Exam Information

The Final Exam will be given on Thursday, March 21 from 10:30 am–12:30 pm. The exam is cumulative and will cover chapters 1.1-1.3, 1.5, 1.6, 2.1-2.6, 3.1-3.6, 4.1-4.5.

The exam is closed book, closed notes, and without calculator. It will be composed of multiple choice and free response questions.

Use the following table to determine your correct exam location:

<table>
<thead>
<tr>
<th>Exam Room</th>
<th>Instructors</th>
<th>Sections</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disque 103</td>
<td>Smith</td>
<td>1, 5, 8</td>
</tr>
<tr>
<td>Disque 108</td>
<td>Li</td>
<td>6, 9</td>
</tr>
<tr>
<td>PISB 120</td>
<td>Daniel, Zefelippo</td>
<td>2, 3, 4, 15</td>
</tr>
</tbody>
</table>

In addition to the course material, there are certain formulas which are pre-requisite knowledge for this course. If you are not familiar with the following formulas, you should review by looking at the inside cover of the textbook as part of your exam studying. Do not expect the following geometry formulas to be given to you:

- Area of a parallelogram
- Area of a triangle
- Area and circumference of a circle
- Volume and surface area of a right circular cylinder
- Volume of a right circular cone
- Volume and surface area of a sphere
- Pythagorean’s Theorem

Exam Taking Tips & Advice

- Know where your exam location is. Visit the room a day or two before the exam to get the lay of the land.
- Arrive early to the exam location. If you arrive late then you get less time, not more time.
Exam Taking Tips & Advice

- Know where your exam location is. Visit the room a day or two before the exam to get the lay of the land.

- Arrive early to the exam location. If you arrive late then you get less time, not more time.

- Bring a watch. Cell phones should be turned off and packed away before the exam begins.

- Bring your student ID card. The cards will be checked when you hand in your exam.

- Use a pencil instead of a pen, and bring more than one with you. Cross out or erase any work that you do not want graded.

- As soon as you receive your exam, write your name and section number on the front page. Exams without names or correct section numbers will result in a loss of 5 points.

- Quickly skim through all of the problems on the exam before doing any one problem. Then do the problems that you think are easiest first.

- Pace yourself. Know when you should stop working on a problem and move on to another one. It is not worth leaving a 10-point problem blank because you struggled for ten minutes on a 5-point problem.

- There is no partial credit for multiple choice questions. You can show as little or as much work as you want, and the work does not have to be very organized. You should try to eliminate incorrect answers and, if necessary, you can guess from the remaining choices. Never leave a multiple choice answer blank.

- For the free response questions you must show all of your work. Be sure that your work is organized and legible and that you do not skip a lot of steps. Answers (even correct ones) will not receive a lot of credit without the necessary work to back them up. Partial credit will be awarded based on how much correct work that you show.

- If you have the time, you should check your answers. (Just be sure to manage your time see #8 above). It doesn't pay to leave 20 minutes early without checking your answers. A lot of silly mistakes can be avoided by going through your work again.
Sample Questions

The following questions are not comprehensive of the material that you are responsible for knowing. These problems are meant to offer you some practice with multiple choice questions. You should refer back through the assigned homework exercises on the main course webpage for a full list of ”Expected Skills” as well as a more comprehensive study guide.

Multiple Choice: Circle the letter of the best answer.

1. What is the natural domain of the function \( f(x) = \frac{(x + 2)^2 \ln(x - 1)}{\sqrt{16 - x^2}} \)?
   
   (a) (1, 4)
   (b) [1, 4)
   (c) (1, 4]
   (d) (−4, 4)
   (e) [−4, 4]

   \[a \text{ is the answer}\].

2. What is \( \lim_{x \to -\infty} \frac{2x - 7}{\sqrt{x^2 + x - 1}} \)?
   
   (a) \(-\infty\)
   (b) \(-2\)
   (c) \(-1\)
   (d) \(0\)
   (e) \(2\)

   \[b \text{ is the answer}\].

3. What is \( \lim_{x \to 1^+} \frac{x}{\ln(x)} \)?
   
   (a) \(0\)
   (b) \(1\)
   (c) \(e\)
   (d) \(e^{-1}\)
   (e) \(+\infty\)

   \[e \text{ is the answer}\].
4. 

\[ f(x) = \begin{cases} 
  x^2, & \text{if } x < -2 \\
  4, & \text{if } -2 < x \leq 1 \\
  6 - x, & \text{if } x > 1 
\end{cases} \]

For the above function \( f \), which of the following statements is true?

