1 INTRODUCTION

The term spatial variation of seismic ground motions denotes the differences in amplitude and phase of seismic motions recorded over extended areas. The spatial variation of seismic ground motions has an important effect on the response of lifelines such as bridges, pipelines, communication transmission systems, etc. Because these structures extend over long distances parallel to the ground, their supports undergo different motions during an earthquake. This differential motion can increase the response of lifelines beyond the response expected if the input motions at the structures’ supports were assumed to be identical. The results of past analyzes reported in the literature indicate that the effect of the spatial variation of seismic ground motions on the response of bridges cannot be neglected, and can be, in cases, detrimental [1]. The incorporation of the spatial variation of seismic motions in lifeline design response spectra has also been attempted [2–5]. It has been recently recognized [6] that the spatial variation of seismic ground motions can have a dramatic effect on the response of extended structures. Presently, in studies performed by the California Department of Transportation (Caltrans), spatially variable ground motions are used as input motions at the supports of various bridges, such as the West Bay Bridge in San Francisco and the Coronado Bridge in San Diego, California [7]. The modeling of spatial variability and its effects on the response of lifelines is a topic under renewed, active investigation, eg, [8–10].

The spatial variability in seismic ground motions can result from relative surface fault motion for recording stations located on either side of a causative fault; soil liquefaction; landslides; and from the general transmission of the waves from the source through the different earth strata to the ground surface. This paper will concentrate on the latter cause for the spatial variation of surface ground motions.

The spatial variation of seismic ground motions has started being analyzed after the installation of dense instrument arrays. Before the installation of these arrays, the spatial variation of the motions was attributed to the apparent propagation of the waveforms on the ground surface, ie, it was considered, at least in engineering applications, that the difference in the motions between two stations was caused solely by a time delay in the arrival of the time history at the further away station, eg, [11–13]. The data recorded at dense seismograph arrays have provided valuable information on additional causes and more detailed descriptions for the spatial variation of the motions.

One of the first arrays installed was the El Centro Differential array [14] that recorded the 1979 Imperial Valley earthquake; the array was linear and consisted of seven stations with a total length of 312.6 m and minimum separation distance of 7.6 m. The array, however, which has provided an abundance of data for small and large magnitude events that have been extensively studied by engineers and seismologists, is the SMART-1 array (Strong Motion ARray in Taiwan), located in Lotung, in the north-east corner of Taiwan.
This 2D array, which started being operative in 1980, consisted of 37 force-balanced triaxial accelerometers arranged on three concentric circles, the inner denoted by \( I \), the middle by \( M \), and the outer by \( O \) (Fig. 1). Twelve equispaced stations, numbered 1-12, were located on each ring, and station C00 was located at the center of the array. Spatial variability studies based on the SMART-1 data started appearing in the literature almost as early as the array recordings became available (eg, [16,17]). Two additional stations, E01 and E02, were added to the array in 1983, at distances of 2.8 and 4.8 km, respectively, south of C00 [18]. The array was located in a recent alluvial valley except for station E02 that was located on a slate outcrop [18]. A smaller scale, 3D (ground surface and down-hole instrumentation) array, the LSST array, was constructed in 1985 within the SMART-1 array close to station M08 (Fig. 1). The LSST array consists of 15 free-surface triaxial accelerometers located along three arms at 120° intervals, and eight down-hole accelerometers at depths up to 47 m [18,19]. Additional arrays, permanent and temporary, have been and are being deployed around the world for studies of the characteristics of seismic ground motions, eg, EPRI Parkfield, USGS Parkfield, Hollister, Stanford, Coalinga, USGS ZAYA, Pinyon Flat [20], Tarzana [21], San Fernando Valley [22], Sunnyvale [23], all in California, USA; Chiba Experiment Station, Tokyo, Japan [24]; Nice, France [25]; Thessaloniki, Greece [26]; L’Aquila, Italy, [27].

This paper presents an overview of the spatial variation of seismic ground motions, its modeling, and the interpretation of the models. It starts in Section 2 with the description of the stochastic approach for the evaluation of the spatial variability from recorded data. Section 3 interprets the stochastic descriptor of the spatial variability, the coherency, in terms of the phase variability it represents. Section 4 presents some of the available empirical and semi-empirical spatial variability models and discusses their validity and limitations; the effect of coherency on the seismic response of lifelines (bridges) is highlighted. Section 5 describes a recent approach that analyzes spatial variability without relying on coherency estimates, and identifies correlation patterns in the amplitude and phase variation of recorded data around a common, coherent component. Section 6 describes simulation techniques for the generation of spatially varying seismic ground motions; the characteristics of simulations generated from different coherency models and by means of different simulation schemes are compared. Finally, Section 7 provides a brief summary, conclusions, and recommendations.

2 STOCHASTIC DESCRIPTION OF THE SPATIAL VARIATION OF SEISMIC GROUND MOTIONS FROM RECORDED DATA

For illustration purposes, consider that seismic ground motions (accelerations), \( a(\vec{r}_j, t) \), in one of the horizontal (N-S or E-W) or vertical direction at a location \( \vec{r}_j = (x_j, y_j, z_j) \) on the ground surface and at time \( t \) can be described by the sum of an infinite number of sinusoids as follows:

\[
a(\vec{r}_j, t) = \sum_{n=1}^{\infty} A(\omega_n, \vec{r}_j, t) \sin[\omega_n t + \delta(\omega_n, t) \cdot \vec{r}_j] \\
+ \phi(\omega_n, \vec{r}_j, t)
\]

where \( \omega_n \) is the (discrete) frequency; \( \delta(\omega_n, t) \cdot \vec{r}_j \) is the wavenumber that specifies the average apparent propagation of the waveforms on the ground surface; and \( A(\omega_n, \vec{r}_j, t) \) and \( \phi(\omega_n, \vec{r}_j, t) \) are the amplitude and phase shift, respectively. For the seismic time history recorded at a different station \( k \), Eq. (1) still holds, but the amplitudes \( A(\omega_n, \vec{r}_k, t) \) and phases \( \phi(\omega_n, \vec{r}_k, t) \) would differ from those at station \( j \). The propagation term \( \delta(\omega_n, t) \cdot \vec{r}_j \) in Eq. (1)) also incorporates station dependent arrival time perturbations around the time delays caused by the average apparent propagation of the motions on the ground surface. The differences in the time histories of Eq. (1) between stations constitute the spatial variation of the motions, and are attributed to source-rupture characteristics, wave propagation through the earth strata, scattering, and local site effects. The physical causes underlying the spatial variation of seismic motions are presented extensively in [28], and are highlighted herein in Sections 4 and 5.

The procedure for the evaluation of the stochastic spatial variation of seismic motions from recorded data considers that the motions are realizations of space-time random fields. In order to extract valuable information from the available limited amount of data, such as the recorded time histories at the array stations during an earthquake, certain assumptions need to be made:

- It is assumed that the random field is homogeneous in space, i.e., all stochastic descriptors of the motions (joint probability distribution functions) are functions of the separation distance between stations, but independent of absolute location. This assumption implies that the frequency content (amplitude) of the seismic motions at different recording stations does not vary significantly. Since
the majority of dense instrument arrays are located on fairly uniform soil conditions, the assumption of homogeneity is valid. Significant variation in the frequency content of the motions can be expected if the stations are located at different local soil conditions (eg, one on rock and the other on alluvium).

- It is further assumed that the time histories recorded at the array stations are stationary random processes. Stationarity implies that the probability functions do not depend on the absolute time, but are functions of time differences (or time lag); in this sense, the time histories have neither beginning nor end. Although this assumption appears to be unrealistic, this is not the case: Most characteristics of seismic ground motions for engineering applications are evaluated from the strong motion shear (S-) wave window, ie, a segment of the actual seismic time history. This strong motion segment from the actual time history can be viewed as a segment of an infinite time history with uniform characteristics through time, ie, a stationary process. For a stationary process, the amplitude and phase of the motions (Eq. (1)) are not functions of time.

- It is also assumed that the stationary time histories at the recording stations are ergodic. A stationary process is ergodic, if averages taken along any realization of the process are identical to the ensemble averages, ie, the information contained in each realization is sufficient for the full description of the process. The assumption of ergodicity is necessary: The evaluation of probabilistic models for the description of the spatially variable seismic ground motions would require, ideally, recordings at the same site from many earthquakes with similar characteristics, so that an ensemble of data can be analyzed and averages of the ensemble can be evaluated. However, in reality, there is only one realization of the random field, ie, one set of recorded data at the array for an earthquake with specific characteristics.

It is obvious that reality does not fully conform to these assumptions, but actual data recorded at dense instrument arrays during the strong motion S-wave window may be viewed as homogeneous, stationary, and ergodic in a limited or weak sense. These necessary, but restrictive, assumptions for the modeling of the spatial variation of seismic ground motions are typically retracted in the simulation of such motions for engineering applications.

The following stochastic descriptors characterize the random field.

### 2.1 Cross spectral density

The random field of seismic ground motions (accelerations) is best described by means of the cross spectral density of the data recorded at two stations (locations) on the ground surface [29]: Let \( u(\vec{r}_j, t) = a_j(t) \) and \( u(\vec{r}_k, t) = a_k(t) \) be two time series in the same direction recorded at locations \( \vec{r}_j \) and \( \vec{r}_k \) on the ground surface, and let \( \xi_{jk} = |\vec{r}_j - \vec{r}_k| \) indicate the station separation distance. Let the duration of the strong motion S-wave window be \( 0 \leq t \leq T, \ T = N\Delta t \), with \( N \) being the number of samples in the recorded time series for the window, and \( \Delta t \) the time step. It is iterated here that this window of the actual time history is assumed to be a segment of an infinite one with uniform characteristics through time (stationarity assumption). The cross covariance function of the motions between the two stations is defined as [30]:

\[
\hat{R}_{jk}(\tau) = \frac{1}{T} \int_0^T a_j(t)a_k(t+\tau)dt \quad |\tau| \leq T
\]

(2)

The cross covariance function is generally smoothed before it is further used as an estimator; the smoothed cross covariance function is:

\[
R_{jk}(\tau) = w(\tau)\hat{R}_{jk}(\tau)
\]

(3)

where \( w(\tau) \) is the lag window, with properties \( w(\tau) = w(-\tau), \ w(\tau=0) = 1 \) and \( \int_{-\infty}^{\infty} w(\tau) = 1 \).

It is customary—for stationary processes—to work in the frequency rather than the time domain. The cross spectral density (or cross spectrum) of the process is defined as the Fourier transform of the covariance function (Eq. (2)). The smoothed cross spectrum is evaluated by Fourier transforming Eq. (3):

\[
S_{jk}(\omega) = \frac{1}{2\pi} \int_{-T}^{T} R_{jk}(\tau) \exp^{-i\omega\tau} d\tau
\]

(4)

with \( i = \sqrt{-1} \) and \( \omega \) being the frequency (in rad/sec). Alternatively, the cross spectral estimates of Eq. (4) can be evaluated directly in the frequency domain as follows: Let \( A_j(\omega) \) and \( A_k(\omega) \) be the scaled discrete Fourier transforms of the time histories \( a_j(t) \) and \( a_k(t) \), respectively, defined as:

\[
A_{j,k}(\omega_n) = \sqrt{\frac{\Delta t}{2\pi N}} \sum_{m=0}^{N-1} a_{j,k}(m\Delta t) \exp^{-i\omega_n m\Delta t}
\]

(5)

The (smoothed) cross spectrum of the series becomes:

\[
S_{jk}(\omega_n) = \sum_{m=-M}^{+M} W(m\Delta\omega) A_j^*(\omega_n + m\Delta\omega)A_k(\omega_n + m\Delta\omega)
\]

(6)

where the spectral window, \( W(\omega) \), is the Fourier transform of the lag window \( w(\tau) \), \( \Delta\omega \) is the frequency step, \( (\Delta\omega = 2\pi/T) \), and \( * \) indicates complex conjugate. The presence of the smoothing windows in the time or frequency domains reduces the variance in the estimates but, also, the resolution of the spectra.

