The Damped Compound Pendulum
A Second Order System

Course Objective

Simulation using “Working Model”
Exponentially Decaying Sinusoid defined by $\omega_n$ and $\zeta$

\[
\ddot{\theta} + 2\zeta \omega_n \dot{\theta} + \omega_n^2 \theta = 0 \quad (1)
\]

\[
\ln \frac{a}{b} = \frac{\zeta 2\pi}{\sqrt{1 - \zeta^2}} = \frac{1}{N} \ln \frac{X_1}{X_{N+1}} \quad (2A)
\]

\[
\frac{2\pi}{T} = \omega_n \sqrt{1 - \zeta^2} \quad (2B)
\]

$\omega_n$  Natural Frequency [rad/s]
$\zeta$  Damping ratio
$T$  Period [sec]
$\theta$  Angle [rad]
Damped Compound Pendulum Equations of Motion

\[ \ddot{\theta} + \frac{c}{J} \dot{\theta} + \frac{m_L g d}{J} \theta = 0 \]  \hspace{1cm} (3)

**Linearized** 2\textsuperscript{nd} order differential equation assumes **small** angles

- **L** \textsuperscript{Bar length [m]}
- **d** \textsuperscript{Pivot to CG distance [m]}
- **m\textsubscript{L}** \textsuperscript{Mass of pendulum [kg]}
- **J** \textsuperscript{Moment of Inertia [kg \cdot m\textsuperscript{2}]}  
- **C** \textsuperscript{Viscous damping coefficient \left[\frac{N\text{ms}}{\text{rad}}\right]}
System Identification by Matching Coefficients

Compare (1) and (3)

\[
\ddot{\theta} + 2\zeta \omega_n \dot{\theta} + \omega_n^2 \theta = 0
\]

\[
\ddot{\theta} + \frac{c}{J} \theta + \frac{m_L g d}{J} \theta = 0
\]

Yields:

\[
\omega_n = \sqrt{\frac{m_L g d}{J}} \tag{4A}
\]

\[
c = 2\zeta \omega_n J \tag{4B}
\]

Now can create a model for simulation
Incremental optical encoders generate two data signals that are electrically 90° out of phase with each other. The term *quadrature* refers to this 90° phase relationship.

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<thead>
<tr>
<th>CCW A-B</th>
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<tbody>
<tr>
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