(a) \( f \) is continuous everywhere.
(b) If \( f(-2) \) were defined to be 4, then \( f \) would be continuous everywhere.
(c) \( f \) is discontinuous at only \( x = -2 \).
(d) \( f \) is discontinuous at only \( x = 1 \).
(e) \( f \) is discontinuous at \( x = -2 \) and \( x = 1 \).

**e is the answer.**

5. If \( f(x) = e^{x^2+2x} \), then \( f'(0) = \)

(a) 1
(b) 2
(c) 3
(d) 4
(e) 6

**b is the answer.**

6. What is the equation of the tangent line to \( f(x) = \tan^{-1}(2x) \) at \( x = 0 \)?

(a) \( y = x \)
(b) \( y = x + 1 \)
(c) \( y = x - 1 \)
(d) \( y = 2x \)
(e) \( y = 2x - 1 \)

**d is the answer.**
7. If \( y = \tan^2(x) \), what is \( \frac{dy}{dx} \)?

(a) \( \sec^2(x) - 1 \)
(b) \( \sec^2(x) \)
(c) \( 2\tan(x)\sec(x) \)
(d) \( 2\tan(x)\sec^2(x) \)
(e) \( 2\tan^2(x)\sec^2(x) \)

\( \text{d is the answer.} \)

8. Which of the following is the best local linear approximation for \( f(x) = \tan(x) \) near \( x = \frac{\pi}{4} \)?

(a) \( 1 + \left( x - \frac{\pi}{4} \right) \)
(b) \( 1 + \frac{1}{2} \left( x - \frac{\pi}{4} \right) \)
(c) \( 1 + \sqrt{2} \left( x - \frac{\pi}{4} \right) \)
(d) \( 1 + 2 \left( x - \frac{\pi}{4} \right) \)
(e) \( 2 + 2 \left( x - \frac{\pi}{4} \right) \)

\( \text{d is the answer.} \)

9. If \( x^2 - y^2 = 10 \), what is \( \frac{d^2y}{dx^2} \)?

(a) \( \frac{y^2 - x^2}{y^3} \)
(b) \( \frac{y - x}{y^3} \)
(c) \( \frac{y - x}{y^2} \)
(d) \( \frac{x}{y} \)
(e) \( -\frac{10}{y^2} \)

\( \text{a is the answer.} \)
10. What is \( \lim_{x \to 0} \frac{e^x - 1}{\tan(x)} \)?

(a) \(-1\)
(b) 0
(c) 1
(d) 2
(e) The limit does not exist.

**c is the answer.**

11. Which of the following is the derivative of \( f(x) = \ln\left(\frac{x + 1}{x}\right) \)?

(a) \(-\frac{(x + 1)}{x^3}\)
(b) \(-\frac{x}{x + 1}\)
(c) \(-\frac{x}{x + 1}\)
(d) \(-\frac{1}{x(x + 1)}\)
(e) \(-\frac{1}{x^2(x + 1)}\)

**d is the answer.**

12. If \( \lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} g(x) = +\infty \) and \( f'(x) = 1 \) and \( g'(x) = e^x \), what is \( \lim_{x \to +\infty} \frac{f(x)}{g(x)} \)?

(a) \(-1\)
(b) 0
(c) 1
(d) \(e\)
(e) The limit does not exist.

**b is the answer.**
13. For the function \( f(x) = \sin^2(x) + \cos(x) \) on the interval \([-\pi, 0]\), which of the following is the \( x \)-coordinate of the absolute maximum?

