Two-Period Consumer Model

Of the expenditure components in the income-expenditure identity, the largest, by far, is consumption - spending by private households on final goods and services. For 2008, GDP was $14.3 trillion and household consumption was $9.9 trillion, or nearly 70%. Therefore, to understand the macroeconomy, it is essential to understand consumption behavior. The flip-side of consumption is savings. So in working to understand consumption we also will learn about the behavior of savings.

1 Savings and Wealth

Before introducing the framework for analytical thinking, it is important to make clear some basic definitions:

**Savings** is the difference between current income and current consumption.  
\[ S = Y - C \]

The **Savings Rate** is the fraction of income saved.

\[ \text{Savings rate} = \frac{S}{Y} = \frac{\text{Savings in a given period}}{\text{Income in same period}} \]

**Wealth** (or Net Worth) is the difference between the sum of all assets (financial and real) and the sum of all liabilities.

That is “savings” is not your pile of money in the bank. Savings is the part of your income over some period of time (e.g. one year) that you do not use for consumption. The percentage of income used for savings is the **savings rate**. The pile of money in your bank is a financial asset and part of your wealth.

Stating this somewhat differently, savings is a *flow* while wealth is a *stock*. A flow is a measure of how much per unit time. A stock is a measure of how much has accumulated at a specific point in time. A straightforward analogy is a bathtub. Suppose you put the stopper in the tub and turn on the faucet. Each minute, 4 gallons of water go through the faucet into the tub. The rate at which the water is entering the tub is the flow and analogous to savings. After 5 minutes you would have 20 gallons of water in the tub, i.e. the stock of water is 20 gallons but the flow rate is 4 gallons per minute. If the faucet continues to add 4 gallons per minute and you then open up the stopper which drains the water at the rate of 3 gallons per second, then the net change per minute is an increase of 1 gallon per minute (that would be the net flow or savings). Starting from the 20 gallons and increasing (saving) the water at the rate of 1 gallon per minute (flow=savings) for another 10 minutes would leave you with 30 gallons (stock=wealth).
Jasmina’s Starting Balance Sheet

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Savings Account $10,000</td>
<td>Credit Card Debt $500</td>
</tr>
<tr>
<td>Checking Account $2,000</td>
<td>Student Loans $6,000</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>$6,500</strong></td>
</tr>
</tbody>
</table>

Wealth = $12,000 - $6,500 = $5,500

**Example 1**: Jasmina has an annual salary of $50,000. She also has $10,000 in her savings account and $2,000 in her checking account. She owes her credit card company $500 and her student loan company $6,000. Her net wealth is the sum of her assets ($10,000 + $2,000 = $12,000) minus the sum of her liabilities ($500 + $6,000 = $6,500) giving her $5,500 in wealth. Now out of her $50,000 salary suppose that Jasmina spends $40,000 this year. Then her savings is $10,000 and her savings rate is $10,000 / $50,000 = 20%. At the end of the year, suppose all of her savings enters her savings account and nothing else changes. Then, her new asset level is her original $12,000 plus the new $10,000 from savings this year, for a total of $22,000. If her liabilities do not change, then her wealth level is $22,000 - $6,500 = $15,500. Notice that her wealth level rose by exactly the amount saved.

Jasmina’s Balance Sheet (Example 1)

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Savings Account $20,000</td>
<td>Credit Card Debt $500</td>
</tr>
<tr>
<td>Checking Account $2,000</td>
<td>Student Loans $6,000</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>$6,500</strong></td>
</tr>
</tbody>
</table>

Wealth = $22,000 - $6,500 = $15,500

Savings Rate = $10,000 / $50,000 = 20%

**Example 2**: Consider Jasmina again from example 1 above. Now instead of savings $10,000, suppose she only saves $2,500. Her savings rate is then $2,500 / $50,000 = 5%. In addition, suppose she sends all of that $2,500 to pay off part of her student loans. So her outstanding debt with the student loan company falls to $3,500. Note that even though she paid off debt - *that is still savings* because the income was used to adjust assets and liabilities and not used for consumption. Her wealth level is her assets ($10,000 + $2,000 = $12,000) minus the sum of her liabilities ($500 + $3,500 = $4,000) which is $8,000. Notice, again, that her wealth level rose by exactly the amount saved.

