Lab I

2D Motion

1 Introduction

In this lab we will examine simple two-dimensional motion without acceleration. Motion in two dimensions can often be broken up into two separate one-dimensional systems, a powerful technique that makes it much easier to analyze a complex system.

“An object at rest will remain at rest, and an object in motion will stay in constant motion unless acted upon by an outside force.” This statement of Newton’s first law is what we intend to test with this lab. To do this, we have to minimize any ‘outside forces’, such as gravity and friction. To help eliminate the influence of gravity, we examine motion on a level table, and to minimize the effects of friction we use a puck filled with dry ice, so that it moves supported on a thin gas layer. The puck’s motion will be captured on video for analysis.

First, the motion (usually a slow drift) of a puck that is started “at rest” will be recorded, then the motion of a puck that is given a light push diagonally across the table. You will see just how difficult it is to have a perfectly level table, or a puck perfectly at rest, when the effects of friction are very, very, small.

2 Theory

2.1 scalars and vectors

A quantity that has both magnitude and direction is called a vector. Displacement (5 m East), velocity (33 m/s ‘down’), and acceleration (2 m/s² 30°clockwise from the
x-axis) are all examples of vectors: they have a magnitude (such as 33 m/s) and direction (i.e., 'East').

A **scalar** only has magnitude, such as distance (30 m) or speed (55 mph) with no direction specified.

Vectors are often written in bold font, such as \( \mathbf{v} \), in books, or with an arrow above the symbol (\( \vec{a} \)). Scalars are written as a 'plain' symbol (\( s \)).

### 2.2 vector addition and components

Figure I.1 shows what happens when we first move by a displacement \( \mathbf{A} \) 10 km to the East and then move by displacement \( \mathbf{B} \) 5 km to the North. The overall result is displacement vector \( \mathbf{D} \):

\[
\mathbf{D} = \mathbf{A} + \mathbf{B}. \tag{I.1}
\]

You don’t ‘add’ vectors like simple (scalar) numbers, but by attaching them ‘tip to tail’ as shown in the figure, and taking the vector from the tail of the first vector to the tip of the last one.

Another way of looking at Fig. I.1 is that \( \mathbf{D} \) can be broken into components along the two axes: \( D_E \) along the ‘East’ axis, and \( D_N \) along the ‘North’ axis. It’s a bit more common to label the axes \( x \) and \( y \), so the components of \( \mathbf{D} \) are \( D_x \) and \( D_y \) along the \( x \) and \( y \) axes respectively.

Figure I.1: Components of a vector displacement.

So, in Fig. I.1, if we make ‘East’ our \( x \)-axis and ‘North’ our \( y \)-axis, we can write
the components of $D$:

$$D_x = A_x + B_x = 10\text{ km} + 0\text{ km} \quad (I.2)$$

$$D_y = A_y + B_y = 0\text{ km} + 5\text{ km} \quad (I.3)$$

where $A$ has 10 km only along $x$, and $B$ has 5 km only along $y$.

### 2.3 absolute value

The *magnitude* of a vector is its overall size, ignoring the direction, and is called the vector’s *absolute value*, for example:

$$|A| = 10\text{ km} \quad (I.4)$$

which is simple because $A$ is along an axis. The value of $|D|$ is the hypotenuse of the triangle in Fig. I.1, so we have to use the Pythagorean theorem:

$$|D| = \sqrt{D_x^2 + D_y^2} \quad (I.5)$$

$$= \sqrt{(10\text{ km})^2 + (5\text{ km})^2} \quad (I.6)$$

$$= 11.2\text{ km}. \quad (I.7)$$

Because the absolute value of a vector does not have a direction, it is a scalar, and (sometimes) gets a different name:

$$|\text{displacement}| = \text{distance}$$

$$|\text{velocity}| = \text{speed}$$

$$|\text{acceleration}| = \text{acceleration}$$

### 2.4 getting components

If you know the absolute value of a vector and its angle from an axis, you can find the components of the vector.

Figure I.1 shows the vector $D$ at an angle $\theta$ from the $x$-axis. Using trig identities:

$$\sin \theta = \frac{D_y}{|D|} \quad (I.8)$$

$$\cos \theta = \frac{D_x}{|D|} \quad (I.9)$$
\[ \tan \theta = \frac{D_y}{D_x} \]  
(I.10)

\[ \theta = \tan^{-1} \left( \frac{D_y}{D_x} \right) \]  
(I.11)

where \(|D|\) is the hypotenuse of the right triangle with adjacent side \(D_x\) and opposite side \(D_y\).