### 2.2 Power spectral density

The power spectral densities of the motions (ie, \( j = k \) in Eqs. (4) and (6)) are estimated from the data of the analysis recorded at each station and are commonly referred to as point estimates of the motions:

\[
S_{jj}(\omega_n) = \sum_{m=-M}^{+M} W(m\Delta\omega)|A_j(\omega_n + m\Delta\omega)|^2
\]

(7)

It is obvious that the Fourier spectra of the motions at the various stations will not be identical. However, the assumption of spatial homogeneity in the random field implies that the power spectrum of the motions is station independent.
Once the power spectra of the motions at the stations of interest have been evaluated, a parametric form is fitted to the estimates, generally through a regression scheme. The most commonly used parametric forms for the power spectral density are the Kanai-Tajimi spectrum [31,32], or its extension developed by Clough and Penzien [33]. The physical basis of the Kanai-Tajimi spectrum is that it passes a white process through a soil filter; the following expression for the power spectral density of ground accelerations results:

\[
S_a(\omega) = S_f \frac{1 + 4 \xi_f^2 \left( \frac{\omega}{\omega_f} \right)^2}{\left[ 1 - \left( \frac{\omega}{\omega_f} \right)^2 \right]^2 + 4 \xi_f^2 \left( \frac{\omega}{\omega_f} \right)^2} \]

(8)

in which \(S_f\) is the amplitude of the white process bedrock excitation, and \(\omega_f\) and \(\xi_f\) are the frequency and damping coefficient of the soil filter. A deficiency of Eq. (8) is that the spectrum produces infinite variances for the ground velocity and displacement. For any stationary process, the power spectral densities of velocity, \(S_v(\omega)\), and displacement, \(S_d(\omega)\) are related to that of acceleration through:

\[
S_v(\omega) = \frac{1}{\omega^2} S_a(\omega); \quad S_d(\omega) = \frac{1}{\omega^2} S_a(\omega)
\]

(9)

It is apparent from Eqs. (8) and (9) that the velocity and displacement spectra of the Kanai-Tajimi spectrum are not defined as \(\omega \rightarrow 0\). Clough and Penzien [33] passed the Kanai-Tajimi spectrum (Eq. (8)) through an additional soil filter with parameters \(\omega_f\) and \(\xi_f\), and describe the power spectrum of ground accelerations as:

\[
S_a(\omega) = S_f \frac{1 + 4 \xi_f^2 \left( \frac{\omega}{\omega_f} \right)^2}{\left[ 1 - \left( \frac{\omega}{\omega_f} \right)^2 \right]^2 + 4 \xi_f^2 \left( \frac{\omega}{\omega_f} \right)^2} \times \left( \frac{\omega}{\omega_f} \right)^4 \left( 1 - \left( \frac{\omega}{\omega_f} \right)^2 \right)^2 + 4 \xi_f^2 \left( \frac{\omega}{\omega_f} \right)^2 \right)^2
\]

(10)

which yields finite variances for velocities and displacements. For the simulation of seismic motions at firm soil conditions, the soil filter parameters assume the values \(\omega_f = 5 \pi \) rad/sec, \(\omega_f = 0.1 \omega_s\), and \(\xi_f = 0.6\) [34]; the values of the parameters for other soil conditions can be found, eg, in [4,35].

The aforementioned spectra model only the effect of the bedrock excitation \(S_f\) in Eqs. (8) and (10)) is a white process. However, seismic ground motions are the result of the rupture at the fault and the transmission of waves through the media from the fault to the ground surface. Alternatively, seismological spectra that incorporate these effects can be used instead of Eqs. (8) and (10) for simulation purposes. For example, Joyner and Boore [36] presented the following description for the stochastic seismic ground motion spectrum:

\[
S(\omega) = CF \cdot SF(\omega) \cdot AF(\omega) \cdot DF(\omega) \cdot IF(\omega) \]

(11)

in which \(CF\) represents a scaling factor which is a function of the radiation pattern, the free surface effect, and the material density and shear wave velocity in the near source region; \(SF(\omega)\) is the source factor that depends on the moment magnitude and the rupture characteristics; \(AF(\omega)\) is the amplification factor described either by a frequency dependent transfer function as, eg, the filters in the Kanai-Tajimi or Clough-Penzien spectra (Eqs. (8) and (10)), or, in terms of the site impedance \(\sqrt{(\rho_0 v_0)/(\rho v_J)}\), where \(\rho_0\) and \(v_0\) are the density and shear wave velocity in the source region and \(\rho\) and \(v\), the corresponding quantities near the recording station; \(DF(\omega)\) is the diminution factor, that accounts for the attenuation of the waveforms; and \(IF(\omega)\) is a filter used to shape the resulting spectrum so that it represents the seismic ground motion quantity of interest.

### 2.3 Coherency

The coherency of the seismic motions is obtained from the smoothed cross spectrum of the motions between the two stations \(j\) and \(k\), normalized with respect to the corresponding power spectra as, eg, [19,37]:

\[
\gamma_{jk}(\omega) = \frac{S_{jk}(\omega)}{\sqrt{S_{jj}(\omega) S_{kk}(\omega)}}
\]

(12)

where the subscript n in the frequency has been dropped for convenience. \(\gamma_{jk}(\omega)\) is a complex number; the square of the absolute value of the coherency, the coherence:

\[
|\gamma_{jk}(\omega)|^2 = \frac{|S_{jk}(\omega)|^2}{S_{jj}(\omega) S_{kk}(\omega)}
\]

(13)

is a real number assuming values \(0 \leq |\gamma_{jk}(\omega)|^2 \leq 1\). Coherence is commonly written as:

\[
\gamma_{jk}(\omega) = \left| \gamma_{jk}(\omega) \right| \exp[i \theta_{jk}(\omega)]
\]

(14)

with

\[
\theta_{jk}(\omega) = \tan^{-1} \left( \frac{\Re(S_{jk}(\omega))}{\Im(S_{jk}(\omega))} \right)
\]

(15)

being the phase spectrum; \(\Re\) and \(\Im\) in Eq. (15) indicate the real and imaginary part, respectively. \(\gamma_{jk}(\omega)\) is commonly referred to as the lagged coherency, and \(\Re(\gamma_{jk}(\omega))\) as the unlagged coherency.

Once coherency estimates are obtained from the recorded time histories, a functional form (model) is fitted through the coherency data points by means of regression analyses. Existing coherency models are extensively discussed in Section 4; Section 3 presents the physical interpretation of coherency.

### 3 Interpretation of Coherency

The complex coherency of Eq. (14) is alternatively expressed as:

\[
\gamma(\xi, \omega) = \left| \gamma(\xi, \omega) \right| \exp[i \theta(\xi, \omega)]
\]

(16)
where the subscripts \( j \) and \( k \) have been dropped; Equation (16) then indicates the variation of coherency for any separation distance \( \xi \) between two locations on the ground surface.

### 3.1 Wave passage effect

The complex term in the above equation, \( \exp[i\theta(\xi, \omega)] \), describes the wave passage effect, i.e., the delay in the arrival of the waveforms at the further away station caused by the propagation of the waveforms. Consider that the ground motions consist of a unit amplitude, monochromatic random wave propagating with velocity \( c \) on the ground surface along a line connecting the stations. The coherency expression for this type of motion would be \([38]\):

\[
\gamma_{wp}(\xi, \omega) = \exp\left[-\frac{i \omega (\xi - \xi')}{c}\right] = \exp\left[-i \frac{\omega \xi}{c}\right] \quad (17)
\]

For a single type of wave dominating the window analyzed, as is most commonly the case for the strong motion S-wave window used in spatial variability evaluations, the consideration that the waves propagate with constant velocity on the ground surface is a valid one \([37]\). Furthermore, since body waves are non-dispersive, except in highly attenuated media, they have the same velocity over a wide range of frequencies \([39]\). Thus, in Eq. (16),

\[
\theta(\xi, \omega) = -\frac{\omega \xi}{c} \quad (18)
\]

where now \( c \) indicates the apparent propagation velocity of the motions along the line connecting the stations. Abrahamson \textit{et al} \([19]\) introduced a correction factor in Eq. (16), that is controlled by the constant apparent propagation velocity of the motions at lower frequencies, but allows random variability in the velocity at high frequencies.

### 3.2 Lagged coherency

The lagged coherency, \( |\gamma(\xi, \omega)| \), is a measure of “similarity” in the seismic motions, and indicates the degree to which the data recorded at the two stations are related by means of a linear transfer function. If, for example, one process can be obtained by means of linear transformation of the other and in the absence of noise \([40]\), coherency is equal to one; for uncorrelated processes, coherency becomes zero. It is expected that at low frequencies and short separation distances the motions will be similar and, therefore, coherency will tend to unity as \( \omega \to 0 \) and \( \xi \to 0 \). On the other hand, at large frequencies and long station separation distances, the motions will become uncorrelated, and coherency will tend to zero. The value of the coherency in between these extreme cases will decay with frequency and station separation distance. This observation has been validated from the analyses of recorded data, and the functional forms describing the lagged coherency at any site and any event are exponential functions decaying with separation distance and frequency.

A significant characteristic of the lagged coherency, that does not become apparent directly from Eq. (12), is that it is only minimally affected by the amplitude variability between the motions at the two stations. Spudich \([41]\) presents a simple example to illustrate this: Consider that the time history at station \( k \) is a multiple of that at station \( j \), i.e., \( a_k(t) = n a_j(t) \); the substitution of this expression in Eqs. (6), (7), and (12) would yield exactly the same coherency for any value of \( n \). Even when the motions at the various stations are not multiples of each other, it can be shown that the absolute value of the coherency is not sensitive to amplitude variations. Thus, lagged coherency describes the phase variability in the data in addition to the phase difference introduced by the wave passage effect (Eq. (18)).

Abrahamson \([42]\) presented the relation between the lagged coherency and the random phase variability, which is summarized in the following: Let \( \phi_j(\omega) \) and \( \phi_k(\omega) \) be the (Fourier) phases at two stations \( j \) and \( k \) on the ground surface, after the wave passage effects have been removed. The relation between \( \phi_j(\omega) \) and \( \phi_k(\omega) \) can be expressed as:

\[
\phi_j(\omega) - \phi_k(\omega) = \beta_{jk}(\omega) \pi \varepsilon_{jk}(\omega) \quad (19)
\]

in which \( \varepsilon_{jk}(\omega) \) are random numbers uniformly distributed between \([-1, +1]\], and \( \beta_{jk}(\omega) \) is a deterministic function of the frequency \( \omega \) and assumes values between 0 and 1. \( \beta_{jk}(\omega) \) indicates the fraction of random phase variability between \([-\pi, +\pi]\) (from the product \( \pi \varepsilon_{jk}(\omega) \)) that is present in the Fourier phase difference of the time histories. For example, if \( \beta_{jk}(\omega) = 0 \), there is no phase difference between the stations, and the phases at the two stations are identical and fully deterministic. In the other extreme case, i.e., when \( \beta_{jk}(\omega) = 1 \), the phase difference between stations is completely random. Based on Eq. (19), and neglecting the amplitude variability of the data, Abrahamson \([42]\) noted that the mean value of the lagged coherency can be expressed as:

\[
E[|\gamma_{jk}(\omega)|] = \frac{\sin(\beta_{jk}(\omega) \pi)}{\beta_{jk}(\omega) \pi} \quad (20)
\]

It is easy to verify from Eq. (20) that when the lagged coherency tends to one, \( \beta_{jk}(\omega) \) is a small number, i.e., only a small fraction of randomness appears in the phase difference between the motions at the two stations; as coherency decreases, \( \beta_{jk}(\omega) \) increases, and, for zero coherency, \( \beta_{jk}(\omega) = 1 \).