Jasmina’s Balance Sheet (Example 2)

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Savings Account $10,000</td>
<td>Credit Card Debt $500</td>
</tr>
<tr>
<td>Checking Account $2,000</td>
<td>Student Loans $3,500</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>$4,000</strong></td>
</tr>
</tbody>
</table>

Wealth = $12,000 - $4,000 = $8,000

Savings Rate = $2,500 / $50,000 = 5%
Example 3: Savings can be negative!! Suppose Jasmina decides to return to school to get a Master’s degree from Anthony’s School of Quantum Mechanics. She finishes her job (having earned $50,000 for the year) and takes out a new loan of $25,000 to pay for tuition in the first year - all of the loan goes right to Anthony. Between school expenses and living costs, she spends all of her $50,000. Her total consumption is actually $50,000 + $25,000 = $75,000. Thus her savings ($S = Y - C$) is negative, $S = $50,000 − $75,000 = −$25,000. Her savings rate is −50%. Her wealth level is, again, total assets ($12,000) minus total liabilities ($6,500 from before plus $25,000 in new student loans) which gives her −$19,500 in net wealth.

Jasmina’s Balance Sheet (Example 3)

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Savings Account $10,000</td>
<td>Credit Card Debt $500</td>
</tr>
<tr>
<td>Checking Account $2,000</td>
<td>Student Loans $6,000</td>
</tr>
<tr>
<td>New Student Loans $25,000</td>
<td></td>
</tr>
<tr>
<td>Total $12,000</td>
<td>$31,500</td>
</tr>
</tbody>
</table>

Wealth = $12,000 - $31,500 = -$19,500
Savings Rate = -$25,000 / $50,000 = -50%

Thought Exercise #1: Use Jasmina’s starting balance sheet. Suppose that Jasmina receives, as a gift, a used car with a market value of $4,000. The used car is a real asset and should be included on the asset side of her balance sheet. Find Jasmina’s new level of assets, liabilities, and wealth.

Thought Exercise #2: a) Suppose that Jasmina gets a $500 bonus from work and deposits the $500 into her checking account. Find Jasmina’s new level of assets, liabilities, and wealth. b) Suppose now that she uses that $500 to pay off her credit card debt. Find Jasmina’s new level of assets, liabilities, and wealth.

Thought Exercise #3: Write down your own balance sheet. Be sure to consider all of your assets (both real - which includes cars, houses, computer, furniture - and financial - cash in your pocket, checking, savings, stock, bonds) and liabilities (car loans, student loans, credit card debt, money you owe your parents, etc.). Find your total level of assets and total level of liabilities and wealth level (note wealth levels can be negative). Next figure out your savings rate for last year. Roughly how much did you spend on consumption in total (keep in mind this includes tuition payments)? Subtract that from the income you earned last year. (For most students savings and wealth are typically negative.) Using your savings figure and income figure, what was your savings rate?
2 The Two-Period Consumer Model

2.1 Set up

Now to think about the decision of consumption versus savings we construct a simple model. This is referred to as the “Two-period Consumer Model.” We make the following simplifying assumptions.

1) There are only two periods of time: period 1 and period 2, which you can think of as "now" and "the future." In a more complicated model, we might have many periods (one period for each year) or an infinite number. However, the number of periods does not change the results and 2 is easier to work with.

2) Our individual consumer receives income today, \( y_1 \), and an income in the future, \( y_2 \). The consumer also begins with some initial level of wealth, \( a \). These figures are taken as given. We could complicate things and allow our consumer to make labor supply decisions which would cause changes in labor income, but to keep the focus on consumption versus savings we treat income and initial wealth as fixed numbers.

3) The individual can save between periods 1 and 2 and earns the real interest rate, \( \rho \), on his/her savings. The same individual could borrow instead at the same real interest rate, \( \rho \).

4) The individual consumer needs to decide the following: how much to consume today (period 1), \( c_1 \), how much to consume in the future (period 2), \( c_2 \), and how much wealth (or debt) to carry between period 1 and period 2. This level of wealth will be denoted as \( b \). \( b \) can be either positive or negative. If its positive (\( b > 0 \)) it means the consumer has some level of wealth. If \( b \) is negative (\( b < 0 \)) then the consumer is borrowing (in debt or negative wealth).

