1. Consider testing $H_0: \mu \geq 40$ vs. $H_1: \mu < 40$ where $\mu$ represents the mean of a normally distributed population having known standard deviation $\sigma = 10$. The test will be based on a sample of size $n = 25$, with significance level $\alpha = 0.01$. In terms of the sample mean $\bar{X}$, what is the decision rule (rejection region) for this test? Your answer should look like ‘reject $H_0$ if $\bar{X} > 84.27$’ or ‘reject $H_0$ if $\bar{X} < 21.33$’, etc. On the $z$-scale, you’d reject $H_0$ if $z < -2.33$, so you’d reject if the sample result is more than 2.33 standard deviations below the mean. On the $\bar{X}$ scale, that translates to $40 - 2.33(10/\sqrt{25}) = 35.34$ (your answer may vary a little due to rounding). So we reject $H_0$ if $\bar{X} < 35.34$.

2. Consider testing $H_0: \mu \leq 60$ vs. $H_1: \mu > 60$ where $\mu$ represents the mean of a normally distributed population having known standard deviation $\sigma = 5$. The test will be based on a sample of size $n = 30$, with significance level $\alpha = 0.05$. The decision rule for this test is to reject $H_0$ if $\bar{X} > 61.50$.

Calculate the power of this test when $H_1$ is true and $\mu$ is actually equal to 62. This is a normal curve probability problem: you want the probability that $\bar{X} > 61.50$, where $\bar{X}$ has a normal distribution with mean equal to 62 and standard deviation $5/\sqrt{30} = 0.9129$. Draw the sketch, calculate $z = -0.55$. Solution is $0.5 + 0.2088 = 0.7088$.

3. Consider testing $H_0: \pi \geq 0.20$ vs. $H_1: \pi < 0.20$, based on a sample of size $n = 240$. Suppose that you're told that, when $H_1$ is true and the actual population proportion is $\pi = 0.16$, the power of the test is 0.6266. This tells us that:

(a) The probability of making a Type II error is 0.6266.

(b) The probability of rejecting $H_0$ when in fact $\pi = 0.16$ is 0.6266.

(c) The probability of not rejecting $H_0$ when in fact $\pi = 0.16$ is 0.6266.

(d) The probability of rejecting $H_0$ when in fact $\pi = 0.20$ is 0.6266.

(e) The probability of not rejecting $H_0$ when in fact $\pi = 0.20$ is 0.6266.
4. Consider testing $H_0: \mu \geq 25$ vs. $H_1: \mu < 25$ where $\mu$ is the mean of a normally distributed population. Which sketch shows the shape of the power function for this test? It’s (a). Power must increase as you move deeper into the “H1 region”, which in this case means as you move further from 25 to the left.

(a) ![Sketch A]

(b) ![Sketch B]

(c) ![Sketch C]

Two different machines are used to automatically fill “16 oz.” boxes of Special Z cereal (the actual fill weights vary, as the machines aren’t perfect). When operating properly, both machines should have the same variation in fill weights. To check this we test $H_0: \sigma_1^2 = \sigma_2^2$ vs. $H_1: \sigma_1^2 \neq \sigma_2^2$. The test will use the following data, taken from a random sample of filled boxes from each machine’s output. Fill weights are assumed to be normally distributed.

$n_1 = 8 \quad \bar{x}_1 = 16.033$ oz. \quad s_1 = 0.47$ oz. \quad n_2 = 10 \quad \bar{x}_2 = 16.219$ oz. \quad s_2 = 0.93$ oz.

5. The calculated value of the appropriate test statistic is: It’s the ratio of the larger to the smaller sample variance: $(0.93^2/0.47^2) = 3.92$.

(a) 0.2554 \hspace{1cm} (b) 0.4177 \hspace{1cm} (c) 0.5054 \hspace{1cm} (d) 0.8321

(e) 1.9787 \hspace{1cm} (f) 2.4476 \hspace{1cm} (g) 3.9153 \hspace{1cm} (h) 4.2388

6. At the $\alpha = 0.05$ level of significance, this test would reject the null hypothesis if the calculated test statistic is greater than: It’s the upper 0.025 tail of the F-distribution with 10 – 1 = 9 DF in the numerator and 8 – 1 = 7 DF in the denominator. That’s 4.82 from the table.

(a) 2.45 \hspace{1cm} (b) 3.29 \hspace{1cm} (c) 3.68 \hspace{1cm} (d) 4.20

(e) 4.82 \hspace{1cm} (f) 5.14 \hspace{1cm} (g) 5.57 \hspace{1cm} (h) 6.63

It is also important that the average fill weight is the same for both machines, so we wish to test: $H_0: \mu_1 = \mu_2$ vs. $H_0: \mu_1 \neq \mu_2$. Suppose that the specified level of significance is $\alpha = 0.05$, and that we can assume that the variability in fill weights is the same for both machines.

(continued next page)
7. The calculated value of the appropriate test statistic is given by: *Just crank out the value of the two-sample t-test statistic.* It works out to -0.51.

<table>
<thead>
<tr>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3.89</td>
<td>-2.88</td>
<td>-1.97</td>
<td><strong>-0.51</strong></td>
</tr>
<tr>
<td>0.27</td>
<td>0.74</td>
<td>1.22</td>
<td>1.76</td>
</tr>
</tbody>
</table>

8. This test would reject $H_0$ if the calculated test statistic is: *It's a two-sample t-test with $8 + 10 – 2 = 16$ DF. Reject in the upper or lower 2.5% tails of the distribution.*

<table>
<thead>
<tr>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;-2.33 or &gt;+2.33</td>
<td>&lt;-1.96 or &gt;+1.96</td>
<td>&lt;-1.645 or &gt;+1.645</td>
<td>equal to 7</td>
</tr>
<tr>
<td>&lt;-2.120 or &gt;+2.120</td>
<td>&lt;-2.110 or &gt;+2.110</td>
<td>&lt;-1.740 or &gt;+1.740</td>
<td>&lt;-1.746 or &gt;+1.746</td>
</tr>
</tbody>
</table>

9. It is becoming more common for bank customers to do the majority of their banking activities on the web. A major bank is interested in testing a manager’s claim that customers who do their banking on the web are happier with the bank than those who do their banking in person.