(a) \(-\pi\)
(b) \(-\frac{\pi}{3}\)
(c) \(\frac{\pi}{6}\)
(d) \(\frac{\pi}{2}\)
(e) \(\pi\)

**b is the answer.**

14. The rational function \( f(x) = \frac{x^2}{x^2 + x - 12} \) has the asymptotes

(a) \(x = -4, x = 3\) only
(b) \(x = -4, x = 3, y = 1\)
(c) \(x = 4, x = -3, y = 1\)
(d) \(x = -4, x = 3\) only
(e) \(x = -4, x = 3, x = 1\)

**b is the answer.**

15. If the function \( f \) is not differentiable at \( x = 0 \), then which of the following MUST be true?

(a) \( f(0) \) is undefined.
(b) \( f \) is NOT continuous at \( x = 0 \).
(c) There is a horizontal tangent line to the graph of \( y = f(x) \) at \( x = 0 \).
(d) There is a vertical tangent line to the graph of \( y = f(x) \) at \( x = 0 \).
(e) \( \lim_{h \to 0} \frac{f(0 + h) - f(0)}{h} \) does not exist.

**e is the answer.**
16. The function $y = x^3 - 6x^2 + 9x - 4$ has a relative maximum at the point $(a, b)$ and a relative minimum at the point $(c, d)$. What is the value of $a + b + c + d$?

(a) −4  
(b) −1  
(c) 0  
(d) 1  
(e) 4

**c is the answer.**

17. The curve $y = -3x^2 \ln(x)$ has a single inflection point at $x = e^k$. What is the value of $k$?

(a) $\frac{-3}{2}$  
(b) $\frac{-1}{2}$  
(c) $\frac{1}{2}$  
(d) $\frac{3}{2}$  
(e) 2

**a is the answer.**
18. The graph above shows the graph of $f'(x)$, the derivative of $f$. For what $x$-values would the graph of $f(x)$ be concave up?

(a) $x < -3$
(b) $x < -2$ and $x > 0$
(c) $-2 < x < 0$
(d) $-3 < x < 0$
(e) $x > 0$

**c is the answer.**

**Free Response:** You must show all work and use correct notation to earn full credit.

19. Using the definition of the derivative, calculate $f'(x)$ for $f(x) = \frac{2}{3 - x}$.

$$f'(x) = \frac{2}{(3 - x)^2}$$

20. In (A) and (B) below, compute $\frac{dy}{dx}$.

**A)** $y = 5x \cot(2x^3)$.

**B)** $x \cos(y) = y^4$.

**A)** $y' = -30x^3 \csc^2(2x^3) + 5 \cot(2x^3)$; **B)** $\frac{dy}{dx} = \frac{\cos y}{4y^3 + x \sin y}$

21. (10 points) Compute $\lim_{x \to +\infty} \left(1 - \frac{5}{x}\right)^x$.

$$e^{-5}$$
22. Find the absolute maximum and minimum values, if any, of \( f(x) = 4x^3 - 3x^4 \) on the interval \((-\infty, +\infty)\).

Absolute maximum of 1 when \( x = 1 \); No absolute minimum

23. An open box is to be made from a 8-inch by 15-inch piece of cardboard by cutting squares of equal size from each of the four corners and bending up the sides. What size are the squares so that the box will have the largest volume possible?

Cut out a square with side length \( \frac{5}{3} \) in

24. On the axes provided on the next page, sketch the graph of the function \( f \) given below, on the grid on the following page, and identify the locations of all critical points, cusp, vertical tangent and inflection points. For your convienience \( f' \) and \( f'' \) are given. Label all intercepts and asymptotes, if any. Hint: \((-4)^{1/3}\) is about \(-1.6\). (10 points)

\[
f(x) = (x^2 - 4)^{\frac{1}{3}} \quad f'(x) = \frac{2x}{3(x^2 - 4)^{\frac{2}{3}}} \quad f''(x) = \frac{-2(x^2 + 12)}{9(x^2 - 4)^{5/3}}
\]

\( x \)-intercepts (2, 0) and \((-2, 0)\); \( y \)-intercept (0, \(-\sqrt[3]{4}\) ); No asymptotes
Critical points \( x = 0, x = -2, \) and \( x = 2 \); No local maxima; Local minimum at \((0, -\sqrt[3]{4})\)
Inflection points \((2, 0)\) and \((-2, 0)\)
25. Water runs into an inverted conical tank at the rate of 10 $ft^3/min$. The tank has a height of 24 feet and a base with radius 6 ft. How fast is the water level rising when the depth of the water is 16 ft?

(Note: The formula for the volume of a cone is $\frac{1}{3}\pi r^2h$)

$$\frac{dh}{dt} = \frac{5}{8\pi} \text{ ft/min}$$