2.2 Budget Constraints

To understand the possible choices the consumer could make, we need to set up budget constraints. There is a budget constraint in each time period. All budget constraints have two components: resources and uses. In the first period, the budget constraint takes the following form:

\[
\text{1st Period Budget Constraint} \\
\begin{align*}
\text{Resources} &= \text{Uses} \\
a + y_1 &= c_1 + b
\end{align*}
\]

which states that the consumer has two possible uses for his/her assets and income. Those uses are consumption in period 1, \( c_1 \), and wealth, \( b \). Note, again that wealth can be negative if the consumer chooses to borrow. SAVINGS, recall, is the difference between current income and current consumption. So here, savings is \( s = y_1 - c_1 \). Thus, savings is also the difference between the new wealth level, \( b \), and the initial wealth level, \( a \): \( s = y_1 - c_1 = b - a \). The savings rate is then \( s = (y_1 - c_1) / y_1 = (b - a) / y_1 \).
The budget constraint in the second period is similar and takes on the following form:

\[
\text{2nd Period Budget Constraint} \\
\begin{align*}
\text{Resources} &= \text{Uses} \\
y_2 + (1 + r) b &= c_2
\end{align*}
\]

In the future period, the consumer has two resources: future income, \( y_2 \), and the wealth, \( b \), plus the interest earned on that wealth, \( r \) times \( b \). For example, if Simon chooses \( b = $10,000 \) and the real interest rate is 5%, then in the future period he gets back \((1 + 0.05)\times 10,000 = $10,500\). By saving, Simon will have a level of future consumption, \( c_2 \), higher than his future income, \( y_2 \).

If, on the other hand, \( b < 0 \) and Simon is borrowing, then \((1 + r) b\) reflects repayment of the loan plus the interest. So if Simon, instead, borrowed $5,000, then \( b = -5,000 \) and then \((1 + r) b = (1.05)\times (-5,000) = -5,250\). That money is taken away from resources and means that consumption in the future, \( c_2 \), must be less than income in the future, \( y_2 \).

Notice that there is no savings or wealth on the uses side of the 2nd period budget constraint. Why? In this model, we are pretending our consumer lives for only two periods. After period 2, the consumer dies and does not care about anything afterwards. Any savings done in the second period would be available for use in the 3rd period - but the consumer has no use for those funds. Therefore, the consumer would never choose savings past period 2. Slightly more realistically, if you think about savings for bequest purposes - to give to future generations - that should be considered as part of consumption.\(^1\)

In addition, we do not allow our consumer to borrow in the second period and run up a debt that could never be paid off.\(^2\)

**Thought Exercise #4:**

a) Find the level of wealth Simon has between periods 1 and 2, \( b \), his savings rate, and future consumption level if he has income of \( y_1 = $50,000 \), \( y_2 = $100,000 \) with initial assets of \( a = $10,000 \). Simon chooses to consume \( c_1 = $55,000 \) while the real interest rate is \( r = 10\% \).  

b) Suppose, instead Simon has the same income and initial assets, but he chooses to consume \( c_1 = $65,000 \). What level of wealth does Simon carry between periods 1 and 2? What is his savings rate and level of second period consumption?

### 2.3 Lifetime Budget Constraint

We can combine the two budget constraints to form the **lifetime budget constraint**. First, notice that the only common element in the first and second

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\(^1\)If we complicated our model with more than 2 periods, there would be savings between each period up until the last one.

\(^2\)Obviously, default on loans does happen in the real world. In more complicated versions of this model, we can incorporate the possibility of defaulting on loans, but even that does not change the basic results here. It does, however, allow us to think more carefully about getting the proper incentives for borrowers and lenders such that the market works efficiently.
period constraints is $b$. Savings is what links our consumption choices and possibilities over time. You should think of savings as a mechanism or "technology" for transferring resources across time. By saving now, we put resources away so that we may consume more in the future. By borrowing now, we are effectively moving income we receive in the future to today so that we can consume more today (If you are taking out student loans, then that is precisely what you are doing).

Rearrange the first period budget constraint to isolate $b$:

$$b = a + y_1 - c_1.$$  

Here the constraint says that wealth in the future is the difference between initial wealth, $a$, current income, $y_1$, and current consumption, $c_1$. If we consume less than our income and assets ($a + y_1 > c_1$), future wealth will be positive ($b > 0$) and we will use that for future consumption. In contrast, if we consume more now than we have in terms of resources ($a + y_1 < c_1$), future wealth is negative ($b < 0$) and we will need to use some of our future income to pay off that debt.

Now substitute this expression for $b$ into the second period budget constraint as follows:

$$c_2 = y_2 + (1 + r) b$$

$$c_2 = y_2 + (1 + r) (a + y_1 - c_1)$$

That last equation is the lifetime budget constraint. Reading it in words it states that future consumption, $c_2$, is equal to future income plus the difference between assets plus income from the first period and consumption in the first period in addition to any interest earned (paid). Here its clear that second period consumption could be higher or lower than second period income depending on whether our consumer is a net saver ($b > 0$) or a net borrower ($b < 0$).

