Part I. Multiple Choice. Please circle the letter corresponding to the correct answer. (2 points each)

1. The constraint "X1 <= 7" in an LP problem having two decision variables:
   A. limits the feasible region to being a line segment
   B. has no effect on the feasible region
   C. would be represented graphically by a horizontal line
   D. would be represented graphically by a vertical line

2. Changing an objective function coefficient in a linear programming problem has what effect on the feasible region?
   A. no effect
   B. the feasible region gets larger
   C. the feasible region gets smaller
   D. there would be no feasible region

3. When you are formulating a linear programming model the first step is to
   A. state the objective function
   B. state the model constraints
   C. define the decision variables
   D. find the solution

4. The region that satisfies all of the constraints in graphical linear programming is called the:
   A. optimum point
   B. region of optimality
   C. profit maximization space
   D. region of non-negativity
   E. feasible region

5. In graphical linear programming, when the objective function line (also known as isoprofit or isocost line) is parallel to one of the constraint lines, then:
   A. the solution is suboptimal
   B. multiple optimal solutions may exist
   C. a single corner point solution exists
   D. no feasible solution exists
   E. none of the above
Part II: Problems

Show your work! Your job is to convince the Professor that you have learned the process needed to solve a problem.

1. (3 points) At the optimal solution point, a linear programming minimization problem currently has an objective function value of $1,000. The dual price for the third constraint is $50. Suppose that the right-hand-side of the constraint will be reduced by three units and we're interested in the impact on the objective function value.

Assuming that this reduction is within the dual price's allowable decrease, what's the new value of the objective function?

2. (20 pts.) Better Products, Inc., is a small manufacturer of three products that it produces on two machines. In a typical week, 40 hours of machine time are available on each machine and 100 hours of labor are available. Unit profit contributions for products 1, 2, and 3 are $40, $90, and $35, respectively. Let $X_1$ represent the number of units of product 1 produced, $X_2$ represent the number of units of product 2 produced, and $X_3$ represent the number of units of product 3 produced. Taking resource utilization information into account, Better Products' production manager is faced with the following product mix problem:

$$\text{MAX} \quad 40 \, X_1 + 90 \, X_2 + 35 \, X_3$$
$$\text{SUBJECT TO}$$
$$0.5 \, X_1 + 2 \, X_2 + 0.75 \, X_3 \leq 40 \quad \text{(Machine #1 Available Hrs.)}$$
$$X_1 + X_2 + 0.5 \, X_3 \leq 40 \quad \text{(Machine #2 Available Hrs.)}$$
$$2 \, X_1 + 5 \, X_2 + 2 \, X_3 \leq 100 \quad \text{(Available Labor Hrs.)}$$
$$X_1, \, X_2, \, X_3 \geq 0$$

Continued.....

Show your work!
Justify your answers!
Here’s the problem formulation and LINDO's solution, with a few results replaced with '?'.
Assume that fractional units are acceptable.

\[
\text{MAX} \quad 40 \, X_1 + 90 \, X_2 + 35 \, X_3 \\
\text{SUBJECT TO} \\
2) \quad 0.5 \, X_1 + 2 \, X_2 + 0.75 \, X_3 \leq 40 \quad \text{Machine \#1} \\
3) \quad X_1 + X_2 + 0.5 \, X_3 \leq 40 \quad \text{Machine \#2} \\
4) \quad 2 \, X_1 + 5 \, X_2 + 2 \, X_3 \leq 100 \quad \text{Labor} \\
\text{END}
\]

LP OPTIMUM FOUND AT STEP 2

OBJECTIVE FUNCTION VALUE
1) \quad 1933.333

VARIABLE \quad VALUE \quad REDUCED COST
X_1 \quad 33.333332 \quad 0.000000
X_2 \quad 6.666667 \quad 0.000000
X_3 \quad 0.000000 \quad ?

