>
<td>14.90</td>
<td>0.000</td>
</tr>
</tbody>
</table>

S = 7.998   R-Sq = 88.8%   R-Sq(adj) = 88.4%

Durbin-Watson statistic = 1.95
Example 2:

The regression equation is
Sales_B = 50.7 + 3.56 Week

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Coef</th>
<th>StDev</th>
<th>T</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
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<td>5.685</td>
<td>8.92</td>
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<tr>
<td>Week</td>
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<td>0.3202</td>
<td>11.11</td>
<td>0.000</td>
</tr>
</tbody>
</table>

S = 15.18     R-Sq = 81.5%     R-Sq(adj) = 80.8%

Durbin-Watson statistic = 0.60