### 2.5 position of an object

The position of an object can be thought of as a displacement from some reference point, usually the origin at \(x = y = 0\). So a position \(P\) will have components \(P_x\) and \(P_y\) along the \(x\)- and \(y\)-axes, and we can often treat the motion along these axes separately.

If our object has a velocity \(v\), we can write the position as a function of time:

\[
P = P_0 + vt, \quad \text{or} \label{eqn:position}
\]

\[
P_x = P_{0x} + v_xt \label{eqn:px}
\]

\[
P_y = P_{0y} + v_yt \label{eqn:py}
\]

where \(P_{0x}\) and \(P_{0y}\) are the components of the object’s position \(P_0\) at \(t = 0\).

Just by looking at Eqn. \ref{eqn:px} you can see that \(P_x\) is a linear function of \(t\), with slope \(v_x\), so that if we plot \(P_x\) vs. \(t\), the result should be a straight line with slope \(v_x\). A deviation from a straight line would mean that the velocity is not constant: there is an acceleration. This also works the same way for \(P_y\), the component of the motion along the \(y\)-axis.

The speed \(s\) of an object is the absolute value (magnitude) of the velocity, and can be calculated from the velocity

\[
s = |v| = \sqrt{v_x^2 + v_y^2} \label{eqn:speed}
\]

where \(v_x\) and \(v_y\) are the components of \(v\).

An object in ‘constant motion’ means that the velocity of the object is the same from one time to another. We can check this by measuring the velocity at two different times and comparing them.

### 2.6 measuring velocity

Speed is distance traveled per unit time \((s = d/\Delta t)\), but speed does not have a direction. To measure a velocity (with direction) one must use ‘displacement’ instead
of ‘distance’:

\[ v = \frac{\Delta P}{\Delta t} = \frac{P_f - P_i}{\Delta t} \quad (I.16) \]

where \( P_i \) is the initial position and \( P_f \) is the final position, over time \( \Delta t \). Equation I.16 gives an *average* velocity between the initial and final points.

If the velocity is constant, we can pick any two points along the object’s path as our \( P_i \) and \( P_f \), and we’ll always get the same velocity. To check if the velocity really is constant, we can try picking different initial and final points (with the \( \Delta t \) travel time between them), and see if we always get the same velocity.

The position can also be written in terms of unit vectors:

\[ X = xi + yj \quad (I.17) \]

where \( i \) and \( j \) are unit vectors pointing along the \( x \)- and \( y \)-directions respectively, and we can also write \( X = (x, y) \) where \( x \) and \( y \) are the components of \( X \). (There are many different choices of notation for unit vectors: \((i, j, k)\), \((\hat{x}, \hat{y}, \hat{z})\), \((e_x, e_y, e_z)\), etc. You can use what you are comfortable with, but you should be able to read the other types of notation as well.)

At time \( t_1 \), we call the position of our object \( X_1 \), and at time \( t_2 \) we call the position \( X_2 \). The displacement between the two positions is:

\[ \Delta X = X_2 - X_1 \quad (I.18) \]

\[ = (x_2 - x_1)i + (y_2 - y_1)j \quad (I.19) \]

where the vector difference has been expanded in terms of components. Because the positions are vectors, the displacement is also a vector with both direction and magnitude.

Using Eqn. I.16 for average velocity and take the usual limit \( \Delta t \to 0 \), gives the instantaneous velocity

\[ v = \lim_{\Delta t \to 0} \frac{\Delta X}{\Delta t} = \frac{dX}{dt} \quad (I.20) \]

\[ = dx \frac{i}{dt} + dy \frac{j}{dt} \quad (I.21) \]

\[ = v_x i + v_y j \quad (I.22) \]

\[ = v_x i + v_y j \quad (I.23) \]
4 Procedure

4.1 Setup

We will use a motion table for this experiment, with the goal of minimizing external forces on our moving objects. To do this we need the table to be accurately level (horizontal) so that gravitational force doesn’t affect the motion, and the objects we use move with very low friction, supported on a gas cushion from evaporating dry ice.