The following peculiarity of the coherency estimate needs to be mentioned: Consider that the smoothing window is not used in the evaluation of the cross covariance function nor, alternatively, in the cross spectrum of the series (Eqs. (3) and (6)). Substitution of Eqs. (6) and (7), which become \( S_{jk}(\omega) = A_j(\omega) A_k(\omega) \), \( S_j(\omega) = |A_j(\omega)|^2 \), and \( S_k(\omega) = |A_k(\omega)|^2 \), into Eq. (12) yields the identity \( |\gamma(\xi, \omega)| \equiv 1 \) for any frequency \( \omega \) and station pair \((j, k)\) regardless of the true coherency of the data. The information about the differences in the phases of the motions at the stations is introduced in the estimate through the smoothing process, which controls the statistical properties of the coherency as well as its resolution. As an example, Fig. 2 presents the lagged coherency evaluated from data recorded at stations J06 and J12 (separation distance of 400 m) and stations C00 and M06 (separation distance of 1000 m) during the strong motion S-wave window of motions recorded in the N-S direction of Event 5 at the SMART-1 array. For the evaluation of the lagged co-

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herencies in the figure, 3-, 7-, 11-, 15-, and 19-point Hamming windows (ie, \( M = 1, 3, 5, 7, \) and 9, respectively, in Eq. (6)) are used. It can be seen from the figure that the more near-neighbor frequencies are used in the process, the higher the loss of resolution in the estimates. Abrahamson et al [19] note that the choice of the smoothing window should be directed not only from the statistical properties of the coherency, but also from the purpose for which it is derived. If the coherency estimate is to be used in structural analysis, for time windows less than approximately 2000 samples and for structural damping coefficient 5% of critical, an 11-point Hamming window is suggested [19]. Recently, Zerva and Beck [43] have proposed an alternative approach for bypassing the strict requirement of coherency smoothing if parametric models of the coherency are to be evaluated: by fitting parametric expressions to unsmoothed cross spectral estimates, parametric models of lagged coherency are recovered indirectly.

The statistics of the coherency estimate \( \gamma_{jk}(\omega) \), as obtained form the recorded data (Eq. (12)), are not simple. Its variance depends on its value as \( \gamma_{jk}(\omega) \) decreases, its variance increases [44]. On the other hand, \( \tanh^{-1}[\gamma_{jk}(\omega)] \) is approximately normally distributed with constant variance inversely proportional to the duration of the strong motion window and the bandwidth of the smoothing spectral window [30]. This transformation is sometimes used (eg, [29,44]), before any parametric form of coherency is fitted to the empirical data. However, the statistical properties for \( \tanh^{-1}[\gamma_{jk}(\omega)] \) assume that the spectrum of the process is approximately constant over the frequency bandwidth of the window; if this is not the case, then an additional source of bias is introduced in the estimate [44]. Coherency functions, as estimates obtained from limited, finite segments of data, are biased estimators: Bias is expected at low frequencies due to the sensitivity characteristics of seismometers and the small intensity of the low frequency components of the ground motions [45–47]. Bias is introduced at higher frequencies and long separation distances due to the use of finite length series that may produce finite values for the estimate when the true coherency is zero [29]. Additional uncertainty is introduced by inaccuracies in recorder synchronization (eg, [45,47,48]), and by imperfect elimination of time lags caused by wave passage effects [47]. The time lags caused by the wave passage effect appear as deterministic (\( \xi/c \)) in Eq. (18). However, the wave passage effect also incorporates random, station dependent time delay fluctuations around these deterministic delays that affect the coherency and should be given proper consideration.

### 3.3 Arrival time perturbations

The apparent propagation velocity of the seismic motions across the seismograph array can be estimated by means of signal processing techniques, such as the conventional method (CV) [49,50], the high resolution method (HR) [50], or the multiple signal characterization method (MUSIC) [51,52]. All techniques evaluate, in different forms, the frequency-wavenumber (F-K) spectrum of the motions, ie, the triple Fourier transform of the recorded time histories, and identify the average propagation velocity of the motions from the peak of the spectrum. An evaluation of the advantages and disadvantages of the techniques can be found, eg, in [53].

Seismic ground motions, however, incorporate random time delay fluctuations around the wave passage delay, that are particular for each recording station. These arrival time perturbations are caused by the upward traveling of the waves through horizontal variations of the geologic structure underneath the array [41] and, also, due to deviations of the propagation pattern of the waves from that of plane wave propagation [54]. They can be so significant that the apparent propagation pattern of the wavefronts between sets of stations can be very different from the estimated overall pattern of constant apparent propagation throughout the array [55].
Whereas the wave passage effects control the complex exponential term of the coherency in Eq. (16), the arrival time perturbations affect its absolute value, namely the lagged coherency. An approach that partially eliminates these effects from coherency estimates is the alignment of the data with respect to a reference station. In this process, the cross correlation of the motions (the normalized cross covariance function of Eq. (3)) relative to the reference station is evaluated. The time corresponding to the highest correlation provides the delay in the arrival of the waves at the various stations relative to their arrival at the reference one. Once the motions are aligned, they become invariant to the reference station selection, but the value of the time delay required for alignment is relative, ie, it is affected by the choice of the reference station. More refined approaches for the estimation of the arrival time perturbations have been proposed by the geophysical community for the analysis of data collected at closely spaced sensors and used for exploration purposes [41]. For example, Rothman [56,57] proposed an approach that views the problem as nonlinear inversion, estimates the cross correlations of the data, but, instead of picking their peak, the correlation functions are transformed to probability distributions; random numbers are generated from the distributions in an iterative scheme that minimizes the objective function, until convergence to the correct time shifts is achieved.

It should be noted, however, that if the motions at the array stations are aligned with respect to a reference station before their lagged coherency is estimated, then special attention should be placed in separately analyzing and estimating these delays due to their significance in the spatial variability of the motions [41,54,55,58]. Boisserie and Vanmarcke [54] modeled these fluctuations from SMART-1 data. They considered an extension of the closure property [37], in which “closure” is checked by relating the lags of all triplets of stations taken two by two. Their model considers that the time lag between two stations is given by:

$$\Delta t_{jk} = \Delta t_{jk}^{\nu} + \Delta t_{jk}^\prime$$  \hspace{1cm} (21)

in which $\Delta t_{jk}^{\nu} = \xi_{jk}/c$ is the time lag between the two stations, $j$ and $k$, due to the average propagation of the waves, and $\Delta t_{jk}^\prime$ represents the random fluctuations. From their analysis of the SMART-1 data they concluded that $\Delta t_{jk}^\prime$ is a normally distributed random variable with zero mean and standard deviation equal to $2.7 \times 10^{-2} + 5.41 \times 10^{-5} \xi_{jk}$, in which $\xi_{jk}$ is measured in m.

4 SPATIAL VARIABILITY MODELS

A mathematical description for the coherency was first introduced in earthquake engineering by Novak and Hindy [34,59] in 1979. The expression, based on wind engineering, was:

$$|\gamma(\xi, \omega)| = \exp\left[ -\kappa \frac{\omega \xi}{V_s} \sin \theta \right]$$  \hspace{1cm} (22)

where $\kappa$ and $\nu$ are constants and $V_s$ is an appropriate shear wave velocity. It should also be noted that Novak and Hindy [34,59] presented the first stochastic analysis of a lifeline system (buried pipeline) subjected to seismic motions experiencing loss of coherency; until that time only the propagation of the motions was considered in deterministic analysis of lifelines eg, [11,12,60], with the exemption of the work of Bogdanoff et al [61], who considered random earthquake-type excitations and Sandi [13], who presented a stochastic analysis of the response of non-synchronous seismic motions. After the installation of strong motion arrays and, in particular, the SMART-1 array, the stochastic description of the seismic motions and the stochastic response analysis of lifelines have been extensively investigated by researchers.

The following two subsections present some of the existing empirical and semi-empirical models for the spatial variation of the seismic ground motions; it is noted, however, that the list of models presented is not exhaustive.

4.1 Empirical coherency models

Because of i) the variability in seismic data recorded at different sites and during different events; ii) the differences in the numerical processing of the data used by various investigators; and iii) the different functional forms used in the regression fitting of a function through data with large scatter, there is a multitude of spatial variability expressions in the literature. The procedures for the removal of the wave passage effects in lagged coherency estimates also vary: some procedures evaluate coherency directly from the data, some remove the apparent propagation effects first and then evaluate the coherency, while others align the data before the coherency is estimated. Some of the developed expressions for the description of the coherency of the seismic ground motions at the SMART-1 array are presented in the following:

- due to Loh [62]:
  $$|\gamma(\xi, \omega)| = \exp\left(-a(\omega/\xi)\right)$$  \hspace{1cm} (23)

with $a(\omega)$ being a function of $\omega$ determined from the data of Event 5;

- due to Loh and Yeh [63]:
  $$|\gamma(\xi, \omega)| = \exp\left(-a(\omega/\xi)^2\right)$$  \hspace{1cm} (24)

with parameters determined from Events 39 and 40;

- due to Loh and Lin [64] for the description of 1D (isotropic) coherency estimates:
  $$|\gamma(\xi, \omega)| = \exp(-a\xi^2)$$  \hspace{1cm} (25)

and due to Loh and Lin [64], again, but for the description of directionally dependent coherency estimates:

  $$|\gamma(\xi, \omega)| = \exp(-a_1 - 2a_2\xi^2)\exp(-a_2 - b_2\omega^2)\exp(-a_3 - b_3\omega^2)$$  \hspace{1cm} (26)

with $\theta$ indicating the angle between the direction of propagation of the waves and the station separation, and the re-
maining parameters depending on SMART-1 data; due to Hao et al [65] and Oliveira et al [66]:

$$|\gamma(\xi_1, \xi_2, \omega)| = \exp(-\beta_1|\xi_1| - \beta_2|\xi_2|)$$

$$\times \exp\left(-\left(\alpha_1(\omega) \sqrt{|\xi_1|} + \alpha_2(\omega) \sqrt{|\xi_2|}\right)\right)$$

$$\times \left[\frac{\omega}{2\pi}\right]^2$$

(27)

with $\alpha_1(\omega)$ and $\alpha_2(\omega)$ given by functions of the form:

$$\alpha_i(\omega) = 2\pi a_i/\omega + b_i\omega/\omega + c_i$$

$\xi_j$ and $\xi_k$ in the equation are projected separation distances along and normal to the direction of propagation of the motions, and the parameters and functions in the equation are obtained through regression analyses of the data.

Most lagged coherency estimates assume that the random field is isotropic, in addition to being homogeneous; isotropy implies that the rotation of the random field on the ground surface will not affect the joint probability density functions. As a consequence of this assumption, most derived expressions for the lagged coherency are functions of separation distance only ($\xi_{jk} = \tilde{r}_j - \tilde{r}_k$) and not direction ($\xi_{ik} = \tilde{r}_i - \tilde{r}_k$). It can also be observed from Eq. (16) that the only directionally dependent term in the coherency is the one representing the wave passage effect (Eq. (17)). The last two expressions, however, are based on the description of the random field as anisotropic and account for the directional dependence of the spatial variation of the motions. Loh and Lin [64] indicate that, since seismic ground motions are path dependent, the spatial variability will be direction dependent, as different directions represent different paths; their evaluation of isocoherence maps for two events recorded at the SMART-1 array suggested that the field is not axisymmetric. Independent analyses of additional SMART-1 data support this observation: Abrahamson et al [19] also observed directional dependence of coherency and presented a possible explanation for this effect, namely that scattering in the forward direction tends to be in phase with the incident wave, whereas scattering to the side tends to loose phase [67]. Ramadan and Novak [47], in order to preserve the simpler representation of the field as isotropic, model its weak anisotropy characteristics by defining the separation distance as $\xi = \xi_1 + \mu_2\xi_2$, in which $\mu_2$ is a separation reduction factor to account for the directional variability in the data. They considered the two expressions (Eqs. (26) and (27)) and the set of data from which they were obtained and concluded that this simple separation distance transformation with variable $\mu_2$ preserves the isotropy of the field [47].

Due to Abrahamson et al [68]:

$$\tanh^{-1}(\gamma(\xi, \omega)) = (2.54 - 0.012\xi)$$

$$\times \left\{\exp\left(-0.115 - 0.00084\xi\right)\right\}$$

$$\times \left[\frac{\omega - 0.878}{3}\right] + 0.35$$

(28)

from nonlinear regression analysis of data recorded during 15 earthquakes; the above expression was derived from data recorded at the LSSTT array and is valid for separation distances less than 100 m.