This budget constraint can be easily graphed for intuition. Put first period consumption, $c_1$, on the x-axis and future consumption, $c_2$, on the y-axis.
The graph above shows the lifetime budget constraint. It is a straight downward sloping line. The downward slope reflects the fact that if you choose more consumption today, $c_1$, you will have less for consumption in the future, $c_2$. At the extreme endpoints we have Ebenezer Scrooge and the Party Animal. In the upper left, where the budget constraint hits the y-axis, that represent a possible choice where $c_1 = 0$ and $c_2 = (1 + r)(a + y_1) + y_2$. This point is where the consumer (Mr. Scrooge) saved absolutely everything in the first period and had zero consumption in period 1. Then, in the future was able to enjoy a very high level of $c_2$ using all savings with interest and future income. In contrast, Mr. Animal would be at the point to the lower right where the budget constraint intersects the x-axis. Here the consumer borrows against his entire future income and spends all current income and initial assets today. Thus, current consumption is very high, however, all income earned in the future is used to pay back the debt and the future consumption level falls to zero. Thus, for Mr. Animal $c_1 = a + y_1 + y_2/(1 + r)$ and $c_2 = 0$.

The budget constraint represents all of the feasible (possible) combinations of consumption now, $c_1$, and consumption in the future, $c_2$, that use all of the resources ($a$, $y_1$, and $y_2$). Above the constraint (everything to the upper right) are combinations of consumption now and consumption in the future that are not possible because the consumer cannot afford that standard of living. Below the budget constraint are points that the consumer can afford, but these are wasteful choices since those levels of consumption would mean the consumer did not fully utilize all of their resources. Thus, the lifetime budget constraint
shows all the points that are both possible and efficient in the sense of using all resources.

The slope of the budget constraint carries important, intuitive information. Mathematically, finding the slope is straightforward from the lifetime budget constraint equation. The slope is how much does $c_2$ change when $c_1$ changes. From the equation, it is $-(1 + r)$, basically the negative of the interest rate. But, what does that mean? The interest rate shows the opportunity cost of current consumption. For each $1$ spent on consumption today, you are giving up $1 + r$ dollars worth of future consumption. That is your opportunity cost. For example if the interest rate is $10\%$ and you spend $100$ today, effectively you just gave up $110$ worth of consumption next year. Another way to say this is the interest rate is showing you the “price” of current consumption in terms of future consumption. What did it cost you to consume $100$ today? It cost you $110$ worth of future consumption. If the interest rate increases, it means that the opportunity cost (the price) of current consumption just went up. Consuming today becomes more costly with higher interest rates because you are giving up more in terms of future consumption.

**Thought Exercise #5** If you borrow $100,000$ to pay for tuition and real interest rate that applies is $8\%$, how much future consumption did you give up? If the interest rate increases are you giving up more or less in future consumption?

### 2.4 Consumption Smoothing

**Definition 1** *Consumption smoothing* is the desire to maintain a steady standard of living as reflected in consumption patterns over time. Changes in income or wealth will be spread out over time to maintain a smooth consumption plan.

Refer back to the figure showing the lifetime budget constraint. The constraint only shows those points that are both feasible and efficient. It does not show us the best or optimal choice for a combination of current and future consumption. The optimal choice depends on an individual’s preferences. However, simple intuition should tell you that most people would choose a point somewhere in the middle where the levels of $c_1$ and $c_2$ are about the same. This behavior is referred to as consumption smoothing. When we look at real world data this is exactly what we see. Consumption levels for individuals do not change much from one year to the next. That is consumption is not volatile, especially when compared with savings which can be very volatile. In addition, relative to other expenditure components in the income-expenditure identity,

\[^3\text{In the Intermediate Macroeconomics course, we specifically model the preferences and can find exactly the optimal levels of } c_1 \text{ and } c_2.\]
consumption displays the least volatility. Intuitively, people prefer their standard of living (which is better shown by consumption than income) to be smooth and not show big changes from one year to the next.

