Math 121: Exam 2 Review Sheet

Exam Information

Exam 2 will be given on Friday, March 1 from 8-8:50 am. The exam will cover chapters 2.3-2.6 and 3.1-3.5.

The exam is closed book, closed notes, and without calculator. It will be composed of multiple choice and free response questions.

All sections will take the exam in the Main Auditorium.

In addition to the course material, there are certain formulas which are pre-requisite knowledge for this course. If you are not familiar with the following formulas, you should review by looking at the inside cover of the textbook as part of your exam studying. Do not expect the following geometry formulas to be given to you:

- Area of a parallelogram
- Area of a triangle
- Area and circumference of a circle
- Volume and surface area of a right circular cylinder
- Volume of a right circular cone
- Volume and surface area of a sphere and cube
- Pythagorean Theorem

Exam Taking Tips & Advice

- Know where your exam location is. Visit the room a day or two before the exam to get the lay of the land.
- Arrive early to the exam location. If you arrive late then you get less time, not more time.
- Bring a watch. Cell phones should be turned off and packed away before the exam begins.
- Bring your student ID card. The cards will be checked when you hand in your exam.
• Use a pencil instead of a pen, and bring more than one with you. Cross out or erase any work that you do not want graded.

• As soon as you receive your exam, write your name and section number on the front page. Exams without names or correct section numbers will result in a loss of 5 points.

• Quickly skim through all of the problems on the exam before doing any one problem. Then do the problems that you think are easiest first.

• Pace yourself. Know when you should stop working on a problem and move on to another one. It is not worth leaving a 10-point problem blank because you struggled for ten minutes on a 5-point problem.

• There is no partial credit for multiple choice questions. You can show as little or as much work as you want, and the work does not have to be very organized. You should try to eliminate incorrect answers and, if necessary, you can guess from the remaining choices. Never leave a multiple choice answer blank.

• For the free response questions you must show all of your work. Be sure that your work is organized and legible and that you do not skip a lot of steps. Answers (even correct ones) will not receive full credit without the necessary work to back them up. Partial credit will be awarded based on how much correct work that you show.

• If you have the time, you should check your answers. (Just be sure to manage your time). It does not pay to leave 20 minutes early without checking your answers. A lot of silly mistakes can be avoided by going through your work

• While last quarter’s exam has been provided this is not the best study guide. The exam will NOT be the same questions with the numbers changed. The best problems to study will be the homework assignments given and this review.

Sample Questions

The following questions are not comprehensive of the material that you are responsible for knowing. These problems are meant to offer you some practice with multiple choice questions. You should refer back through the assigned homework exercises on the main course webpage for a full list of ”Expected Skills” as well as a more comprehensive study guide.
Multiple Choice: Circle the letter of the best answer.

1. The derivative of \( y = x^2 \cos \left( \frac{1}{x} \right) \) is
   
   (a) \( 2x \cos \left( \frac{1}{x} \right) - x^2 \sin \left( \frac{1}{x} \right) \)
   
   (b) \( \frac{2}{x} \sin \left( \frac{1}{x} \right) \)
   
   (c) \( -2x \sin \left( \frac{1}{x} \right) \)
   
   (d) \( 2x \cos \left( \frac{1}{x} \right) + \sin \left( \frac{1}{x} \right) \)
   
   (e) \( \sin \left( \frac{1}{x} \right) \)

   [D]

2. At how many points on the interval \([-\pi, \pi]\) is the tangent line to the graph of \( y = 2x + \sin x \) parallel to the secant line which passes through the graph endpoints of the interval?

   (a) 0
   
   (b) 1
   
   (c) 2
   
   (d) 3
   
   (e) None of these

   [C]

3. What is the 101st derivative of \( y = \sin x \)?

   (a) 0
   
   (b) \( \sin x \)
   
   (c) \( -\sin x \)
   
   (d) \( \cos x \)
   
   (e) \( -\cos x \)

   [D]
4. For \( y = \ln (\cos x) \), which of the following is \( \frac{dy}{dx} \)?