Due to Harichandran and Vanmarcke [37] and Harichandran [29,55]:

$$|\gamma(\xi, \omega)| = A \exp\left(-\frac{2B|\xi|}{v(\omega)}\right) + (1 - A) \exp\left(-\frac{2B|\xi|}{v(\omega)}\right)$$

(29)

$$v(\omega) = k\left[1 + \left(\frac{\omega}{2\pi f_0}\right)^{b/2}\right]^{-1/2}$$

$$B = (1 - A + aA)$$

$$|\gamma(\xi, \omega)| = A \exp\left(-\frac{2|\xi|(1 - A)}{ak}\right)$$

$$\times \left[1 + \left(\frac{\omega}{2\pi f_0}\right)^{b/2}\right] + (1 - A)$$

(30)

with the parameters determined from the data of Event 20 [37] and Events 20 and 24 [29]; Harichandran [29] introduced the second coherency expression (Eq. (30)) to account for the difference in the behavior of the coherency at longer separation distances and higher frequencies.

The dependence of the functional form of the coherency on shorter and longer separation distances was noted by the last two coherency models. In particular, Abrahamson et al [19] investigated the same events recorded at the SMART-1 array (separation distances of 200–4000 m) and the LSST array (separation distances of 0–100 m). They noted that coherency values extrapolated at shorter separation distances from the SMART-1 data tended to overestimate the true coherency values obtained from the LSST data. Riepl et al [69] made a similar observation from their analyses of an extensive set of weak motion data recorded at the EUROSEISTEST site in northern Greece: the loss of coherency with distance for their data was marked by a “cross over” distance, that distinguished coherency for shorter (8–100 m) and longer (100–5500 m) separation distances.

Another point worth noticing regarding the behavior of coherency at shorter and longer separation distances is the following: Abrahamson et al [68], Schneider et al [20], and Vernon et al [70] from analyses of data at close by distances (<100 m) observe that coherency is independent of wavelength [20], and decays faster with frequency than with separation distance. On the other hand, independent studies of data at longer separation distances (>100 m) by Novak and his coworkers [47,71], and Toksöz et al [72] observe that the decay of coherency with separation distance and frequency is the same: Novak [71] and Ramadan and Novak [47] observed that if coherency is plotted as function of a normalized separation distance with wavelength ($\xi/\lambda$), with $\lambda = V_s/(2\pi\omega)$ being the wavelength and $V_s$ an appropriate shear wave velocity, then the coherency plots at various frequencies collapse onto the same curve; they used data from the 1979 Imperial Valley earthquake and Events 20 and 40 at the SMART-1 array. Similarly, Toksöz et al [72], from their analysis of data recorded at three arrays in Fennoscandia, Finland, noted that the curves of coherency decay with fre-
quency were very similar if separation distance was scaled with wavelength. The observed coherency dependence on wavelength implies that coherency is a function of the product \((\xi \omega)\), i.e., that the decay with frequency and separation distance is the same. It appears then that different factors control the loss of coherency in the data at shorter and longer separation distances.

Figure 3 presents the decay with frequency at separation distances of 100, 300, and 500 m of the coherency models of Harichandran and Vanmarcke [37] (Eq. (29)) and of Hao et al [65] (Eq. (27)). The parameters of Harichandran and Vanmarcke’s model are \(A = 0.736\), \(a = 0.147\), \(k = 5210\) m, \(f_o = 1.09\) Hz, and \(b = 2.78\); their values were determined from the data of Event 20 at the SMART-1 array. For the coherency model of Hao et al [65] it is considered that \(\xi_l = \xi, \xi_t = 0\), i.e., the stations are located on a line along the epicentral direction; the parameters of the model are \(\beta_1 = 2.25 \times 10^{-4}\), \(a_1 = 106.6 \times 10^{-4}\), \(b_1 = 0.265 \times 10^{-4}\), and \(c_1 = -0.999 \times 10^{-4}\), which were determined from the data of Event 30 at the SMART-1 array. In both expressions (Eqs. (27) and (29)), frequency \((\omega)\) is measured in rad/sec and separation distance \((\xi)\) in m; however, in all figures, for consistency with subsequent results, frequency is given in Hz. The differences in the representation of coherency for the same site but different events is obvious from the figure.

For comparison purposes, Fig. 3 presents the coherency decay with frequency at the shorter separation distances (50 and 100 m) of the LSST array as proposed by Abrahamson et al [68].

The expressions in Eqs. (23–30) represent the lagged coherency of motions recorded at the alluvial site of the SMART-1 array. It should be emphasized at this point that motions recorded at rock sites also experience loss of coherence that is attributed to the near surface rock weathering and cracking [73]. Cranswick [74], Toksz [72], Menke et al [75], and Schneider et al [20] estimated coherency from data at rock sites and observed that it also decays exponentially with frequency and station separation distance. Cranswick [74] presented a thorough investigation of the response (spectral amplitudes and spatial variability) of rock sites, and explained the drop of coherence in terms of the rock properties and layer formation. Schneider et al [20] compared coherency estimates at alluvial and rock sites, and concluded that, for the separation distances examined (<100 m), coherencies computed at rock sites were lower than those recorded on alluvium.

### 4.2 Semi-empirical models

Semi-empirical models for the spatial variation of the seismic ground motions, i.e., models for which their functional form is based on analytical considerations but their parameter evaluation requires recorded data, have also been introduced.

Somerville et al [76] proposed a model that attributes the spatial variation of the motions to the wave propagation effect, the finite source effect, the effect of scattering of the seismic waves as they propagate from the source to the site, and the local site effects. It has been shown, however, from data analyses [19] and seismological observations [41], that earthquake magnitude may not particularly influence coherency estimates. Abrahamson et al [19], from their analysis of 15 events of small \((M < 5)\) and large \((M \geq 6)\) magnitudes

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**Fig. 3** Variation of empirical coherency models based on SMART-1 data with frequency at separation distances of 100, 300, and 500 m, and on LSST data at separation distances of 50 and 100 m.
recorded at the LSST array, observed that there was no consistent trend indicating dependence of coherency on extended faults. Spudich [41] gives a possible explanation for the reason why the source finiteness may not significantly affect coherency estimates: For large earthquakes of unilateral rupture propagation, the waves radiating from the source originate from a spatially compact region that travels with the rupture front, and, thus, at any time instant, a relatively small fraction of the total rupture area radiates. Unilateral rupture at the source constitutes the majority of earthquakes [41]; Spudich [41] cautions, however, that bilateral rupture effects on spatial ground motion coherency are yet unmeasured. Der Kiureghian [58] developed a stochastic model in which the total spatial variation of the seismic motions is composed of terms corresponding to wave passage effects, bedrock motion coherence effects, and site response contribution; in the model evaluation, Der Kiureghian [58] also noted that the attenuation of the waveforms does not affect coherency estimates.

Perhaps the most quoted coherency model was introduced by Luco and Wong [77], and is based on the analysis of shear waves propagating a distance $R$ through a random medium:

$$\gamma(\xi, \omega) = e^{-\alpha \xi / v_{rm}} = e^{-a^2 \omega^2 \xi^2}$$

(31)

where $v_{rm}$ is an estimate for the elastic shear wave velocity in the random medium, $r_0$ the scale length of random inhomogeneities along the path, and $\mu$ a measure of the relative variation of the elastic properties in the medium. $\alpha$, the coherency drop parameter, controls the exponential decay of the function; the higher the value of $\alpha$, the higher the loss of coherence as separation distance and frequency increase. With appropriate choices for the coherency drop parameter, the model has been shown to fit the spatial variation of recorded data, and has been used extensively by researchers in their evaluation of the seismic response analysis of lifelines (e.g., [4,77–79]). Figure 4a presents the exponential decay of the model with frequency at separation distances of 100, 300, and 500 m for a median value $\alpha = 2.5 \times 10^{-4}$ sec/m from the ones suggested by Luco and Wong [77]. It is noted that this analytically based model produces coherencies equal to 1 at zero frequency for any separation distance, and tends to zero—rather than to a finite value—as frequency and separation distance increase (cf. Fig. 4a with Figs. 3a and b, that present empirical coherency models). Luco and Wong’s model also considers that the exponential decay with separation distance and frequency is the same.

Zerva and Harada [80] introduced a semi-empirical model for the coherency that approximates the site topography by a horizontally extended layer with random characteristics overlaying a half-space (bedrock). The model includes the effects of wave passage with constant velocity on the ground surface, the loss of coherence due to scattering of the waves as they travel from the source to the site by means of Luco and Wong’s [77] expression (Eq. (31)), and the local site effects, approximated by vertical transmission of shear waves through a horizontal layer with random properties. The random layer properties are evaluated from the spatial variation of the soil profile for the site under investigation. The resulting coherency for a soft random layer is shown in Fig. 4b; its decay with separation distance and frequency is identical to that of Luco and Wong’s model (i.e., the loss of coherency due to the scattering of the waves in the bedrock), except for a drop at the stochastic layer predominant frequency. Such drops in coherency have been observed from analyses of recorded data between two stations: Cranswick [74] notices that perturbations with small deviations in the layer characteristics will produce the greatest changes in the site response functions, and, since coherency is a measure of similarity of the motions, it will be low at the resonant frequencies. Although such a drop in coherency (Fig. 4b) may seem insignificant in terms of the overall coherency decay, it controls seismic ground strains, which in turn control the seismic response of buried pipelines [80]. In a similar approach, Kanda [9], using finite element modeling for a layered medium with irregular interfaces and random spatially variable incident motions, analyzed coherency and amplitude variability on the free surface of the site.
Zerva, Ang, and Wen [81,82] developed an analytical model for the estimation of the spatial coherency. The model is based on the assumption that the excitation at the source can be approximated by a stationary random process, which is transmitted to the ground surface by means of frequency transfer functions, different for each station. The frequency transfer functions are the Fourier transform of impulse response functions determined from an analytical wave propagation scheme and a system identification technique in the time domain. The resulting surface ground motions are stationary random processes specified by their power and cross spectral densities, from which the spatial variation of the motions is obtained. The approach was applied to the site of the SMART-1 array for Event 5; the analytical coherency captured the trend of the recorded data, even though the wave propagation scheme did not consider local site effects.

4.3 Effect of spatial variability in lifeline seismic response

The significance of the effect of the spatial variation of seismic ground motions on the response of lifelines, and, in particular, bridges, has been recognized since the mid 1960s [11,13,60,83], and has been continuously investigated with increased interest. Examples include: Abdel-Ghaffar [84,85] analyzed the response of suspension and cable-stayed bridges using random vibration theory; Harichandran and Wang [86,87], and Zerva [88,89] examined simple bridge configurations in random vibration analysis using various spatial variability models; Harichandran et al [90] analyzed suspension and arch bridges subjected to simulated spatially variable motions; Monti et al [91] examined the response of multi-span bridges to simulated spatially variable excitations; recently, Deodatis et al [8] and Saxena et al [1] investigated the nonlinear response of highway bridges subjected to simulated spatially variable seismic ground motions.

The most definite outcome that can be drawn from all past studies is that the use of identical motions as excitations at the structures supports will not always yield a conservative response; indeed—in cases—the response induced by identical motions can be grossly unconservative [8]. Specific conclusions regarding the effect of spatially variable motions on the response of bridges compared to that induced by identical excitations cannot be easily drawn: This effect depends on the structural configuration and properties, as well as on the ground motion characterization, i.e., the apparent propagation velocity of the motions, the soil conditions at the bridge supports, and the choice of the coherency model from the multitude of models existing in the literature. The time delays caused by the apparent propagation velocity result in out-of-phase motions at the structures’ supports. The consideration of variable soil conditions at the bridge supports affect the amplitude variation of the motions, and produce generally higher response than if the soil conditions were assumed to be identical [1,8]. In order to analyze the effect of the selection of a coherency model on the seismic response of extended structures, Zerva [5,79] isolated the contribution of the different coherency models to the quasi-static and dynamic response of linear, generic models of lifelines. It was shown [79] that the root-mean-square (rms) quasi-static response of lifelines is proportional to the rms differential displacement between the supports, and the rms contribution to the excitation of individual modes is proportional to the differential response spectrum. The differential response spectrum was defined as the rms response of a single-degree-of-freedom oscillator resting at two supports, which were subjected to spatially variable seismic ground motions. A normalized Clough-Penzien spectrum (Eq. (10)) and the models of Harichandran and Vanmarcke [37] (Fig. 3a) and Luco and Wong [77] (Fig. 4a) were used in the comparison; no wave passage effects were included, so that the variability due to the lagged coherency estimates could be clearly observed. Figure 5 shows the comparison of the lifeline response to the two different models: it can be seen that, in cases, the response can as much as double depending on the exponential decay in the models. Thus, the selection of a particular coherency model in the seismic response evaluation of lifelines can have an important effect on the seismic resistant design of the structures.

