Shear Forces and Bending Moments in Beams
Sign Convention

+ Shear

- Shear

+ Moment

- Moment
Shear Forces and Bending Moments in Beams

Reactions

For \( 0 \leq x \leq a \)

\[
\sum F_y = A_y - V_x = 0 \Rightarrow V_x = A_y = \left(1 - \frac{a}{L}\right)P
\]

\[
\sum M_x = -A_y \cdot x + M_x = 0 \Rightarrow M_x = A_y \cdot x = \left(1 - \frac{a}{L}\right)Px
\]

For \( a \leq x \leq L \)

\[
\sum F_y = A_y - P - V_x = 0 \Rightarrow V_x = -\left(\frac{a}{L}\right)P
\]

\[
\sum M_x = -A_y \cdot x + P(x-a) + M_x = 0 \Rightarrow M_x = \left(1 - \frac{x}{L}\right)Pa
\]
Shear Forces and Bending Moments in Beams

For \( 0 \leq x \leq a \)

\[
\sum F_y = A_y - V = 0 \Rightarrow V_x = A_y = \left(1 - \frac{a}{L}\right)P
\]

\[
\sum M = -A_y \cdot x + M_x = 0 \Rightarrow M_x = A_y \cdot x = \left(1 - \frac{a}{L}\right)Px
\]

For \( a \leq x \leq L \)

\[
\sum F_y = A_y - P - V = 0 \Rightarrow V_x = -\left(\frac{a}{L}\right)P
\]

\[
\sum M = -A_y \cdot x - Pa + M_x = 0 \Rightarrow M_x = \left(1 - \frac{x}{L}\right)Pa
\]
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Reactions

\[ \sum M_A = B_y (L) - q(L)(L/2) = 0 \Rightarrow B_y = \frac{qL}{2} \]
\[ \sum F_y = A_y + B_y - qL = 0 \Rightarrow A_y = \frac{qL}{2} \]
\[ \sum F_x = A_x = 0 \]

\[ \sum F_y = A_y - qx - V_x = 0 \Rightarrow V_x = q\left(\frac{L}{2} - x\right) \]
\[ \sum M = -A_y \cdot x + \frac{qx^2}{2} + M_x = 0 \Rightarrow M_x = \frac{qx}{2} (L - x) \]
Example

Reactions

\[ \sum M_{\text{about } A} = R_B \cdot 24 - P \cdot 9 - q \cdot (30) \cdot (30/2) = 0 \]
\[ \Rightarrow R_B = 9 \text{ k}; \quad R_A = 11 \text{ k} \]

Free-body diagram #1

\[ \sum F_y = 11 - 0.2 \cdot (15) - 14 - V = 0 \Rightarrow V = -6 \text{ k} \]
\[ \sum M = -V_x \cdot (15) - 14 \cdot (9) - 0.2 \cdot (15) \cdot (15/2) + M = 0 \]
\[ \Rightarrow M = 58.5 \text{ ft} \cdot \text{k} \]

Free-body diagram #2

\[ \sum F_y = V + 9 - 0.2 \cdot (30 - 15) = 0 \Rightarrow V = -6 \text{ k} \]
\[ \sum M = -M + 9 \cdot (9) - 0.2 \cdot (15) \cdot (15/2) = 0 \Rightarrow M = 58.5 \text{ ft} \cdot \text{k} \]
Shear Forces and Bending Moments in Beams

Relations among \( w, V, \) and \( M \)

\[
\sum F_y = V + wdx - (V + dV) = 0
\]

or \( dV = wdx \)

\[
\frac{dV}{dx} = w
\]

\[
\int_{x_1}^{x_2} wdx = \int_{V_1}^{V_2} dV = V_2 - V_1
\]

\[
V_2 = \int_{x_1}^{x_2} wdx + V_1
\]
Shear Forces and Bending Moments in Beams

Relations among $w$, $V$, and $M$

$$\sum M_o = -M - Vdx - \frac{w(dx)^2}{2} + (M + dM) = 0$$

or $$dM = Vdx + \frac{w(dx)^2}{2}$$

$$\frac{dM}{dx} = V$$

$$\int_{x_1}^{x_2} Vdx = \int_{M_1}^{M_2} dM = M_2 - M_1$$

$$M_2 = \int_{x_1}^{x_2} Vdx + M_1$$
Shear-Force and Bending Moment Diagrams

\[ w = \frac{dV}{dx}, \quad V = \frac{dM}{dx} \]
Shear-Force and Bending Moment Diagrams

One Concentrated Load

Several Concentrated Loads
Shear-Force and Bending Moment Diagrams

Uniform load over the entire span

Uniform load over part of the span.
Shear-Force and Bending Moment Diagrams

Cantilever beam with two concentrated loads.

Cantilever beam with a uniform load.
Shear-Force and Bending Moment Diagrams

Beam with an overhang and a concentrated moment
Shear Forces and Bending Moments in Beams

\[ \sum M_B = -R_A (10) + 2000 (13) + 1000 (6) (5) + 2000 (2) = 0 \]

\[ R_A = 6000 \text{ lb} \]

\[ \sum F_y = -2000 + 6000 - 1000 (x - 2) - V = 0 \]

\[ \sum M_{a-a} = 2000 (3 + x) - 6000 x + 1000 \left( \frac{(x - 2)^2}{2} \right) + M = 0 \]

\[ V = -1000 x + 6000 \text{ lb} \]

\[ M = -500 x^2 + 6000 x - 8000 \text{ ft-lb} \]
Shear Forces and Bending Moments in Beams

\[ w = -q \quad V_0 = \frac{qL}{2} \quad M_0 = 0 \]

\[ \int_0^x w \, dx = \int_0^x (-q) \, dx = -qx = V_x - V_0 \]

\[ V_x = q \left( \frac{L}{2} - x \right) \]

\[ \int_0^x V \, dx = \int_0^x q \left( \frac{L}{2} - x \right) \, dx = q \left( \frac{Lx}{2} - \frac{x^2}{2} \right) = M_x - M_0 \]

\[ M_x = \frac{qx}{2} (L - x) \]