1. Set up the video system, with the camera centered above the motion table, pointing down. Start the video capture software on your lab PC, and focus the camera. (See Appendix A for more information on the video setup.) There should be a meter stick on the motion table, in the camera’s field of view (See Fig. I.2). You will use this to set the image scale later.

Figure I.2: Camera set up above the motion table, with meter stick in view.

Once the puck is loaded with dry ice, you will have several minutes before the dry ice evaporates. Prepare the video system and be ready to capture before loading dry ice in the puck.

CAUTION: USE GLOVES TO HANDLE DRY ICE AND PUCKS LOADED WITH DRY ICE TO PREVENT INJURY FROM EXTREME LOW TEMPERATURES. DO NOT BLOCK THE VENT HOLE IN THE PUCKS.
Figure I.3: Filling the puck with dry ice. Use gloves and a plastic spoon to avoid injuries.

2. Make sure that the table of cleared of any debris. Using gloves to handle a puck, fill the puck with dry ice, without covering the hole. Cap the puck. (See Fig. I.3)

3. GENTLY Place the puck in the center of the motion table to avoid scratching the puck or the table and increasing the friction.

4. Check the camera focus: the ‘X’ on the puck should be sharp.

5. If the level of dry ice gets too low, the puck won’t float very well: refill if needed, before measurements are taken.

4.2 Data: An object at rest

1. Put the puck on the table near its center, hold it still, and try to release it gently without any push.

2. If the puck begins to move to the side, it generally means that the motion table is not completely level. Use the adjustment screws under the motion table to get the motion table level, and for fine adjustments a few pieces of paper under the feet of the table can be used.

3. Once you are satisfied with the adjustment (it’s very hard to get it perfect), return the puck to the center of the table, start the video capture, and release
the puck at rest.

4. Capture at least 10 seconds of video. You will use any ‘drift’ of the puck over 10 seconds to give a measure of how well the effect of outside forces have been removed.

5. Before analyzing the video, stop the video capture, and save the video file (.TIF format) for later analysis.

Continue the next part of the procedure promptly to avoid having the dry ice evaporate before finishing.

4.3 Data: An object in motion

In this section you will give the puck a gentle push diagonally across the table, and capture a video of its motion.

- Practice launching the puck diagonally across the motion table with a moderate velocity. You want the puck to take roughly 1 s to cross the table, and its path to be approximately diagonal, to give you the maximum path length and maximum data without having the motion so slow that extraneous forces are significant. Make sure you can see the puck on the video.

- Start capturing video, and launch as you practiced. Save the video file, with a different filename from previously.

4.4 Analysis: puck at rest

For this part of the lab, you will compare the position of the puck to its position 10 sec later.

1. Start the image analysis software, and import the video image file of the puck at rest on the table.

2. Invert the $y$-axis, use video frames with the meter stick to set the scale, and change the measurement to '$xy$-center' (for more detail on using the software, see Appendix A). Press $\text{Ctrl-2}$ to show the measurement results window.
3. Start with a video frame soon after the puck was released; measure the $xy$-position of the center of the puck by putting the arrow on the center of the puck and pressing $\text{Ctrl-1}$. Record $x$ and $y$ positions and the image time (at the bottom of the image) in Table I.1.

4. Skip forward through the image file by approximately ten seconds (push ‘>’, and use the time at the bottom to keep track). Again measure the puck’s position and time, entering them in Table I.1.

5. Calculate the time difference between your two measurements, and the displacement between the two $xy$-positions, as well as the magnitude of the displacement (distance) between the two positions. Record your calculations in Table I.1.

### 4.5 Analysis: puck in motion

Next analyze the puck moving across the motion table. Clear your previous measurements from the results window by pressing $\text{Ctrl-3}$. Import the video file of the moving puck.

1. Skip forward in the video (press ‘>’ to move forward one frame) until a time after the puck has clearly been released. You only want to use frames after release, and before the puck hits the edge of the table.

2. Measure the puck $xy$ positions for the images ($\text{Ctrl-1}$) and record the time in Table I.2. You need times in seconds: just take the seconds (and fraction) from the video frame time, adding sixty seconds if the minutes go up. These results will later be entered in Excel.

3. Skip forward frame by frame and record $xy$ positions and enter the times in Table I.2. **Please note:** it is possible for the capture software to skip frames, so the times might not be evenly spaced. Don’t assume even spacing, but record the time on all the frames you measure.