The question that arises, then, is which one of the coherency models available in the literature should be used in lifeline earthquake engineering. From one side, empirical coherency models are essentially event-, or mostly site-, specific. It was recognized, since the early studies of the spatial
variation from SMART-1 data, that there are large differences in the correlations between events [17], so that the same parametric expression may not describe the spatial variability of different events at the same site [37]; this observation can also be made from the comparison of the coherency models in Figs. 3a and b. As such, empirical coherency models cannot be reliably extrapolated to different sites and events. From the other side, semi-empirical models are based on certain simplifying assumptions that may not always capture reality. It is being examined recently whether a generic model can reproduce the spatially variable nature of seismic motions at different sites and various events. Some studies [47,68] suggest that such models may be feasible, whereas others [20] suggest that they may not. The underlying difficulty in establishing appropriate coherency expressions for any site and any event is that coherency, as a purely statistical measure, is not related to physical parameters.

An additional consideration for the description of spatially variable seismic ground motions is their amplitude variability. The amplitude variability has not been investigated as widely as the phase variability: Abrahamson et al [68] and Schneider et al [20] evaluated the variation of the amplitude spectra of the data by considering that the Fourier amplitude spectrum is lognormally distributed; it was observed that the variance of the difference between the logarithms of spectral amplitudes between stations increased with frequency and tended to constant values at higher frequencies.

It becomes then apparent, from the aforementioned discussion, that the description of the spatial variation of the seismic motions ought to incorporate both the amplitude and the phase variation in the data. It should also be tied to physical parameters, so that it can be reliably extrapolated to any site. An alternative approach [53,92] that deals directly with time histories rather than coherency estimates, recognizes—qualitatively—physical patterns in the spatial variability of the data, and analyzes simultaneously the amplitude and phase variability in the motions is presented in the following.

5 CORRELATION PATTERNS IN THE AMPLITUDE AND PHASE VARIATION OF THE MOTIONS

The approach [92] models the seismic ground motions as superpositions of sinusoidal functions described by their amplitude, frequency, wavenumber, and phase. For each event and direction (horizontal or vertical) analyzed, it identifies a coherent, common component in the seismic motions over extended areas. The common component represents a coherent wavetrain that propagates with constant velocity on the ground surface and approximates to a satisfactory degree the actual motions. The spatial variation of the seismic motions is determined from the differences between the recorded data and the coherent estimates of the motions. The methodology was applied to data recorded at the SMART-1 dense instrument array in Lotung, Taiwan, and is presently applied to additional recorded data. For illustration purposes, the strong motion S-wave window (7.0–12.12 sec actual time in the records with a time step of 0.01 sec) in the N-S direction of Event 5 at the SMART-1 array is presented herein.

Identification of propagation characteristics of the motions—Signal processing techniques, eg, [49,53], are initially applied to the data recorded at all stations in order to identify the propagation characteristics of the motions. The application of the conventional method [50] with slowness stacking [39] to the data identified the slowness of the dominant broad-band waves in the window as \( \tilde{s} = (0.1 \text{ sec/km}, \ldots, -0.2 \text{ sec/km}) \), i.e., the waves impinge the array at an azimuth of 153° with an apparent propagation velocity of 4.5 km/sec. The (horizontal) wavenumber, \( \tilde{k} = \omega \tilde{s} \), with \( \omega \) indicating frequency.

Identification of amplitude and phase of the signals—The seismic ground motions are then described by \( M \) sinusoids and expressed as:

\[
\hat{a}(\tilde{r},t) = \sum_{m=1}^{M} A_m \sin(\tilde{k}_m \cdot \tilde{r} + \omega_m t + \phi_m)
\]  

(32)

in which \( \tilde{r} \) indicates location on the ground surface and \( t \) is time. Each sinusoidal component is identified by its discrete frequency and wavenumber \((\omega_m, \tilde{k}_m)\); \( A_m \) and \( \phi_m \) are its amplitude and phase shift, respectively. It is noted that no noise component is superimposed to the ground motion estimate of Eq. (32). The subscript \( m \) in the parameters of Eq. (32) indicates frequency dependence, and the number of sinusoids, \( M \), used in the approach depends on the cut-off frequency, above which the sinusoids do not contribute significantly to the seismic motions. The amplitudes, \( A_m \), and phases, \( \phi_m \), of the sinusoidal components can then be determined from the system of equations resulting from the least-squares minimization of the error function between the recorded time histories \( a(\tilde{r},t) \) and the approximate ones \( \hat{a}(\tilde{r},t) \) (Eq. (32)) with respect to the unknowns \( A_m \) and \( \phi_m \) [53]:

\[
E = \sum_{l=1}^{L} \sum_{n=1}^{N} \left[ a(\tilde{r}_l,t_n) - \hat{a}(\tilde{r}_l,t_n) \right]^2
\]  

(33)

evaluated at discrete locations (stations) \( l \) and times \( n \). Any number \( L \) of stations—ranging from one to the total number of recording stations—can be used for the evaluation of the signal amplitudes and phases. When \( L > 1 \) in Eq. (33), the identified amplitudes and phases represent the common signal characteristics at the number of stations considered; when \( L = 1 \), the amplitudes and phases correspond to the motions at the particular station analyzed.

Reconstruction of seismic motions—Five stations \((L=5)\) are used in Eq. (33) for the identification of their common amplitudes and phases; the stations are C00, I03, I06, I09 and I12 (Fig. 1). Once the common characteristics are identified, they are substituted in Eq. (32), and an estimate of the motions (reconstructed motions) at the stations considered is obtained. The comparison of the recorded motions with the reconstructed ones is presented in Fig. 6; no noise (random) component is added to the reconstructed signals. Since amplitudes and phases at each frequency are identical for all
five stations considered, the reconstructed motions represent a coherent waveform that propagates with constant velocity on the ground surface. Figure 6 indicates that the reconstructed motions reproduce, to a very satisfactory degree, the actual ones and, although they consist only of the broad-band coherent body wave signal (Eq. (32)), they can describe the major characteristics of the data. The details in the actual motions, that are not matched by the reconstructed ones, constitute the spatially variable nature of the motions, after the (deterministic) wave passage effects have been removed.

**Variation of amplitudes and phases**—When only one station at a time \( (L = 1 \text{ in Eq. (33)}) \) is used in the evaluation of amplitudes and phases at different frequencies for that particular station, the reconstructed motion is indistinguishable from the recorded one. This does not necessarily mean that the analyzed time histories are composed only of the identified broad-band waves, but rather that the sinusoidal functions of Eq. (32) can match the sinusoidally varying seismic time histories, i.e., Eq. (33) becomes essentially compatible to a Fourier transform. The comparison of the results at the individual stations with the common ones provides insight into the causes for the spatial variation of the motions.

Figure 7 presents the amplitude and phase variation of the sinusoidal components of the motions with frequency; the continuous, wider line in these figures, as well as in the subsequent ones, indicates the common signal characteristics, namely the contribution of the identified body wave to the motions at all five stations, whereas the thinner, dashed lines represent the corresponding amplitudes and phases when one station at a time is considered in Eq. (33). In the lower frequency range \( (<1.5 \text{ Hz}) \), amplitudes and phases identified at the individual stations essentially coincide with those of the common component. In the frequency range of 1.5–4 Hz, the common amplitude represents the average of the site amplification and phases start deviating from the common phase. It is noted that phases were restricted in the range \([0,2\pi)\), and, therefore, jumps of approximately \(2\pi\) do not indicate drastic variations in their values. At higher frequencies, the common amplitude becomes lower than the ones identified at the stations and phases vary randomly.

**Alignment of time histories**—Part of the variabilities around the common component in Fig. 7 are due to the fact that the time history approximation (Eq. (32)) does not allow for the random arrival time perturbations at the array stations (Section 3.3). Their effect can be noted in the comparison of the recorded and reconstructed motions in Fig. 6, when, e.g., the reconstructed motion at I12 arrives later than the recorded one. In the present approach, these arrival time perturbations are partially eliminated through the alignment of the seismic motions with respect to C00 (Fig. 1). As expected, the arrival time delays evaluated from the alignment process between individual sets of stations exhibited a random behavior around the delays determined from the apparent propagation of the motions identified from the constant slowness of the broad-band waves. However, the average
pattern of wave propagation across the array was consistent with the one identified from the signal processing technique [92].

**Common components in aligned motions**—For the identification of the amplitude and phase variation of the aligned motions, the error function (Eq. (33)) is used again in the minimization scheme but, in the sinusoidal approximation of the motions (Eq. (32)), the term $\tilde{k}_r \cdot \tilde{r}$ is set equal to zero, since the aligned motions arrive simultaneously at all array stations. Figure 8 presents the variation of the amplitudes and phases determined from the application of the least-squares minimization scheme to the aligned motions at four inner ring stations ($I_{03}$, $I_{06}$, and $I_{09}$, and $I_{12}$) and the center station $C_{00}$, together with the amplitude and phase variation identified using the aligned motions recorded at one station at a time. The variability range of amplitudes and phases around the common component is reduced in the aligned data results compared to those of the actual motions (Figs. 7 and 8), indicating the significance of the consideration of the spatial arrival time perturbations in the approach. Figure 9 presents the results of the application of the approach to the aligned seismic motions recorded at four middle ring stations, $M_{03}$, $M_{06}$, $M_{09}$, and $M_{12}$, and the reference station $C_{00}$.

The common amplitude and phase identified from the inner and the middle ring station data (Figs. 8 and 9), shown together for comparison purposes in Fig. 10, are very similar, particularly considering the facts that separate analyses were performed for the two sets of stations, and that the longest separation distance for the middle ring stations is 2 km whereas that of the inner ring ones is 400 m. Their differences are an expected consequence of the larger scatter in the data at the further away stations due to attenuation of waves and more significant variations in site topography. The agreement of the common amplitudes and phases over an extended area of radius of 1 km strongly suggests the existence of the coherent component in the data. The common amplitude can be viewed as a mean value representing the average amplification of the motions at the site and is associated with the common phase variability with frequency, that resembles

![Amplitude and phase variation of the aligned motions at the center and inner ring stations](image1.png)

![Amplitude and phase variation of the aligned motions at the center and middle ring stations](image2.png)
random distribution between \([0, 2\pi]\). The spatial variability of the motions, in addition to their propagation effects already considered, results from deviations in both amplitudes and phases at the individual stations around the common components (Figs. 8 and 9), which are described in terms of differential amplitudes and phases in Figs. 11 and 12.

**Differential amplitude and phase variability in aligned motions**—The normalized differential amplitudes (Figs. 11 and 12) are obtained by subtracting at each frequency the common amplitude from the amplitudes identified at the individual stations and dividing by the common amplitude. The normalized differential amplitudes are cut off at a maximum value of 7.5; their actual values, which are not important for the subsequent analysis, can be significantly high, because the common component can assume low values at certain frequencies (Figs. 8 and 9). Furthermore, the normalized differential amplitudes, from their definition, cannot assume values lower than \((-1\)). The differential phases (Figs. 11 and 12) are obtained by subtracting at each frequency the common phase from the phases identified at each station; the differential phases (Figs. 11 and 12) are allowed to vary between \([-\pi, +\pi]\) rather than between \([0, 2\pi]\), as was the case in Figs. 7–10.