(a) \( \ln x (-\sin x) + (\cos x) (\ln x) \)
(b) \( -\tan x \)
(c) \( \cot x \)
(d) \( \sec x \)
(e) \( \frac{1}{\ln (\cos x)} \)

B

5. Evaluate \( \frac{d}{dx} (\cos^2 x - \sin^2 x) \)

(a) \( \sin^2 x + \cos^2 x \)
(b) \( 2 \cos x - 2 \sin x \)
(c) \( -2 \sin (2x) \)
(d) \( 2 \cos (2x) \)
(e) \( 0 \)

C

6. Which of the following is \( \frac{d}{dx} \left( \frac{\sin x \cos x}{\sin 2x} \right) \)?

(a) \( \cos 2x \)
(b) \( \sin^2 2x \)
(c) \( \sin^2 (2x) \cos^2 (2x) \)
(d) \( \frac{\sin x \cos x}{\sin^2 2x} \)
(e) \( 0 \)

E
7. Find the value of the constant $A$ in $y = A \sin 4t$ so that $d^2y/dt^2 + 7y = \sin 4t$.

(a) $\frac{1}{9}$
(b) $\frac{1}{23}$
(c) $-\frac{1}{9}$
(d) $-9$
(e) $-23$

8. If $f(x) = ax^2 + bx + c$, $f(2) = 4$, $f'(2) = -3$, and $f''(2) = -2$, find $\frac{a + b + c}{3}$.

(a) $-2$
(b) $-1$
(c) $0$
(d) $1$
(e) $2$

9. If $f(t) = \ln(t^5) + (\ln t)^8$, then $f'(1)$ is:

(a) $0$
(b) $1$
(c) $5$
(d) $8$
(e) $13$

[C]
10. If \( f(t) = \sin 3x + \cos 2x \), then \( f'' \left( \frac{\pi}{4} \right) \) is:

(a) \( -\frac{3}{\sqrt{2}} - 2 \)
(b) \( -\frac{1}{\sqrt{2}} - 1 \)
(c) \( -\frac{3}{\sqrt{2}} \)
(d) \( -\frac{9}{\sqrt{2}} \)
(e) \( \frac{3}{\sqrt{2}} \)

11. What is the derivative of \( f(x) = \sec^2(\sqrt{x}) \)?

(a) \( 2 \sec (\sqrt{x}) \)
(b) \( 2 \sec^2 (\sqrt{x}) \tan (\sqrt{x}) \)
(c) \( (x^{-1/2}) \sec^2 (\sqrt{x}) \tan (\sqrt{x}) \)
(d) \( \sec (\sqrt{x}) \tan (\sqrt{x}) \)
(e) \( 2 \sec (\sqrt{x}) \tan (\sqrt{x}) \)

12. For the curve defined by \( x^4 + y^4 = 17 \), what is the slope of the tangent line at the point \( (1, -2) \)?

(a) \( -2 \)
(b) \( -1 \)
(c) \( -\frac{1}{8} \)
(d) \( \frac{1}{8} \)
(e) \( 1 \)

D
13. Compute \( \frac{d}{dx}(4^{\sin x}) \)

(a) \(-\cos x)(4^{\sin x})(\ln 4)\)
(b) \(\cos x)(4^{\sin x})(\ln 4)\)
(c) \(\cos x)(4^{\sin x})\)
(d) \(\frac{\cos x)(4^{\sin x})}{\ln 4}\)
(e) \(-\frac{\cos x)(4^{\sin x})}{\ln 4}\)

14. Consider the triangle shown below.

If \( \theta \) increases at a constant rate of 3 radians per minute, at what rate is \( x \) increasing in units per minute at the instant when \( x \) equals 3 units?

(a) 3
(b) \(\frac{15}{4}\)
(c) 4
(d) 9
(e) 12
15. which of the following is the local linear approximation of \((2x^2 + x - 1)^5\) near \(x = 0\)?

(a) 0
(b) \(5x - 1\)
(c) \(20x + 1\)
(d) \(5x + 1\)
(e) \((2x^2 + x - 1)^4(4x - 1)\)

D

Answer the following questions as True or False.

1. \(\frac{d^{10}}{dx^{10}}(\sin (2x)) = 1024 \sin (2x)\)
   
   F

2. The tangent line to \(y = \tan^{-1} x\) at the point \((0, 0)\) is \(y = x\).
   
   T

3. The derivative of \(y = x^x\) is \(y' = x \cdot x^{x-1}\)
   
   F

4. If \(f, g,\) and \(h\) are differentiable everywhere, then the derivative of \(y = f(g(h(x)))\) is \(y' = f'(g(h(x)))g'(h(x))h'(x)\)
   
   T

5. \(\frac{d}{dx}(\sin x \cdot \cos x) = \cos 2x\)
   
   T

6. \(\frac{d}{dx}(\sin^2 x + \cos^2 x) = 0\)
   
   T

7. \(\frac{d}{d\theta}(e^{\ln(\sin \theta)}) = \cos \theta\)
   