**Example 4:** Suppose our consumer, Amy, has an income today of $50,000, a future income of $105,000. She has no initial assets and the real interest rate is 10%. Thus, \( y_1 = 50,000 \), \( y_2 = 102,500 \), \( a = 0 \), and \( r = 0.10 \). Suppose Amy wants perfectly smooth consumption such that \( c_1 = c_2 \). What levels of consumption in each period will she choose and what level of savings, \( b \)? Use the lifetime budget constraint to solve for the consumption levels.

\[
c_2 = y_2 + (1 + r)(a + y_1 - c_1)
\]

Since \( c_1 = c_2 \), we can replace both with \( \bar{c} = c_1 = c_2 \)

\[
\bar{c} = y_2 + (1 + r)(a + y_1 - \bar{c})
\]

Solving for \( \bar{c} \), we have:

\[
\bar{c} + (1 + r)\bar{c} = y_2 + (1 + r)(a + y_1) \\
(2 + r)\bar{c} = y_2 + (1 + r)(a + y_1) \\
\bar{c} = \frac{1}{2 + r} [y_2 + (1 + r)(a + y_1)]
\]

Using the numbers given, we get:

\[
\bar{c} = \frac{1}{2.10} [102,500 + (1.1)(0 + 50,000)] \\
\bar{c} = \frac{1}{2.10} (157,500) \\
\bar{c} = 75,000
\]

Since Amy is choosing to consume $75,000 worth of goods in period 1, \( c_1 = 75,000 \), then her level of savings is \( s = y_1 - c_1 = 50,000 - 75,000 = -25,000 \). So she is borrowing $25,000 and \( b = -25,000 \). Therefore in the second period she must pay back the loan with interest for a total of \( (1 + r)b = (1 + 0.10)(25,000) = 27,500 \). Thus, her second period budget constraint looks like this: \( c_2 = y_2 + (1 + r)b = 102,500 - (1.1)(25,000) = 75,000 \).

**Thought Exercise #6:** If Amy wants perfectly smooth consumption such that \( c_1 = c_2 \), find Amy’s levels of first and second period consumption \( (c_1 \text{ and } c_2) \), level of wealth between period 1 and 2 \( (b) \), savings \( (s) \) and savings rate if: \( y_1 = 40,000 \), \( a = 30,000 \), \( y_2 = 48,500 \), and \( r = 15\% \). BE CAREFUL, \( b \) is not the same as savings. Savings is the difference between income and consumption, \( y_1 - c_1 \).
2.5 Factors that Change Consumption and Savings

Now consider how changes in income, wealth, and the interest rate affect consumption and savings behavior. It is important to recognize that consumption both now and in the future are normal goods. That is, the more income a person has the higher the level of consumption in both periods they will want to choose. So, consider first the effect of an increase in first period income, \( y_1 \). The figure below shows what happens.

**Effect of an Increase in \( Y_1 \)**

\[
c_2 = y_2 + (1+r)(a + y_1 - c_1)
\]

The increase in current income shifts the budget constraint outward, indicating that higher levels of consumption now and in the future are possible. If the consumer’s initial choice of \( c_1 \) and \( c_2 \) was at point A, then the new choice will occur at a location like point B where both first period and second period consumption increase. This is a pure income effect. There are no substitution effects. Substitution effects only occur when a relative price changes - in this case the relative price is the interest rate. The income effect says that an increase in income (or wealth) leads to an increase in the demand for normal goods. Both \( c_1 \) and \( c_2 \) are normal goods, therefore they both increase with an increase in \( y_1 \).

So, what happens to savings? Notice that both \( y_1 \) and \( c_1 \) increase. However, in order for the consumer to increase \( c_2 \), it must mean that they have increased savings. Thus, although both \( y_1 \) and \( c_1 \) increase, \( c_1 \) does not increase as much.
as $y_1$. This is the result of consumption smoothing. Some of that increase in current income is used to raise consumption today, but another portion of it is saved to increase consumption in the future. Thus, with an increase in $y_1$, savings rises.

Another way to see the effect is through rearranging the lifetime budget constraint where we move all resources to one side and all uses to the other.

$$\begin{align*}
c_2 &= y_2 + (1 + r) (a + y_1 - c_1) \\
(1 + r) c_1 + c_2 &= (1 + r) (a + y_1) + y_2 \\
c_1 + \frac{c_2}{1 + r} &= a + y_1 + \frac{y_2}{1 + r}
\end{align*}$$

The expression above is still the lifetime budget constraint. However, on the left-side of the equals sign we have the 'Present Value of Lifetime Consumption' and on the right-side we have the 'Present Value of Lifetime Resources.'