Envelope functions, drawn by eye, containing the amplitude and phase differential variability range are also shown in the figures. The phase envelope functions are symmetric with respect to the zero axis. The envelope functions for the amplitudes are symmetric only in the lower frequency range, as their values are restricted in the range greater than \((-1\)). Isolated peaks within the dominant site amplification frequency range are excluded from both the amplitude and phase envelope functions. The trend of the positive envelopes of both amplitudes and phases is very similar, implying that the amplitude and phase variability of the data around their respective common component characteristics are correlated. Since amplitude variability is easier visualized and

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**Fig. 10** Comparison of the common component amplitude and phase identified from the analysis of the inner and middle ring station aligned data

**Fig. 11** Differential amplitude and phase variability with respect to the common component of the center and inner ring station aligned motions; envelope functions containing the variability are drawn by eye
attributed to physical causes than phase variability, the variation of both amplitudes and phases can be—qualitatively—associated with physical parameters: In the low frequency range, the envelope functions are at close distance to the zero axis, that increases consistently for both amplitudes and phases as separation distance between stations increases (inner vs middle ring results); this is attributed to the long wavelength of the contributing waves at low frequencies, that do not “see” the site irregularities, particularly for the inner ring stations. As frequency increases within the dominant site amplification frequency range, the distance of the amplitude and phase envelope functions from the zero axis increases gradually with a slower rate for the inner ring stations than the middle ring ones. In this range, the common amplitudes reproduce the average of the site amplification (Figs. 8 and 9) implying that the motions are controlled by the signal that is modified in amplitude and phase as it traverses the horizontal variations of the layers underneath the array. The increase in the variabilities of amplitudes and phases around the common component as frequency increases may also be associated with the decreasing wavelength of the signals at increasing frequencies, which is more obvious for the middle ring results, and to the more significant contribution of scattered energy. At higher frequencies, past the dominant site amplification frequency range, wave components in addition to the broad-band signal, and, mainly, scattered energy (noise) dominate the motions. Because these wave components propagate at different velocities, phases at the individual stations (Figs. 8 and 9) deviate significantly from the common phase, and the common signal amplitude no longer represents the average of the site amplification and becomes lower than the amplitudes identified at the individual stations. Consequently, the phase differences vary randomly between \([-\pi, +\pi]\), i.e., the differential phase envelopes are parallel to the zero axis at a distance equal to \(\pi\), and the normalized differential amplitudes assume high values. Noise is also the cause of the isolated peaks in the dominant frequency range of the motions; this variability, however, may not be of significant consequence for the modeling or the simulation of spatially variable seismic ground motions: it occurs when amplitudes are low within the dominant frequency range of the motions.

**Differential phase variability and spatial coherency**—The envelope functions in Figs. 11 and 12 contain the variability range of amplitudes and phases with respect to the common component for motions recorded at separation distances less than the maximum station separation distance for the area considered. Thus, the envelope functions, as upper limits, are functions of frequency and maximum station separation distance, i.e., 400 m and 2 km for the inner and middle ring data, respectively. Constrained within the envelope functions, amplitudes and phases vary randomly, suggesting that the variability can be described by the product of the envelope function and a random number uniformly distributed within a specific range. Differential amplitudes and phases between stations also vary randomly within the bounds of envelope functions that have trends very similar to those of the envelope functions with respect to the common component; this observation can also be made, indirectly, from Figs. 11 and 12. It is recalled from Eqs. (19) and (20) that differences in phases between stations are directly related to the lagged coherency through envelope functions, \(\beta_{jk}(\omega)\pi\), containing their random variability [42]. Thus, the differential phase variability identified by means of this methodology is equivalent to conventional coherency estimates. However, the shape of the envelope functions, and, consequently, the associated coherency (Eq. (20)) can now be related to physical parameters: In the low frequency range, where the envelope functions are close to the zero axis, coherency assumes values close to one; this behavior is a consequence of the long signal wavelength at low frequencies and the distance between stations. Within the dominant site amplification frequency range of the motions, where the envelope functions increase with frequency, coherency decreases; this may be attributed to the decreasing wavelength of the motions, which is more prominent for the further away stations, and to

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**Fig. 12** Differential amplitude and phase variability with respect to the common component of the center and middle ring station aligned motions; envelope functions containing the variability are drawn by eye.
the more significant contribution of scattered energy. In the higher frequency range, where phases vary randomly between \([-\pi, +\pi]\) and noise dominates, coherency assumes zero values. It is noted that coherency evaluated from recorded data (Eq. (12)) can never reach zero in the higher frequency range, although the true coherency of the motions tends to zero \([29,47]\). The qualitative association of coherency with physical parameters introduced in this approach sets the bases for the possible physical modeling of coherency.

### 6 SIMULATIONS OF SPATIALLY VARIABLE GROUND MOTIONS

The spatially variable seismic ground motion models described in Sections 2, 3, and 4 can be used directly as input motions at the supports of lifelines in random vibration analyses; this approach, however, is applicable to relatively simple structural models and for the linear response of the structures. For complex structural lifeline systems, and for their nonlinear seismic response evaluation, only the deterministic seismic response analyses; this approach, however, is applicable to relatively simple structural models and for the linear response of the structures. For complex structural lifeline systems, and for their nonlinear seismic response evaluation, only the deterministic seismic response analyses. Thus, Monte Carlo simulations become the most versatile tool for such problems.

An extensive list of publications addressing the topic of simulations of random processes and fields has appeared in the literature. The following list is by no means exhaustive: One of the techniques that has been widely applied in engineering problems is the spectral representation method \([93,94]\). Simulations based on AR (auto-regressive), MA (moving-average) and ARMA (auto-regressive-moving-average) techniques have also been used extensively \(\text{eg,} [95–99]\). Simulations by means of the local average subdivision method \([100]\), the turning bands method \([101]\), and based on wavelet transforms \([102,103]\) have been generated. Simulations of non-stationary time series by means of processes modulated by time varying functions \(\text{eg,} [104–106]\) have also been reported. More specifically, spatially variable seismic ground motions have been simulated by means of a wide variety of techniques. Examples include, but are not limited to: covariance matrix decomposition \([65,107]\); spectral representation method \([78,79,108]\); envelope functions containing random phase variability \([42]\); coherency function approximation by a Fourier series \([109,110]\); ARMA approximation \([111]\); FFT \([112]\) and hybrid DFT and digital filtering approach \([113]\) for non-stationary random processes; and conditional simulations \([114,115]\) and interpolation \([116]\).

The appropriate simulation technique for the particular problem at hand depends on the characteristics of the problem itself \([117]\). The main objectives are that the characteristics of the simulated motions match those of the target field, and that the computational cost for the simulations is not excessive. An additional consideration in the simulation of spatially variable ground motions is which one of the extensive list of coherency models should be used in the analysis. In order to illustrate these considerations, the first subsection herein simulates spatially variable seismic ground motions based on two commonly used spatial coherency models by means of the spectral representation method; the similarities and differences in the resulting motions are described. The second subsection presents two simulation techniques that are commonly used in lifeline earthquake engineering; the differences in the simulated motions resulting from the use of the particular simulation scheme are addressed. In both subsections, the motions are simulated as stationary, ie, no deterministic modulation function is used to transform the simulated time history to a non-stationary one. The reason for this is to preserve the characteristics of the coherency model and the simulation scheme in the comparison. For the same reason, the contribution of the propagation of the motions in the simulated ones is only illustrated in an example in Section 6.1. Section 6.3 presents approaches for the modification of the simulated time histories, so that they reproduce more closely actual seismic ground motions.

#### 6.1 Selection of coherency model

Simulations resulting from two coherency models commonly used in lifeline earthquake engineering, namely the models of Harichandran and Vanmarcke \([37]\) (Eq. (29)) and of Luco and Wong \([77]\) (Eq. (31)) are compared. The spectral representation method is used for the generation of the simulated time histories.

The concept of representations of Gaussian random processes was introduced by Rice in 1944 \([93]\) (reprinted in \([118]\)), but the use of the approach in generating simulations of random processes and fields originates from Shinozuka \([94,119,120]\). The methodology has been presented in a comprehensive article by Shinozuka \([121]\), and was more recently elaborated upon in Shinozuka and Deodatis \([122,123]\) and references therein. In its initial formulation, the spectral representation method dealt with the summation of large numbers of weighted trigonometric functions, which is computationally, not efficient. Yang \([124]\) introduced the Fast Fourier Transform (FFT) technique in simulating envelopes of random processes, and Shinozuka \([125]\) extended the approach to random processes and fields. The use of the FFT dramatically reduces the computational requirements for simulations. Improvements on the approach and evaluations of its properties and capabilities have been reported by various researchers over the last few years \(\text{eg,} [78,126–129]\). Mignolet and Harish \([129]\) compared the performance of the spectral representation algorithm, the randomized spectral representation scheme \([120]\) and the random frequencies algorithm \([119]\), and concluded that, irrespectively of computational effort, the latter performs generally better in terms of first and second order distributions than the other two. As computational efficiency is a significant consideration, the simulation scheme described in the following utilizes the spectral representation method at fixed frequencies.

Consider a homogeneous space-time random field with zero mean, space-time covariance function \(R(\xi, \tau)\), \(\xi\) being the separation distance, and \(\tau\) being the time lag, and...
frequency-wavenumber (F-K) spectrum $S(\kappa, \omega)$, in which $\kappa$ indicates the wavenumber and $\omega$ indicates frequency. The frequency-wavenumber spectrum and the space-time covariance function are Wiener-Khintchine transformation pairs and possess the same symmetries [38]. The F-K spectrum is obtained through the Fourier transform of the cross spectral density of the motions. Simulations of spatially variable seismic ground motions can then be generated by means of the spectral representation method [78,121]:

$$f(x,t) = \sqrt{2} \sum_{j=0}^{J-1} \sum_{n=0}^{N-1} \left[ 2 S(\kappa_j, \omega_n) \Delta \kappa \Delta \omega \right]^{1/2} \times \cos(\kappa_j x + \omega_n t + \phi_j^{(1)}) + \left[ 2 S(\kappa_j, -\omega_n) \Delta \kappa \Delta \omega \right]^{1/2} \times \cos(\kappa_j x - \omega_n t + \phi_j^{(2)})$$

(34)

in which $\phi_j^{(1)}$ and $\phi_j^{(2)}$ are two sets of independent random phase angles uniformly distributed between $(0, 2\pi)$, $\kappa_j = (j + \frac{1}{2}) \Delta \kappa$ and $\omega_n = (n + \frac{1}{2}) \Delta \omega$ are the discrete wavenumber and frequency, respectively, and $\Delta \kappa$ and $\Delta \omega$ are the wavenumber and frequency steps. Simulations based on Eq. (34) can be obtained if there exists an upper cut-off wavenumber $\kappa_u = J \times \Delta \kappa$ and an upper cut-off frequency $\omega_u = N \times \Delta \omega$, above which the contribution of the F-K spectrum to the simulations is insignificant for practical purposes. The FFT algorithm can then be introduced in Eq. (34) to improve the computational efficiency of the method. In order to accommodate the fact that the FFT algorithms start at $\kappa_0 = 0$ and $\omega_0 = 0$, rather than $\kappa_0 = \frac{1}{2} \Delta \kappa$ and $\omega_0 = \frac{1}{2} \Delta \omega$, as is required in the present case, Eq. (34) is rewritten as [78]:

$$f_{rs} = \sqrt{2} \mathfrak{R} \left[ e^{i \pi r/M} e^{i \pi s/L} \sum_{j=0}^{M-1} \sum_{n=0}^{L-1} \left( 2 S(\kappa_j, \omega_n) \Delta \kappa \Delta \omega \right)^{1/2} \times e^{i \phi_j^{(1)}} e^{i 2 \pi j/M} e^{i 2 \pi n/L} \right]$$

$$+ v2 \mathfrak{R} \left[ e^{i \pi r/M} e^{-i \pi s/L} \sum_{j=0}^{M-1} \sum_{n=0}^{L-1} \left( 2 S(\kappa_j, -\omega_n) \Delta \kappa \Delta \omega \right)^{1/2} \times e^{i \phi_j^{(2)}} e^{i 2 \pi j/M} e^{-i 2 \pi n/L} \right]$$

(35)

in which, $f_{rs} = \mathfrak{F}(x_r, t_s)$; $x_r = r \Delta x$; $\Delta x = 2 \pi / M \Delta \kappa$; $r = 0, \ldots, M - 1$; $t_s = r \Delta t$; $\Delta t = 2 \pi / L \Delta \omega$; $s = 0, \ldots, L - 1$; and $\mathfrak{R}$ indicates the real part. The FFT is applied to the two expressions in the braces in Eq. (35). $M$ and $L$ are powers of 2, and ought to satisfy the inequalities $M \geq 2 J$ and $L \geq 2 N$, so that aliasing effects can be avoided; ie, for $J < j < M$ and $N < n < L$ the value of the F-K spectrum in Eq. (35) is zero. The following characteristics are inherent in the simulations: i) they are asymptotically Gaussian as $J, N \to \infty$ due to the central limit theorem; ii) they are periodic with period $T_o = 4 \pi / \Delta \omega$ and wavelength $L_o = 4 \pi / \Delta \kappa$; iii) they are ergodic, at least up to second moment, over an infinite time and distance domain or over the period and wavelength of the simulation; and iv) as $J, N \to \infty$, the ensemble mean, covariance function and frequency-wavenumber spectrum of the simulations are identical to those of the random field.