   F

8. If \(f(x) = \cos x\), then \(f^{(17)}(x) = \sin x\)
   
   F
9. \( \frac{d}{dx}(\ln |x|) = \frac{1}{x} \)

T

10. If \( f, g, \) and \( h \) are differentiable functions, then
\[
\frac{d}{dx}(f(x)g(x)h(x)) = f(x)g(x)h'(x) + f(x)g'(x)h(x) + f'(x)g(x)h(x)
\]

T

11. If \( f(x) = \sqrt[4]{x} + 7x \), then \( f'(x) = \frac{\sqrt[4]{x}}{4} + 7. \)

F

12. If \( f(x), g(x), \) and \( h(x) \) are differentiable function and \( h \neq 0 \) anywhere on its domain, then
\[
\frac{d}{dx} \left( \frac{f(x)g(x)}{h(x)} \right) = \frac{f(x)g'(x)h(x) + f'(x)g(x)h(x) - f(x)g(x)h'(x)}{(h(x))^2}
\]

T

The following problems are free-response. You should show all work and use correct notation to provide a clear and comprehensive solution for each problem.

1. A cube is expanding so that its surface area increases at the constant rate of 36 square inches per second. How fast is the volume changing at the instant when the surface area is 216 square inches? Your answer should include the appropriate units.

54 cubic inches per second

2. A shark, looking for dinner, is swimming parallel to a straight beach and 120 feet offshore. The shark is swimming at the constant speed of 13 feet per second. At time \( t = 0 \), the shark is directly opposite a lifeguard station at the shoreline. How fast is the shark moving away from the lifeguard station when the distance between them is 130 feet?

5 feet per second

3. Differentiate
   (a) \( y = \frac{x^4 - 16}{x^2 + 4} \)
   \[
y' = 2x
\]
   (b) \( y = x^4 \cos x \)
   \[
y' = 12x^2 \cos x - 8x^3 \sin x - x^4 \cos x
\]
   (c) \( f(x) = (x^4 - 2)(x^4 + 2) \)
   \[
y' = 8x^7
\]
(d) \( y = (x^2 + 1)^{\ln x} \)  
\[ y' = (x^2 + 1)^{\ln x} \left( \frac{2x \ln x + \ln (x^2 + 1)}{x^2 + 1} \right) \]

(e) \( f(x) = x^\tan x \)
\[ x^\tan x \left( \ln x \sec^2 x + \frac{\tan x}{x} \right) \]

4. Solve for \( \frac{dy}{dx} \) if \( x^3 e^y + y^2 e^{-5x} = \csc x \)
\[ y' = \frac{5y^2 e^{-5x} - 3x^2 e^y - \csc x \cot x}{x^3 e^y + 2ye^{-5x}} \]

5. Use logarithmic differentiation to find \( y' \) if \( y = \frac{(x^2 + x + 1)^{5/2}(2x^2 + 1)^3}{\sqrt{x^4 + x^2 + 1}} \)
\[ y' = \left( \frac{(x^2 + x + 1)^{5/2}(2x^2 + 1)^3}{\sqrt{x^4 + x^2 + 1}} \right) \left( \frac{10x + 5}{2x^2 + 2x + 2} + \frac{12x}{2x^2 + 1} - \frac{4x^3 + 2x}{3x^4 + 3x^2 + 3} \right) \]

6. Use logarithmic differentiation to find \( y' \) if \( y = \sqrt[8]{x + 9} \)
\[ y' = \frac{1}{8} \left( \sqrt[8]{\frac{x + 9}{x + 7}} \right) \left( \frac{1}{x + 9} + \frac{1}{x + 7} \right) \]

7. Use an appropriate local linear approximation to estimate \( \sqrt[8]{17} \)
\[ L(x) = \sqrt[8]{16} + \frac{1}{4(\sqrt[8]{16})^3} (x - 16) \] so \( \sqrt[8]{17} \) is approximated by \( L(17) = 2 + \frac{1}{32} (17 - 16) = \frac{65}{32} \)

8. Consider the curve defined by \( x^3 + 3y^2 + xy = 3 \).

(a) Calculate \( \frac{dy}{dx} \) in terms of \( x \) and \( y \).
\[ \frac{dy}{dx} = \frac{-y - 3x^2}{6y + x} \]

(b) What is \( \frac{d^2 y}{dx^2} \) evaluated at \((x, y) = (0, 1)\)?
\[ \frac{d^2 y}{dx^2} \bigg|_{(x, y) = (0, 1)} = \frac{1}{36} \]