Quantitative x-ray differential-interference-contrast microscopy with independently adjustable bias and shear

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We present a quantitative x-ray phase imaging method that can be readily implemented on existing x-ray microscopy facilities. This technique utilizes Fresnel zone plates both as imaging optical elements for magnification and as second-order grating structures for phase-shifting interferometry. By making high-resolution quantitative x-ray phase information widely available, we expect this work to have significant impact on nanoscale biological and material studies.

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I. INTRODUCTION

X-ray phase contrast imaging enables the visualization of highly transparent objects that cannot otherwise be imaged by conventional absorption contrast microscopes [1]. Numerous x-ray phase techniques have been proposed and demonstrated, including interferometry [2–4], ptychography [5,6], and transfer-function methods [7,8]. Characteristics of these imaging techniques span a wide spectrum in terms of field of view, spatial resolution, penetration depth, and exposure time. Modalities capable of nanometer-resolution quantitative phase imaging using existing x-ray microscope facilities are of particular interest, especially in the area of biological and magnetic studies [9].

Differential-interference-contrast (DIC) microscopy is widely used in biological sciences to image highly transparent samples. At shorter wavelengths, the combination of DIC and x-ray microscopy [10–13] allows one to observe transparent phase objects with nanometer resolution [14,15]. Such a combination was first demonstrated by Wilhein et al. using a twin zone plate as the DIC objective lens [15]. Several other DIC objectives were also designed and implemented, although existing x-ray DIC microscopes have not yet demonstrated quantitative phase imaging as did their visible-light counterpart [14,16–18]. Here we introduce a phase-shifted two-zone-plate design capable of quantitative x-ray phase and amplitude imaging. The proposed DIC objective consists of two laterally shifted zone plates, each with an arbitrary amount of phase shift defined by its relative ring positions. Since a zone plate is essentially a second-order grating structure, shifting its rings alters the phase at the focus while keeping the focal intensity distribution unchanged. By varying the ring positions, this design permits independent phase control and, as a result, enables phase-shifting interferometry for quantitative imaging of complex objects at the x-ray wavelengths.

II. ZONE-PLATE DOUBLET AS DIC OBJECTIVE

DIC microscopy is an interferometric imaging modality where image intensity is derived from interfering two laterally sheared and often phase-shifted images of the object. Accordingly, the quality of the DIC image is affected by two fundamental parameters: lateral shear and bias retardation, which designate the separation in space and the added phase shift between the two sheared images, respectively [19–21]. Phase objects undetectable by conventional absorption-contrast microscopy become visible due to interference. Smaller lateral shears, approximately the size of the microscope’s spatial resolution, are desired for sharper images. Optimization of image contrast can be achieved by adjusting the bias retardation. In addition, by incorporating controlled amounts of bias retardation into the DIC images, phase-shifting interferometry can be employed for quantitative phase retrieval [22,23].

Several DIC objectives, such as the twin zone plate and the XOR pattern have been implemented on x-ray microscopes for qualitative phase imaging [15,24]. For the twin-zone-plate design, two exactly identical zone plates are used, and the effect of bias retardation was not explored [15]. The XOR objective, although allowing independent control over bias retardation, cannot be used for interferometry due to the existence of multiple first-order foci [24]. In contrast, the proposed DIC objective produces only two first-order foci with designed amounts of bias retardation and therefore permits direct application of the phase-shifting interferometry algorithms for quantitative phase extraction.

At the x-ray wavelengths, Fresnel zone plates are commonly employed as the focusing element. Although for a single Fresnel zone plate the phase of its wave front at the focus is essentially irrelevant in terms of image formation, the relative phase difference between the foci of two laterally sheared zone plates, i.e., a zone-plate doublet, does have a significant effect on the resultant interference image. To illustrate this point, a Fresnel zone plate can be expressed mathematically as [24]

$$Z(x, y) = \frac{1}{2} \left[ 1 + \text{sgn} \left[ \cos \left( \frac{\pi r^2}{\Delta r (D - \Delta r)} + \phi \right) \right] \right]$$