Seismic ground motion simulations (displacements) based on the spatial variability models of Harichandran and Vanmarcke [37] (Eq. (29)) and that of Luco and Wong [77] (Eq. (31)) are generated by means of Eq. (35). The parameters of the Harichandran and Vanmarcke model were $A = 0.736$, $a = 0.147$, $k = 5210m$, $f_o = 1.09$ Hz, and $b = 2.78$. Two different values for the coherence drop parameter of the Luco and Wong model were used: a low value, $\alpha = 2 \times 10^{-4}$ sec/m, that reproduces slow exponential decay in the coherency as frequency and separation distance increase, and a high value, $\alpha = 10^{-3}$ sec/m, that represents a sharp exponential decay. The power spectral density considered was the Clough-Penzien spectrum (Eqs. (9) and (10)) with parameters: $S_o = 1 \text{ cm}^2/\text{sec}^3$, $\omega_g = 5 \pi \text{ rad/sec}$, $\omega_f = 0.1 \omega_g$, and $\xi_g = \xi_f = 0.6$, ie, firm soil conditions. The simulations were performed at a sequence of six locations along a straight line on the ground surface with separation distance between consecutive stations of 100 m; the resulting time histories are presented in Figs. 13a, b, and c. For comparison purposes, the same number of wavenumber and frequency points ($J$ and $N$), the same discrete wavenumbers and frequencies, and the same seeds for the generation of the random phase angles were used in the simulations. As a result, the simulations at $x = 0$ look similar for all cases (Figs. 13a, b, and c). There are, however, significant differences as $x$ increases: Figure 3b suggests that simulations based on the Luco and Wong model with low coherency drop, $\alpha = 2 \times 10^{-4}$ sec/m, are essentially unchanged at all locations, whereas the ones based on the other two models (Figs. 13a and c) vary with distance. It is also noted that the variability in the time histories of the motions for the high coherency drop Luco and Wong model ($\alpha = 10^{-3}$ sec/m) in Fig. 13c is dominated by high frequency components, whereas of the Harichandran and Vanmarcke model (Fig. 13a) by lower frequency ones. These differences in the time histories are caused by the behavior of the models in the low frequency range that controls the displacements (0–2 Hz). The model of Harichandran and Vanmarcke is only partially correlated even at zero frequency (Fig. 3a) yielding low frequency variation in the time histories between the stations. On the other hand, Luco and Wong’s model is fully correlated as $\omega \to 0$ for any value of the coherency drop parameter (Eq. (31)). For low values of $\alpha$ (Eq. (31) and Fig. 4a), it results in high values for the coherency in the lower frequency range, thus yielding essentially identical displacement simulations (Fig. 13b). For higher values of $\alpha$ (Eq. (31)), it decreases rapidly with frequency and separation distance, and the variability in the time histories shown in Fig. 13c is caused by higher frequency components. It needs to be mentioned, however, that acceleration time histories based on Luco and Wong’s model for low values of $\alpha$ show variability in space: acceleration time histories are controlled by a higher frequency range than that of displacements; in this higher frequency range (Fig. 4a), the model decreases with frequency and separation distance. The comparison of the simulated time histories in Figs. 13a–c.
indicates that the exponential decay of the coherency model affects the characteristics of the ground motions, which, in turn, affect the seismic response of lifelines.

It should also noted from Figs. 13a–c that the seismic ground motions do not propagate on the ground surface. Indeed, it can be shown that for quadrant-symmetric space time fields (i.e., \( S(\kappa, \omega) = S(-\kappa, \omega) = S(\kappa, -\omega) = S(-\kappa, -\omega) \)), such as the random fields resulting from both the Harichandran and Vanmarcke and the Luco and Wong models, Eq. (34) can be rewritten as:

\[
f(x,t) = 2\sqrt{2} \sum_{j=0}^{J-1} \sum_{n=0}^{N-1} \left[ S(\kappa_j, \omega_n) \Delta \kappa \Delta \omega \right]^{1/2} \times \cos \left( \kappa_j x + \frac{\phi_{jn}^{(1)} + \phi_{jn}^{(2)}}{2} \right) \cos \left( \omega_n t + \frac{\phi_{jn}^{(1)} - \phi_{jn}^{(2)}}{2} \right)
\]

which represents the superposition of standing waves, and reflects correctly the characteristics of the lagged coherency. If the apparent propagation of the motions is included in the description of the random field, then it is reflected in the simulated motions. Figure 13d presents displacement simulations based on Luco and Wong’s model for \( \alpha = 2 \times 10^{-4} \) sec/m and an apparent propagation velocity \( c = 500 \) m/sec (Eqs. (17) and (18)). It can be observed from the figure that the seismic ground motions propagate along the \( x \) direction with the specified apparent propagation velocity.

6.2 Selection of simulation scheme

Another important consideration in the generation of artificial spatially variable ground motions is the selection of the simulation scheme, so that the resulting seismic motions at all locations on the ground surface possess the same properties. To this effect, Katafygiotis et al. [108] and Zerva and Katafygiotis [107] compared commonly used simulation schemes in lifeline earthquake engineering. Their results— for two techniques—are summarized in the following:

Let \( \omega_n \) be a cut-off frequency, above which the contribution of the spectrum to the simulations is insignificant for.
practical purposes. The interval \([0, \omega_d]\) is then divided into \(N\) equal parts, each having length \(\Delta \omega = \omega_d / N\). Let \(\omega_k\) be some point in the interval \([(k-1)\Delta \omega, k\Delta \omega]\) chosen in such a way that for any \(k \geq 1\), \(\omega_k - \omega_{k-1} = \Delta \omega\).

**The Cholesky decomposition with random amplitudes (CDRA) method**—The random field at a point \(\hat{x}\) on the ground surface is simulated as the sum of trigonometric terms with random space-dependent coefficients:

\[
f(\hat{x}, t) = \sqrt{\Delta \omega} \sum_{k=1}^{N} \left[ \eta_{1k}(\hat{x}) \cos(\omega_k t) + \eta_{2k}(\hat{x}) \sin(\omega_k t) \right]
\]

(37)

where \(\eta_{jk}(\hat{x})\), \(j = 1, 2\), is a sequence of independent random fields. Equation (37) is, essentially, the classical spectral representation simulation scheme: For the simulation of random processes, ie, \(\hat{x}\) is fixed, \(\eta_{1k}(\hat{x})\) and \(\eta_{2k}(\hat{x})\) are independent random variables having Gaussian distribution [118], or, the trigonometric functions of Eq. (37) are combined into a single sinusoidal function with a random phase, that has uniform distribution over the interval \([0, 2\pi]\) [94].

If simulations at \(M\) locations on the ground surface are to be generated, \(\eta_{1k}(\hat{x}_i)\) and \(\eta_{2k}(\hat{x}_i), i = 1, ..., M, (\text{Eq. (37)})\), are written in vector form as:

\[
\mathbf{n}_{kj} = [\eta_{1k}(\hat{x}_1), \ldots, \eta_{1k}(\hat{x}_M)]^T, \quad 1 \leq k \leq N, \quad j = 1, 2
\]

(38)

The elements of the autocovariance matrix \(\mathbf{R}_{n_j n_j}\) of the random vector \(\mathbf{n}_{kj}\) are assigned the form:

\[
[\mathbf{R}_{n_j n_j}]_{lm} = \mathbb{E}[\eta_{lj}(\hat{x}_l) \eta_{lk}(\hat{x}_m)] = S(\omega_k) |\gamma(\xi_{lm}, \omega_k)|
\]

(39)

where \(\xi_{lm}\) is the distance between the two points \((\hat{x}_l, \hat{x}_m)\), \(S(\omega_k)\) is the power spectral density of accelerations, velocities, or displacements, and \(|\gamma(\xi_{lm}, \omega_k)|\) is the lagged coherency. If \(\mathbf{R}_{n_j n_j} = \mathbf{C}_k^T \mathbf{C}_k\) is the Cholesky decomposition of the autocovariance matrix of the random vector \(\mathbf{n}_{kj}\), then \(\mathbf{n}_{kj}\) can be simulated as:

\[
\mathbf{n}_{kj} = \mathbf{C}_k \mathbf{\Theta}_j
\]

(40)

where the random vector \(\mathbf{\Theta}_j\) consists of \(M\) independent components having normal distribution and unit variance. The generated \(\mathbf{n}_{kj}\) are substituted into Eq. (37) and the simulated time histories are obtained. The characteristics of the simulated field relative to the target random field are discussed extensively in Katayefiotis et al [108]. It is noted herein, that it will be elaborated upon later, that the simulated motions are sums of trigonometric functions with random amplitudes, which, for station \(l\) and frequency \(k\), take the form:

\[
A_{kl} = \sqrt{\Delta \omega} (\eta_{1k}^2(\hat{x}_l) + \eta_{2k}^2(\hat{x}_l))
\]

(41)

**The Hao-Oliveira-Penzien (HOP) method**—The simulated time histories according to this method [65] are superpositions of sinusoids, each of which has a deterministic amplitude and a random phase, and corresponds to the frequency \(\omega_k\) in the partitioned frequency domain \([0, \omega_d]\). Consider that simulations need to be generated at a sequence of locations. For each of these locations, the number of sinusoids that needs to be superimposed depends on the order of the particular location in the sequence. Hence, for the first location, only one sinusoid is used per frequency, for the second location in the sequence, two sinusoids are superimposed for each frequency, and so on. The amplitudes of these sinusoids convey the coherency relation of each point in the sequence with all the previous ones. Each sinusoid also contains a random phase that is uniformly distributed between \([0, 2\pi]\).

Let \(f_i(t), i = 1, \ldots, M\) denote the simulations of \(M\) random processes corresponding to \(M\) prescribed locations \(\hat{x}_i\), \(i = 1, \ldots, M\), of the random field. According to the HOP method, the simulation of these processes is achieved according to:

\[
f_i(t) = \sum_{m=1}^{i} \sum_{k=1}^{N} A_{im}(\omega_k) \cos(\omega_k t + \phi_{mk}), \quad i = 1, \ldots, M
\]

(42)

where \(A_{im}(\omega_k)\) are the amplitudes and \(\phi_{mk}\) the random phases, which are independent random variables uniformly distributed between \([0, 2\pi]\). The amplitudes at frequency \(\omega_k\) are calculated by Cholesky decomposition of the cross spectral density matrix, \(\mathbf{S}(\omega_k)\), where

\[
[\mathbf{S}(\omega_k)]_{im} = S(\omega_k) |\gamma(\xi_{im}, \omega_k)|
\]

(43)

as \(\mathbf{S}(\omega_k) = \mathbf{L}(\omega_k) \mathbf{L}(\omega_k)^T \mathbf{S}(\omega_k), 0 < k < N; \) in this decomposition, \(\mathbf{L}(\omega_k)\) is the lower triangular matrix. The amplitudes in Eq. (42) become

\[
A_{im}(\omega_k) = l_{im}(\omega_k) \sqrt{2 S(\omega_k) \Delta \omega}
\]

(44)

with \(l_{im}(\omega_k)\) being the elements of \(\mathbf{L}(\omega_k)\) and depending on the coherency. It is noted that the HOP method allows for a sequential simulation at the locations of interest. Although the phases for all locations are chosen randomly, the amplitudes for each station are chosen in a deterministic manner, so as to ensure that the currently generated time history satisfies the necessary coherency relations with all previously generated time histories.