**Definition 2** *Present Value* is the equivalent of a future value in today’s terms.

To understand what that means consider a simple example. How much is a payment of $1,000 dollars that comes one year from today worth in today’s money? It depends on the real interest rate. Another way to put the question is, to get to $1,000 one year from now how much would you need today? If the real interest rate is 10%, then the present value of $1,000 in one year is:

$$PV(\text{1,000 paid in one year}) = \frac{1,000}{1 + 0.10} = \$909.09.$$ 

What that means is if you are given $909.09 today, save it at the 10% interest rate, you will have $1,000 in one year. Thus, $909.09 is the *present value* of $1,000 paid in one year. Suppose the $1,000 payment is due in three years. What is the present value? Well, the equivalent payment today would earn interest in each of the three years, so we would have:

$$PV(\text{1,000 paid in 3 years}) = \frac{1,000}{(1.1)^3} = \$751.31.$$ 

**Thought Exercise # 7:** a) What is the present value of $1,000 to be paid in 5 years if the interest rate is 10%? b) What is the present value of $1,000 to be paid in 5 years if the interest rate is 5%? Which is worth more between (a) and (b)? Why?

**Thought Exercise # 8:** Suppose you are the Vice-President for finances of a large pharmaceutical firm. A new project to develop a medication is expected to yield profits of $15,000,000, but those profits do not come until 5 years from now. The project itself costs $9,000,000 which must be paid today. If the expected real interest rate is 10%, is the project profitable? What if the expected real interest rate is 15%? 8%? Trickier, can you figure out the
real interest rate that makes the company hit the break even point where the 
present value of the profits exactly equals the costs paid today?

**Thought Exercise #9:** Suppose we transport you back in time to re-
consider your decision to attend University. To go to University, you will need
to take out loans of $35,000 each year for the next five years. Assume the real 
interest rate is 5%. If the first loan happens today it has a present value of 
$35,000. Then the second loan would have a present value of $35,000/(1.05) 
and so on. What is the present value of all your loans at the time you en-
ter college? Now assume that starting in the 6th year you get a job paying 
$50,000/year. How many years would you need to work, such that the present 
value (at the time of entering school) of your future salary is at least as large 
as the present value of the loans you took out? Finally, suppose that instead 
of going to college, you could have taken a job paying $25,000 per year. At 
what point (i.e. how many years into the future) will you be better off having 
gone to college accounting for both what you needed to pay in tuition ($35,000 
a year for 5 years) and for giving up the $25,000 per year for just working and 
skipping school altogether?

Returning to the lifetime budget constraint,
\[
c_1 + \frac{c_2}{1 + r} = a + y_1 + \frac{y_2}{1 + r}
\]
any increase in \(a, y_1, \) or \(y_2\) will increase the present value of lifetime resources and 
the consumer will be effectively wealthier and able to afford higher consumption 
today and higher consumption in the future. Thus, increases in either \(a\) or \(y_2\) 
will also shift the budget constraint up and in a parallel manner just as it did 
with an increase in \(y_1\).

However, consider an increase in \(y_2\) where the graph would look the same 
as above (higher \(y_2\) means more resources, so the consumer can afford more of 
both \(c_1\) and \(c_2\). What happens to savings in this case? Well, consumption 
smoothing is still there as both current and future consumption are normal 
goods. That means the consumer wants to increase both \(c_1\) and \(c_2\). Since the 
increase comes through future income, in order to increase current consumption, 
the consumer must reduce savings (or borrow more) in order to fund more 
consumption today. Effectively they are borrowing against the higher future 
income to smooth consumption over time.

Suppose there is a decrease in future income, \(y_2\) (or suppose that people 
become concerned about the economy and worry about their future income - 
that is effectively the same as a loss in future income). The figure below shows 
what happens.
Effect of a Decrease in $Y_2$

$$c_2 = y_2 + (1+r)(a+y_1-c_1)$$

A decrease in $y_2$ shifts the budget constraint downward and in a parallel manner. It means that the consumer can only afford less of both $c_1$ and $c_2$. The parallel shift is because the interest rate did not change and therefore the slope did not change.

The budget constraint shifts down indicating that less consumption now and in the future is affordable. This remains a pure income effect. However, the direction is opposite. Our consumer has become poorer and therefore will reduce demand for normal goods, so consumption now and in the future will fall (both $c_1$ and $c_2$ decline). What about savings? Consumption smoothing means that the negative impact will be smoothed over time. The consumer reduces consumption in both periods, therefore, since income today did not change ($y_1$ remained the same) but current consumption fell, savings increased. The consumer, recognizing the fall in future income, saves more to buffer against the fall in income coming in the future. They are willing to sacrifice some consumption today to keep future consumption from falling too much.