Let $\pi_1$ represent the proportion of ‘web’ customers who are highly satisfied with the bank’s services, and let $\pi_2$ represent the proportion of ‘in person’ customers who are highly satisfied with the bank’s services. We’d like to collect perform a study to see if sample results support the manager’s claim. The appropriate hypothesis test setup is…..

*It’s (c). The manager’s claim is that $\pi_1 > \pi_2$. We want to know if the data supports the claim, so it goes in $H_1$.***

<table>
<thead>
<tr>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0 : \pi_1 - \pi_2 \geq 0$ vs. $H_1 : \pi_1 - \pi_2 &lt; 0$</td>
<td>$H_0 : \pi_1 - \pi_2 &gt; 0$ vs. $H_1 : \pi_1 - \pi_2 \leq 0$</td>
<td>$H_0 : \pi_1 - \pi_2 \leq 0$ vs. $H_1 : \pi_1 - \pi_2 &gt; 0$</td>
<td>$H_0 : \pi_1 - \pi_2 &lt; 0$ vs. $H_1 : \pi_1 - \pi_2 \geq 0$</td>
</tr>
<tr>
<td>$H_0 : \pi_1 - \pi_2 = 0$ vs. $H_1 : \pi_1 - \pi_2 \neq 0$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

10. To determine whether treatment effects exist, the analysis of variance procedure compares variation caused by two different sources. These sources are:

<table>
<thead>
<tr>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>variation due to a treatment effect and variation due to random error</strong></td>
<td>variation due to a treatment effect and variation due to measurement error</td>
<td>variation due to measurement error and variation due to random error</td>
<td>variation due to a data entry error and variation due to random error</td>
<td>variation due to a treatment effect and variation due to dosage level error</td>
</tr>
</tbody>
</table>
In order to determine which of four store displays best motivates customers to buy more of a product, an experiment is conducted by a large chain of grocery stores. Six stores are randomly selected to receive Display 1, six to receive Display 2, and so on. The displays are put up for one week, and the response variable of interest is the number of cases of the product which are sold. An analysis of variance is then conducted.

Here's a partial Minitab printout for this problem:

**One-way Analysis of Variance**

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor</td>
<td>3</td>
<td>9.70</td>
<td>3.2333</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>20</td>
<td>51.42</td>
<td>2.5710</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>23</td>
<td>61.13</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We’re naturally interested in testing $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$ vs. $H_1$: Not all means equal.

11. This test would reject the null hypothesis if the calculated F-statistic exceeds what value? (Write correct answer on solution sheet.) I should have given you an $\alpha$-value here (oops!). Let’s use 0.05. You’d reject in the upper 5% tail of the F-distribution having 3 and 20 DF. That’s equal to 3.10.

12. The value of the test statistic is equal to: The ‘mean squares’ are $9.70/3 = 3.2333$ and $51.42/20 = 2.5710$. Take the ratio to obtain 1.26.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.14</td>
<td>0.19</td>
<td>0.84</td>
<td>1.04</td>
<td><strong>1.26</strong></td>
<td>1.88</td>
<td>2.73</td>
<td>3.15</td>
<td>4.28</td>
</tr>
</tbody>
</table>
Solutions:

1. Reject H₀ if ___________________________

2. _____  3. _____  4. _____  5. _____  6. _____


12. _____
Formulas:

Hypothesis testing:

\[ z = \frac{\overline{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{\overline{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} ; \quad \overline{X} = \mu_0 + z \frac{\sigma}{\sqrt{n}} \]

Two-Sample Tests:

\[ t = \frac{\overline{x}_1 - \overline{x}_2}{\sqrt{s^2_p \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}} \quad \text{where} \quad s^2_p = \frac{(n_1 - 1)s^2_1 + (n_2 - 1)s^2_2}{n_1 + n_2 - 2} \quad \text{DF} = n_1 + n_2 - 2 \]

or

\[ z = \frac{\overline{x}_1 - \overline{x}_2}{\sqrt{s^2_1 / n_1 + s^2_2 / n_2}} \quad \text{or} \quad z = \frac{\overline{x}_1 - \overline{x}_2}{\sqrt{\frac{\sigma^2_1}{n_1} + \frac{\sigma^2_2}{n_2}}} \]

\[ t = \frac{\overline{d}}{s_d / \sqrt{n}} \quad \text{Degrees of freedom:} \quad n - 1 \]

\[ z = \frac{p_1 - p_2}{\sqrt{\overline{p}(1 - \overline{p}) \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}} \quad \text{where} \quad \overline{p} = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} \]

\[ F = \max \left\{ \frac{s^2_1}{s^2_2}, \frac{s^2_2}{s^2_1} \right\} \]

One-Way Analysis of Variance:

\[ F = \frac{MSTR}{MSE} = \frac{SSTR / (t - 1)}{SSE / (N - t)} \quad \text{Degrees of freedom:} \quad (t - 1, N - t) \]

(Formulas for randomized block designs are not given: we’ll just work with computer output.)

Standard deviation:

\[ s_d = \sqrt{\frac{\sum (d_i - \overline{d})^2}{n - 1}} = \sqrt{\frac{\sum d_i^2 - n \overline{d}^2}{n - 1}} \]