It is also noted that the ergodicity properties of the time histories at the various locations as simulated by Eq. (42) differ, whereas the time history simulated at the first location is ergodic over the period of the simulations, the time histories at the subsequent locations are not. A way to remedy this deficiency is by utilizing a location-dependent frequency discretization that amounts to double-indexing of the frequencies [130]: The frequency domain is discretized at frequencies \(\omega_k, k = 1, \ldots, N\), for the simulation at the first location and the first term of the summation for all subsequent locations (Eq. (42)). The domain is then discretized at different frequencies \(\omega_{km}, k = 1, \ldots, N\), in the second term of the summation for the simulation at the second and all subsequent stations, and so on. Hence, frequencies are double-indexed, ie, \(\omega_{km}, m = 1, \ldots, i\), in Eq. (42). With a proper selection of \(\omega_{km}, m = 1, \ldots, i\), the simulated motions are ergodic over the longest period that results from the discretization [130].
Comparison of simulation techniques—For the simulation of ground accelerations based on the two techniques, the Clough-Penzien spectrum (Eq. (10)) and the coherency expression of Luco and Wong (Eq. (31)) are used again; the parameters in this case are: $S_0 = 1 \text{m}^2/\text{sec}^4$, $\omega_g = 5\pi \text{ rad/sec}$, $\omega_f = 0.1 \omega_g$, $\zeta_f = \zeta = 0.6$, i.e., same firm soil conditions as in the previous subsection, and $\alpha = 2.5 \times 10^{-4} \text{sec/m}$.

The averages of the unsmoothed power spectral densities (PSD) and cross spectral densities (CSD) of 100 realizations at a sequence of three locations along a straight line were evaluated by means of the two techniques; the distance between consecutive stations was 500 m. The mean PSD and CSD functions obtained from the simulations are compared with the target ones in Figs. 14 and 15. In particular, Fig. 14 presents the PSD at the two first stations and the CSD between the 1st and 2nd stations, and the 2nd and 3rd stations (500 m apart) evaluated by the CDRA method. Figure 15 presents the corresponding results for the simulations generated by means of the HOP method. The CDRA method (Fig. 14) reproduces well the prescribed spectral and cross spectral density functions as random functions with mean value equal to the corresponding target spectra and non-zero variance over the entire frequency domain. However, the HOP method has the interesting inherent feature that it produces “quasi-deterministic” spectra, meaning that, for some frequency range, the spectral values it produces are random numbers with mean equal to the target values, while, for another range of frequencies, they are deterministic (zero variance) and equal to the target values. In particular, the PSD at station 1 generated by the HOP method fully coincides with the target PSD; the PSD at station 2 (and, similarly, at station 3), gen-

![Fig. 14 Comparison of power (PSD) and cross (CSD) spectral estimates generated by the CDRA method with target ones](image1)

![Fig. 15 Comparison of power (PSD) and cross (CSD) spectral estimates generated by the HOP method with target ones](image2)
erated by HOP, produces non-deterministic spectra only in the frequency range from 0.3–3.0 Hz. This behavior is directly related to the variability in the amplitudes of the simulations generated by the HOP method:

Figures 16 and 17 present the mean values of the amplitudes of the 100 realizations at the first two stations as generated by the CDRA method (Fig. 16) and the HOP method (Fig. 17). In all subfigures, the target amplitude of the Clough-Penzien spectrum, evaluated as \( A(\omega) = \sqrt{2S(\omega)\Delta\omega} \), is also presented. It is also noted that, where the amplitudes behave as random variables, the mean amplitude is lower than the target one. This is because the amplitudes are random variables with non-zero variance; e.g., for the CDRA method, they result from the square root of the PSD function (Eqs. (37) and (41)), which follows a \( \chi^2 \) distribution with two degrees of freedom. For the CDRA method, the mean values of the amplitudes at both stations 1 and 2 (and, also, station 3) behave consistently as random variables. This is not the case for the mean amplitudes generated by the HOP method: The amplitudes of all simulations at station 1 coincide with the target amplitudes. For station 2 (and, similarly, station 3), the range where the amplitudes exhibit random characteristics is limited in the frequency range 0.3–3.0 Hz; outside this range, they are deterministic and identical to the target amplitudes. This may be explained as follows [107]: The time series at station 1 (first simulation in the sequence, \( i = 1 \) in Eq. (42)) is composed of only one cosine term at each frequency \( \omega_k \) with random phase and deterministic amplitude, given by: \( A_{11}(\omega_k) = l_{11}(\omega_k)\sqrt{2S(\omega_k)\Delta\omega} = \sqrt{2S(\omega_k)\Delta\omega} \) (Eq. (44)), as the value of the coherency of a time history with itself is equal to one, resulting in \( l_{11}(\omega_k) = 1 \). The time series at station 2 (second simulation in the sequence, \( i = 2 \) in Eq. (42)) is composed of two cosines at each frequency \( \omega_k \), with amplitudes...
A_{21}(\omega_k) = l_{21}(\omega_k) \sqrt{S(\omega_k)} \Delta \omega = \gamma_{21}(\omega_k) \sqrt{S(\omega_k)} \Delta \omega \quad \text{and} \quad A_{22}(\omega_k) = l_{22}(\omega_k) \sqrt{S(\omega_k)} \Delta \omega = 1 - \gamma_{21}(\omega_k) \sqrt{S(\omega_k)} \Delta \omega \quad \text{(Eq. (44))}

Above 3 Hz, and for the given values of the coherency drop parameter \( \alpha \) and the separation distance \( \xi \), the coherency becomes very low, so that \( l_{21}(\omega_k) = \gamma_{21}(\omega_k) \rightarrow 0 \).

The amplitude of the first cosine term, \( A_{21}(\omega_k) \) in Eq. (42), tends to zero, and the series is comprised, essentially, of only one cosine term at every frequency with amplitude \( A_{22}(\omega_k) = \sqrt{S(\omega_k)} \Delta \omega \). Consequently, the spectral densities and the amplitudes of the generated motions (Figs. 15 and 17) become identical to the target ones, as was the case for station 1.

Thus, the amplitudes of the simulated motions exhibit different properties depending on the order in which they are simulated. The amplitude variability in the motions generated by the HOP method, as well as their ergodicity properties discussed earlier, suggest that seismic ground motions simulated by this technique do not possess the same characteristics at all locations on the ground surface.

It should be noted that the behavior of the amplitude variability in the motions simulated by the HOP method (Fig. 17) is also, not consistent with the amplitude variability of actual recorded data, which vary randomly around the common component amplitude in the dominant frequency range of the motions (Figs. 8 and 9). On the other hand, the CDRA method, as well as the spectral representation method for the simulation of random fields described in the previous section, produce simulated motions with characteristics consistent with those of the recorded data [108]; an extensive discussion on the differential amplitude and phase variability resulting from the simulated motions evaluated by means of these techniques is presented in [107] and [108]. It is therefore cautioned that the characteristics of the simulation scheme should be carefully evaluated before any technique is used in the seismic response evaluation of lifelines.

6.3 Simulations of spatially variable seismic ground motions in engineering practice

The approaches described in the previous two subsections form the basis for the simulation of spatially variable seismic ground motions. The simulated motions, if properly evaluated, should reproduce the characteristics of the random field from which they were generated; as a consequence, simulated motions are stationary and homogeneous. These properties of the random field, although necessary for the evaluation of spatial coherency, are not desirable in the evaluation of seismic motions to be used in engineering applications as input motions at the supports of a lifeline. The following modifications can then be made, so that the simulated time histories exhibit spatially variable characteristics, but are also compatible with actual seismic ground motions:

**Introduction of non-stationarity in simulated motions**—Stationary time histories have neither beginning nor end (Section 2). In order to introduce finite duration in the generated motions, the time histories simulated by any of the aforementioned techniques (eg, Eq. (34)) are multiplied by modulating (envelope) functions, \( \alpha(t) \), namely,

\[
\tilde{f}(x,t) = \alpha(t) f(x,t)
\]

where the modulating function can be given by, eg [61]:

\[
\alpha(t) = \alpha_1 t \exp(-\alpha_2 t), \quad t \geq 0
\]

The parameters \( \alpha_1 \) and \( \alpha_2 \) in the above equation can be selected so that the function’s maximum is equal to 1 and occurs at the desired time during the strong motion part of the motions. Different modulating functions introduce explicitly the duration of the seismic motions: a model developed by Jennings et al [131] increases sharply at the beginning of the time history, remains constant for the duration of the strong motion, and decays exponentially thereafter. On a more sophisticated level, the modulating function can also be frequency dependent [132]. The generation of non-stationary, spatially variable simulated time histories has been reported, among others, in [7,109,112,132].

**Introduction of non-homogeneity in simulated motions**—As has already been indicated (Section 2), the random field of seismic data recorded at dense instrument arrays is considered as homogeneous, ie, the characteristics of the motions, and, subsequently, the coherency, are independent of absolute location and functions of separation distance only. Since the majority of the dense instrument arrays are located on fairly uniform soil conditions, the assumption of homogeneity is valid. However, the supports of an extended structure, such as a bridge, can be located in different soil conditions that are characterized by different frequency content. In this case, the point estimates of the motions become functions of absolute location as well. The variations in the frequency content of the point estimates of the motions have a very significant effect on the seismic response of extended structures [1,8]. In order to accommodate the necessity of having this fact reflected in the characteristics of the simulated input motions at the bridge supports, the simulations are generated as random vector processes rather than random fields [132]; this consideration allows the power spectral densities of the motions to vary at different locations.

**Compatibility with seismic codes**—As indicated in Section 2.2, the point estimates of the motions can be described by engineering models, such as the Kanai-Tajimi [31,32] or the Clough-Penzien spectrum [33], or seismicological ones [36]. In many cases, however, simulated seismic ground motions ought to be compatible with appropriate response spectra specified in design codes [133]. The simulation of spatially variable, response spectrum compatible seismic ground motions requires, generally, an iterative scheme, in which the power spectral density of the simulated motions is upgraded so as to match, to the degree possible, the prescribed seismic response spectra, eg, [1,8,132].

**Wave passage effects**—The wave passage effects can be introduced in the simulated time histories by either imposing a time delay compatible with the average apparent propagation velocity after the motions have been generated, eg, [7,132], or by incorporating the wave passage term of the spatial variability (Eq. (17)) in the description of the random field before the simulations are generated, eg, [78]. It should
be noted, however, that the arrival time perturbations discussed in Section 3.3 are not being considered in the simulation of seismic ground motions.

It needs to be emphasized at this point that the coherency functions used in all applications are not modified; they are used as they were developed from the recorded data, namely as stationary and homogeneous.

7 SUMMARY AND CONCLUSIONS

The spatial variation of seismic ground motions denotes the differences in the seismic time histories at various locations on the ground surface. Its significance for the seismic response of extended structures has been recognized since the early 1960s; at the time, however, it was attributed only to the wave passage effect. The modeling of the spatial variability, and, particularly, the coherency, initiated with the analysis of the first recorded data that became available from the SMART-1 array in Lotung, Taiwan.

The spatial variability of seismic ground motions, as understood from analyses of data recorded at dense instrument arrays, was described in this paper. The stochastic estimation of the spatial variability by means of the coherency, and the interpretation of the coherency in terms of the phase variability of the data was presented. Empirical and semi-empirical models for the coherency, their similarities, and, mostly, differences, as well as their effect on the seismic response of lifelines were briefly described. An alternative approach for the investigation of the spatial variability of seismic ground motions, that views spatial variability as deviations of amplitudes and phases at individual stations around a coherent approximation of the seismic motions was described. Simulations of spatially variable seismic ground motions to be used in the seismic response of lifelines were presented; the variability in the simulated motions resulting from the use of a particular spatial coherency model as well as from the use of the simulation scheme itself have been illustrated. Approaches for the modification of the simulated seismic time histories, so that they become compatible with actual ones, were also highlighted.

The investigation of the spatial variability and its effects on lifelines has recently attracted renewed interest, that can be partially attributed to the lessons learned during the devastating earthquakes of the last decade [6]. Unresolved issues involving spatial variability include its physical modeling and its use in simplified design recommendations for all extended structures. Present basic and applied research on the topic is moving in this direction.

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