4. Start Excel, and enter the frame times in a column. Copy from the results window to Excel, matching up frame times with corresponding measurements in the same row.

5. Make a plot (and linear fit, see Appendix B) of the $x$ coordinates vertically vs. $t$ horizontally. Enter the slope and intercept of the fit in Table I.2.

6. Do the same with the $y$ coordinates.
7. Turn in both fit plots with your data. Use the slopes from your fits to fill in values for the velocity vector in Table I.2.

8. Now to calculate how much your puck sped up or slowed during the measurement: Take the \(xy\) position and time for your first two images and enter them in Table I.3; calculate the \(\Delta x\), \(\Delta y\) and \(\Delta t\) between the two and enter them in the table.

9. Enter the measurements from the last two images in Table I.3, and again calculate \(\Delta x\), \(\Delta y\) and \(\Delta t\).

10. Copy the first image measurement and last image measurements to the last two rows of Table I.3, and again calculate \(\Delta x\), \(\Delta y\) and \(\Delta t\).

11. From the \(\Delta x\), \(\Delta y\) and \(\Delta t\) values in Table I.3, calculate velocity components \(v_x\), \(v_y\) and then calculate the speed \(|v|\).
### 5 Data

Table I.1: Motion starting from rest

<table>
<thead>
<tr>
<th>Initial</th>
<th>Final</th>
<th>(Final-Initial)</th>
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<tbody>
<tr>
<td>$x_i$:</td>
<td>$x_f$:</td>
<td>$\Delta x$:</td>
</tr>
<tr>
<td>$y_i$:</td>
<td>$y_f$:</td>
<td>$\Delta y$:</td>
</tr>
<tr>
<td>$t_i$:</td>
<td>$t_f$:</td>
<td>$\Delta t$:</td>
</tr>
</tbody>
</table>

Distance $d = \sqrt{\Delta x^2 + \Delta y^2}$: __________

Average $v_x = \Delta x / \Delta t$: __________

Average $v_y = \Delta y / \Delta t$: __________

Speed $|v| = d / \Delta t$: __________
Table I.2: Uniform motion

(Record times here or enter directly in Excel. Record fit results below.)

<table>
<thead>
<tr>
<th>Image#</th>
<th>t [s]</th>
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<tbody>
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<td>15</td>
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</tbody>
</table>

(graph results)

\[ |v| = \sqrt{v_x^2 + v_y^2} \]

from slopes:
Table I.3: Velocity change during motion

| Meas.\# | x | y | t | Δx | Δy | Δt | vx | vy | |v| |
|---------|---|---|---|----|----|----|----|----|--|--|
| 1       |   |   |   |    |    |    |    |    |   | |
| 2       |   |   |   |    |    |    |    |    |   | |
| next to last | | | | | | | | | |
| last    | | | | | | | | | |
| 1       |   |   |   |    |    |    |    |    |   | |
| last    | | | | | | | | | |

Name: _________________  Sec/Group: ____  Date: ____
6 Conclusions

1. Your data in Table I.1 tells you an average puck drift speed \( s \) when started from rest. How far would the puck drift over the \( \Delta t \) of your measurement in Table I.3?

2. Do the two techniques for finding the velocity of a moving puck (linear fit in Tab. I.2 and average velocity in Tab. I.3) give results that are consistent with one another? Calculate the difference in \( v_x \) and \( v_y \) between the two measurements, and the difference in speed.

3. Did you observe uniform motion of the puck? What plots or data support your conclusion?
4. If the puck is started from rest, \( v_i = 0 \). After some time \( \Delta t \), the final velocity is \( v_f = a\Delta t \). The velocity you calculate in Tab. I.1 is the average of these two:
\[
    v_{\text{avg}} = \frac{v_f + v_i}{2} = \frac{v_f}{2}
\]
Use this to calculate a value for the acceleration.

5. You may see a small acceleration for a puck ‘at rest’ because the motion table is not perfectly level. The acceleration of gravity is \( g = 9.8 \text{ m/s}\) vertically, but the component along the table is \( a_{\text{side}} = g \sin \theta \), where \( \theta \) is the angle of the table from horizontal. Calculate the angle \( \theta \) that would explain the acceleration you calculate.