Let $Z$ represents the standard normal random variable. Calculate the following probabilities to four decimal places.

1. $P\{Z < 1.77\} = 0.4616 + 0.5000 = 0.9616$
2. $P\{-1.34 < Z < 1.88\} = 0.4099 + 0.4699 = 0.8798$

Correct answers to multiple choice questions are in italics!

Suppose that savings account balances at First International Bank are normally distributed with a mean of $\mu = 242$ and a standard deviation of $\sigma = 74$.

3. What proportion of accounts have a balance of at least $100$? Calculate $z = (100 – 242)/74 = -1.92$. Solution is $0.4726 + 0.5000 = 0.9726$.

<table>
<thead>
<tr>
<th>A</th>
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</tr>
</thead>
<tbody>
<tr>
<td>0.0274</td>
<td>0.1328</td>
<td>0.1827</td>
<td>0.4726</td>
<td>0.5188</td>
<td>0.7172</td>
<td>0.8334</td>
<td><strong>0.9726</strong></td>
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</tbody>
</table>

4. Find the 85th percentile for the distribution of account balances. (Values are in $\$$.) Draw sketch: you need a cutoff with 0.85 area below so the ‘inner’ area is 0.35. Corresponding $z$-value is 1.04 standard deviations above the mean. Solution is $242 + 1.04(74) = 318.96$.

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<tbody>
<tr>
<td>114.72</td>
<td>208.98</td>
<td>270.86</td>
<td><strong>318.96</strong></td>
<td>422.18</td>
<td>457.22</td>
<td>488.13</td>
<td>505.71</td>
</tr>
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</table>

5. A random sample of $n = 16$ accounts is taken. What is the probability that the mean account balance for the sample is at least $260$? It’s a question about the sample mean $\bar{X}$. $\bar{X}$ has a normal distribution (because individual balances do) with mean of $242$ and standard error $74/\sqrt{16} = 18.50$. Calculate $z = (260 – 242)/18.50 = 0.97$. Solution is $0.5000 – 0.3340 = 0.1660$.

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<tbody>
<tr>
<td><strong>0.1660</strong></td>
<td>0.3178</td>
<td>0.4052</td>
<td>0.5948</td>
<td>0.6144</td>
<td>0.8340</td>
<td>0.9172</td>
<td>0.9876</td>
</tr>
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</table>
6. 80% of all graduating seniors in the United States feel that starting salary is the most important factor when choosing among several offers for their first full-time job. If we take a random sample of 200 graduating seniors, what is the probability that at least 75% of the seniors in the sample feel that starting salary is the most important factor? It’s a question about a sample proportion $p$. Population proportion $\pi = 0.80$. Sample size $n = 200$. Since $n\pi$ and $n(1-\pi)$ both exceed 5, $p$ is approximately normally distributed with mean 0.80 and standard error $\sqrt{(0.8(0.2)/200)} = 0.02828$. Calculate $z = (0.75 - 0.80)/0.02828 = -1.77$. Solution = 0.5000 + 0.4616 = 0.9616.

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<th>I</th>
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</tr>
</thead>
<tbody>
<tr>
<td>0.0114</td>
<td>0.0288</td>
<td>0.0384</td>
<td>0.2188</td>
<td>0.3177</td>
<td>0.6823</td>
<td>0.7812</td>
<td><strong>0.9616</strong></td>
<td>0.9712</td>
<td>0.9886</td>
</tr>
</tbody>
</table>

7. A production process has been used for many years and is known to produce goods at the average rate of 1.2 minutes per item. A recently hired MBA with an emphasis in operations has come up with a proposal, involving rebalancing the line and reassigning tasks, which she claims will reduce the current average. Management is skeptical, and will not switch to the proposed approach unless it is presented with convincing evidence that a reduction does indeed take place.

Let $\mu_{\text{new}}$ represent the production rate using the proposed new approach. The correct hypothesis test setup for this situation is:

(a) $H_0: \mu_{\text{new}} = 1.2$ vs. $H_1: \mu_{\text{new}} \neq 1.2$
(b) $H_0: \mu_{\text{new}} \leq 1.2$ vs. $H_1: \mu_{\text{new}} > 1.2$
(c) $H_0: \mu_{\text{new}} < 1.2$ vs. $H_1: \mu_{\text{new}} \geq 1.2$
(d) $H_0: \mu_{\text{new}} \geq 1.2$ vs. $H_1: \mu_{\text{new}} < 1.2$  
   We’re putting the ‘burden of proof’ on the data to convince us that a reduction has occurred. Therefore, ‘reduction occurred’ goes in $H_1$.
(e) $H_0: \mu_{\text{new}} > 1.2$ vs. $H_1: \mu_{\text{new}} \leq 1.2$