**Thought Exercise #10:** Suppose that a typical consumer, Jared, wants his consumption pattern to be such that the present value of future consumption equals the present value of current consumption, $c_1 = c_2/(1+r)$. Jared has $a = 2,000$, $y_1 = 18,000$, and $y_2 = 22,000$. The real interest rate is 10%. a) Find Jared’s level of current consumption, future consumption, and savings. b) Suppose Jared’s future career prospects improve and his future income is now $y_2 = 33,000$ and he still desires the present value of consumption to be equal over time. Find Jared’s level of current consumption, future consumption, and savings.
Thought Exercise #11: Consider Jared again from TE#10. Jared has \( a = 2,000, y_1 = 18,000, \) and \( y_2 = 22,000 \). The real interest rate is 10%. Suppose that Jared’s initial wealth, \( a \), is all held as shares of XYZ company. That company announces a research breakthrough and the value of those shares triples. Jared’s initial wealth level goes to $6,000. Find Jared’s level of current consumption, future consumption, savings (remember that savings is current income minus consumption), and the amount of wealth he carries from period 1 to period 2.

Suppose that the interest rate increases. The effects are more complicated now. First, we need to figure out how the budget constraint moves so we can see what choices become possible that were not available before and what choices are no longer available. An increase in \( r \) means the slope of the budget constraint changes, specifically it becomes steeper (a fall in the interest rate means it becomes flatter). Moreover, as the budget constraint becomes steeper it rotates through a very specific point on the budget constraint. In the diagram below, this is point E, which is explained next.

**The Effect of an Increase in the Real Interest Rate**

![Diagram showing the effect of an increase in the real interest rate on the budget constraint](image)

Point E is the no borrowing/no lending point. This point represent a possible choice by the consumer to have exactly zero wealth between periods 1 and 2. That is the consumer will spend all of their first period resources, \( a + y_1 \), on consumption in the first period, \( c_1 \), such that \( c_1 = a + y_1 \). Furthermore,
future consumption will simply be equal to future income, \( c_2 = y_2 \). Since this point is always a possible choice and it does not depend on the interest rate, this particular point is exactly where the old and new budget constraints intersect. With the increase in the interest rate, the budget constraint becomes steeper rotating through the no borrowing / no lending point E. Note that point E is always possible, we are not saying it is the optimal (or best) choice, merely that it always exists as a possible choice.

With the increase in \( r \), that means the opportunity cost of consumption today changes and, in effect, the price of current consumption changes. Therefore, we will have substitution effects. Substitution effects occur with any relative price change (e.g. price of coffee goes up while tea prices stay the same; wages go up making leisure more expensive). So, naturally what would you expect people to do when the interest rate rises in terms of choosing consumption now versus savings? They will choose to increase savings and decrease consumption now in order to gain more from the higher interest rate. That is, consumers will substitute more \( c_2 \) for less \( c_1 \). Because the interest rate rose, the opportunity cost of consuming today just increased, thus consuming today is, in effect, more expensive. In sum, the substitution effect says that an increase in the real interest will lead to lower consumption now \( (c_1 \text{ declines}) \) higher savings \( (b \text{ increases}) \), and with increased savings that means more resources are available in the future and future consumption rises \( (c_2 \text{ increases}) \).

But, that is not all. Changes in the interest rate also have income effects. The direction of the income effect depends on where our consumer started before the interest rate change. The Figure below illustrates:
The Effect of an Increase in the Real Interest Rate: Income Effects

Recall that point E was labelled the no borrowing / no lending point. It means that if a person chooses that combination of first and second period consumption they will have no savings and they will not be borrowing \((b = 0)\). However, on the original budget constraint before the increase in the real interest rate, consider someone at point A. From point A going down we find the level of first period consumption, \(c_1\). That level is below first period income plus initial assets, \(a + y_1\), which would be directly below point E. Thus, a person choosing a consumption pattern at point A is a net saver, because they are, overall, saving resources for future consumption. The level of future consumption, \(c_2\), they will have exceeds second period income, \(y_2\). In contrast consider a person who chooses a point like W. For them, their first period consumption exceeds initial assets and income. As a result they must be borrowing to support that consumption level. Thus, the level of \(c_2\) they receive is less than future income.