ROW SLACK OR SURPLUS DUAL PRICES
2) \quad ? \quad ?
3) \quad 0.000000 \quad 6.666667
4) \quad 0.000000 \quad 16.666666

NO. ITERATIONS = 2

RANGES IN WHICH THE BASIS IS UNCHANGED:

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>CURRENT COEF</th>
<th>ALLOWABLE INCREASE</th>
<th>ALLOWABLE DECREASE</th>
</tr>
</thead>
<tbody>
<tr>
<td>X_1</td>
<td>40.000000</td>
<td>49.999996</td>
<td>4.000000</td>
</tr>
<tr>
<td>X_2</td>
<td>90.000000</td>
<td>9.999999</td>
<td>4.999996</td>
</tr>
<tr>
<td>X_3</td>
<td>35.000000</td>
<td>1.666665</td>
<td>INFINITY</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ROW</th>
<th>CURRENT RHS</th>
<th>ALLOWABLE INCREASE</th>
<th>ALLOWABLE DECREASE</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>40.000000</td>
<td>INFINITY</td>
<td>10.000000</td>
</tr>
<tr>
<td>3</td>
<td>40.000000</td>
<td>9.999999</td>
<td>20.000000</td>
</tr>
<tr>
<td>4</td>
<td>100.000000</td>
<td>20.000000</td>
<td>19.999998</td>
</tr>
</tbody>
</table>

Show your work!
Justify your answers!
Use the results on the previous page to answer the following questions. Assume that, whatever the situation, the profit-maximizing profit mix will naturally be used. If you do not have enough information to answer a question, say "Not enough info" and also, when possible, tell me what you CAN say, based on the information given. **Be sure to justify your answers: I need to know that you know what printout values to use to reach your conclusions. No justification, no credit!**

(a) Suppose that we can acquire 10 additional hours of labor time at current costs. What is the effect on the value of the objective function?

(b) Suppose that we're told that 25 hours of Machine #2 time has been reassigned to another department and is no longer available to us. What is the effect on the value of the objective function?

(c) What are the values for the slack and for the dual price for the first constraint?

    Slack _____________       Dual Price _____________

(d) What is the reduced cost for Product 3?

(e) For each of the following changes, give the impact on the solution point and on the objective function value. Changes are all from the original base values,... they are **not** cumulative.

(i) Product 1 profit contribution changes from $40 to $30.

(ii) Product 2 profit contribution changes from $90 to $95.

*Show your work!*  
*Justify your answers!*
(iii) The number of available Machine 2 hours increases by 5 and the number of available labor hours decreases by 15.

(f) Suppose that, by managerial decree, at least 25% of the total production (measured in units) must be product 2 units. To take this into account a new constraint would need to be added to the problem.

(i) State the appropriate constraint. For full credit, you'll need to put it in a form that is acceptable to LINDO.

(ii) Will adding this constraint change the optimal solution point? Why or why not? (Note: You might be able to answer this, even if you couldn't answer Part (i).)
3. (6 pts.) Solve the following linear programming problem graphically. Be sure to clearly indicate the feasible region and to show all of your work. (I should be able to determine which solution procedure you’re using and that you know how to apply it.) To help you out, two of the constraint lines have already been drawn in.

Maximize: \[ 3X_1 + 4X_2 \]

Subject To:  
(A) \[ 4X_1 + 8X_2 \geq 32 \]  
(B) \[ 2X_1 + 3X_2 \leq 31 \]  
(C) \[ 2X_1 + X_2 \leq 13 \]  
(D) \[ X_2 \leq 6 \]  
\[ X_1, X_2 \geq 0 \]

Solution Point: ________________   OV: ________________

Show your work!  
Justify your answers!
4. (10 pts.) Consider the following product mix problem:

\[ X_1 = \text{number of units of Product #1 to produce} \]
\[ X_2 = \text{number of units of Product #2 to produce} \]

Maximize Profit ($): \[ Z = 18X_1 + 12X_2 \]

Subject to:

\[ 20X_1 + 10X_2 \leq 200 \quad \text{(200 lbs. of steel are available)} \]
\[ 9X_1 + 22X_2 \leq 195 \quad \text{(195 hours of labor are available)} \]
\[ X_2 \leq 8 \quad \text{(maximum forecasted demand for Product 2 is 8 units)} \]
\[ X_1 + X_2 \geq 4 \quad \text{(Must make at least four units of product)} \]
\[ X_1, X_2 \geq 0 \]

The solution to this problem is given by: \( X_1 = 7, X_2 = 6 \) for a profit of $198. Here’s the graph:

Show your work!
Justify your answers!
(a) Re-write the LP formulation in standard form.

(b) The current profit contribution for Product #2 is $12. What would this value have to change to in order for the optimal strategy to change to a solution that makes more units of Product #1 and fewer units of Product #2? Show your work!

(c) Find the dual price for the steel constraint. Show your work!

Show your work!
Justify your answers!