8. In hypothesis testing, a Type I error is made if we:

(a) Reject the null hypothesis when in fact it is true
(b) Fail to reject the null hypothesis when in fact it is false
(c) Take too few samples to reach a meaningful conclusion
(d) Take a sample which is not random, leading to the possibility of a biased conclusion
9. Currently, V6 engines built by the Acme Engine Company (AEC) have a mean lifetime that is known to be 122,000 miles. An engineering team has come up with a new design which uses fewer moving parts and will therefore be much less expensive to build. AEC is predisposed to switch to the new design, and will do so unless testing of prototypes provides evidence that the new design reduces the mean lifetime. Therefore, the appropriate hypothesis test setup is: $H_0: \mu \geq 122,000$ vs. $H_1: \mu < 122,000$.

In this situation, if we make a Type II error then we:

(a) conclude that the new engine has a lower life when it fact it doesn't. We miss the opportunity to switch to a less expensive design when we should switch.

(b) conclude that the engine actually increases engine life when in fact it reduces it. We switch to the new design and advertise that the new engines last longer than the old ones, when in fact they don't.

(c) conclude that the new engine does not reduce engine life when it fact it does. We go ahead and switch to a new engine that doesn't last as long as the old one. (We conclude $H_0$ when in fact $H_1$ is true.)

Consider testing $H_0: \mu \leq 40$ vs. $H_1: \mu > 40$ based on a random sample of size $n = 40$ from a population having known standard deviation $\sigma = 2.5$.

10. At the $\alpha = 0.05$ level of significance, this test would reject the null hypothesis if the calculated $z$-statistic is: Look up the upper 5% tail of the z-distribution: critical value is +1.645.

(a) < -1.28 (b) > 1.28 (c) < -1.645 (d) > 1.645 (e) < -1.96 (f) > 1.96 (g) < -2.575 (h) > 2.575

11. If the 40 observations are made and the resulting sample mean is $\bar{X} = 40.37$, the value of the calculated $z$-statistic is: $\text{Calculate } z = (40.37 - 40)/(2.5/\sqrt{40}) = 0.936$.

(a) -0.148 (b) 0.148 (c) -2.347 (d) 2.347 (e) -1.283 (f) 1.283 (g) -0.936 (h) 0.936
12. When a production process is working properly, piston rings made using the process have a mean diameter of 48.5000 mm. If there is a problem with the process, this mean may shift to some other value. To determine whether there is a problem, we test $H_0: \mu = 48.5000$ vs. $H_1: \mu \neq 48.5000$. The test will be based on a sample of $n = 12$ rings, and will be conducted at the $\alpha = 0.01$ level of significance. Piston ring diameters are normally distributed.

The test results in the following sample results: $\bar{X} = 48.5016; s = 0.0032$. The test conclusion is to: It’s a $t$-test since the population standard deviation $\sigma$ is unknown and must be estimated by the sample standard deviation $s$. Calculate $t = (48.5016 - 48.5)/(0.0032/sqrt(12)) = 1.73$. Critical value for this two-sided test comes from $t$-table with $12 - 1 = 11$ DF. Reject in the upper and lower 0.005 tails, so the critical values are $\pm 3.106$. So we do not reject $H_0$ since $1.73 < 3.106$.

(a) do not reject $H_0$ since $z = 1.732 < 2.326$
(b) do not reject $H_0$ since $z = 1.732 < 2.576$
(c) do not reject $H_0$ since $t = 1.732 < 2.681$
(d) do not reject $H_0$ since $t = 1.732 < 3.055$
(e) do not reject $H_0$ since $t = 1.732 < 2.718$
(f) do not reject $H_0$ since $t = 1.732 < 3.106$

**Quiz 1 Formulas:**

**Normal Distribution Conversion Formulas:**

$$Z = \frac{X - \mu}{\sigma} \quad X = \mu + \sigma Z$$

**Sampling Distributions**

<table>
<thead>
<tr>
<th>$\mu_X = \mu$</th>
<th>$\sigma_X = \frac{\sigma}{\sqrt{n}}$</th>
<th>$z = \frac{\bar{X} - \mu_X}{\sigma_X}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_p = \pi$</td>
<td>$\sigma_p = \sqrt{\frac{\pi(1 - \pi)}{n}}$</td>
<td>$z = \frac{p - \pi}{\sigma_p}$</td>
</tr>
</tbody>
</table>

**Hypothesis testing:** $z = \frac{\bar{X} - \mu_0}{\sigma \bar{X}} = \frac{\bar{X} - \mu_0}{\sigma \sqrt{\frac{1}{n}}}$; $t = \frac{\bar{X} - \mu_0}{s \sqrt{\frac{1}{n}}}$ (Degrees of freedom for $t = n - 1$)