Now, consider how the increase in the real interest rate affects the net saver and the net borrower differently. For the net saver, because they had some savings already earning interest, when \(r\) rises it means they will earn even more on their savings making them effectively wealthier. Since the net saver is wealthier, the income effect says they will want to increase both current and future consumption. However, since savings is current income minus consumption, increasing current consumption means that savings goes down. Notice in the Figure above, that Point A (the net saver’s original choice) is still a feasible
point. Now, though, the net saver can choose points like B where both \( c_1 \) and \( c_2 \) are higher.

For the net borrower, the effects are opposite. The net borrower already has outstanding debt. An increase in the real interest rate means they will have to pay more back in interest and that effect makes them poorer. Since the net borrower is poorer, the income effect says they will choose less current and future consumption. If current consumption declines, that means savings must be rising. From the graph, the net borrower’s initial choice of W is no longer feasible using the new budget constraint. The original choice is not possible because the net borrower has been made poorer and therefore must reduce both \( c_1 \) and \( c_2 \). The net borrower then must choose a point like Z instead of W.

Important: When the real interest rate changes, both income and substitution effects are present. The discussion above separated out the income and substitution effects of an increase in the real interest rate (and remember that for changes in \( y_1, y_2, \) and \( a \) there are only income effects; no substitution effects). However, both effects happen simultaneously and sometimes work in opposing directions leaving the end result ambiguous. Furthermore, while the income effect depends on whether the person began as a net saver or a net lender, the substitution effect works in the same direction no matter where the person started. To see the big picture, the table below summarizes:

<table>
<thead>
<tr>
<th>Effect</th>
<th>Who</th>
<th>Variable</th>
<th>( c_1 )</th>
<th>( c_2 )</th>
<th>( b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Substitution Effect</td>
<td>Net Borrower</td>
<td>( \downarrow )</td>
<td>( \uparrow )</td>
<td>( \uparrow )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Net Saver</td>
<td>( \downarrow )</td>
<td>( \uparrow )</td>
<td>( \uparrow )</td>
<td></td>
</tr>
<tr>
<td>Income Effect</td>
<td>Net Borrower</td>
<td>( \downarrow )</td>
<td>( \downarrow )</td>
<td>( \uparrow )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Net Saver</td>
<td>( \uparrow )</td>
<td>( \uparrow )</td>
<td>( \uparrow )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Total Effect: Substitution and Income Together</th>
<th>( c_1 )</th>
<th>( c_2 )</th>
<th>( b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net Borrower</td>
<td>( \downarrow )</td>
<td>( ? )</td>
<td>( \uparrow )</td>
</tr>
<tr>
<td>Net Saver</td>
<td>( ? )</td>
<td>( \uparrow )</td>
<td>( ? )</td>
</tr>
</tbody>
</table>

In words, let’s look at the net borrower. An increase in the interest rate means that the substitution effect will decrease current consumption because savings is more attractive. The income effect also says that the net borrower will decrease current consumption. In this case, both the income and substitution effects work in the same direction for the net borrower on current consumption. However, for future consumption the effects work in opposite directions. The substitution effect for the net borrower says \( c_2 \) will increase since savings is more attractive. But the income effect says the net borrower is poorer and has to reduce their future consumption. The effects work in the opposite direction.
and the end result is then ambiguous (indicated by ‘?’ in the table above). For an individual, whether future consumption increases or decreases depends on which effect, income or substitution, is stronger.

**Thought Exercise #12:** Draw a typical budget constraint and consider an interest rate decrease. Show the new budget constraint and the no borrowing/no lending point. Furthermore, show the original and likely new positions on the graph for net savers and net borrowers.

**Thought Exercise #13:** Using both income and substitution effects - a) Work through what happens to current consumption, future consumption and savings for a net saver if the interest rate decreases. b) Work through what happens to a net borrower if the interest rate decreases.

**Thought Exercise #14:** Going back to TE#10, suppose that a typical consumer, Jared, wants his consumption pattern to be such that the present value of future consumption equals the present value of current consumption, \( c_1 = c_2/(1 + r) \). Jared has \( a = 2,000 \), \( y_1 = 18,000 \), and \( y_2 = 22,000 \). The real interest rate is 10%. a) Find Jared’s level of current consumption, future consumption, and savings (this repeats TE #10a). Now suppose the real interest rate increases to \( r = 60\% \). Find Jared’s level of current consumption, future consumption, and savings. c) Suppose the real interest rate falls to zero, \( r = 0 \). Find Jared’s level of current consumption, future consumption, and savings.