(1)

where $r = \sqrt{x^2 + y^2}$, $\phi$ designates the shifting of its rings in the radial direction, and $\Delta r$ and $D$ are the outermost zone width and diameter of the zone plate, respectively. First-order focal length $f_1$ of the zone plate is $\Delta r (D - \Delta r)/\lambda$, where $\lambda$ is the wavelength [1]. Note that varying amounts of $\phi$ translate...
FIG. 1. (Color online) Zone-plate doublet (ZPD) as a DIC objective. The zone-plate doublet is obtained by multiplying two laterally sheared and phase-shifted zone plates, i.e., \( Z_1, Z_2 \). Its lateral shear \( \Delta s \) is defined by the center-to-center separation between the two component zone plates, and its bias retardation \( 2\Delta \theta \) equals the relative phase difference between \( Z_1 \) and \( Z_2 \).

to spatially shifting zone-plate rings, and the phase of the field at the first-order focus is, indeed, given by \( e^{-j\phi} \). Following the same notation, two laterally sheared and phase-shifted Fresnel zone plates, \( Z_1 \) and \( Z_2 \), can be written as [25],

\[
Z_{1,2}(x,y) = \frac{1}{2} \left[ 1 + \text{sgn} \left( \cos \left( \gamma r_{1,2}^2 \pm \Delta \theta \right) \right) \right], \tag{2}
\]

where \( \Delta s \) is the lateral shear, \( r_{1,2}^2 = (x \pm \Delta s/2)^2 + y^2 \), and \( \pm \Delta \theta \) are the phase shifts of the two zone plates. Note that the notation \( \gamma = \pi/\left( \Delta r (D - \Delta r) \right) \) has been used for simplicity. As defined above, the two zone plates are laterally separated by \( \Delta s \), and each has a phase shift of \( \pm \Delta \theta \), respectively. The DIC objective is obtained by multiplying these two zone plates \( Z_1 \) and \( Z_2 \) into one diffractive pattern (Fig. 1). The relative phase difference between the two zone plates, \( 2\Delta \theta \), is now, by definition, the bias retardation of the DIC objective, and \( \Delta s \) is the lateral shear. Independent control over bias retardation and lateral shear can be achieved by specifically selecting the two design parameters \( \Delta \theta \) and \( \Delta s \).

The zone-plate doublet \( Z_D(x,y) \), defined as the multiplication of \( Z_1 \) and \( Z_2 \), can therefore be expressed as (see the Appendix),

\[
Z_D(x,y) \triangleq Z_1(x,y)Z_2(x,y) = \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \sin(m\pi/2) \sin(n\pi/2) \frac{\sin(m\pi/2)}{m\pi} \frac{\sin(n\pi/2)}{n\pi} e^{-j(m+n)\gamma(x^2+y^2)} e^{-j(m-n)\gamma(\Delta y x + \Delta \theta)} e^{-j(m+n)\gamma(\Delta \theta)^2/4}. \tag{3}
\]

As seen in Eq. (3), even orders of the zone-plate doublet vanish except when \( (m,n) = (0,0) \). The two first-order foci are given by \( (m,n) = (0,1) \) and \( (1,0) \), respectively. Numerical simulation shows that this DIC optical element indeed produces two first-order foci separated by \( \Delta s \) at the first-order focal plane, and the bias retardation is, as expected, the phase difference between the two component zone plates (Fig. 2). As will be demonstrated below, these two first-order foci enable the use of phase-shifting interferometry on x-ray DIC microscopes for quantitative x-ray phase imaging.

III. PHASE-SHIFTING INTERFEROMETRY USING THE ZONE-PLATE DOUBLET AS THE OBJECTIVE LENS

Like Fresnel zone plates [24], the first-order foci of the zone-plate doublet are used for imaging. Equation (3) can

FIG. 2. (Color online) Point-spread functions of the zone-plate doublet: (top) two-dimensional intensity distributions, (middle) cross-sectional view of the intensity, and (bottom) cross-sectional view of the phase distributions at the first-order focal plane. Numerical calculations are performed using \( \lambda = 13.4 \) nm, \( D = 686 \) \( \mu \)m, \( \Delta r = 20 \) nm, \( \Delta s = 2\Delta r = 40 \) nm, and (a) \( \Delta \theta = 0 \), (b) \( \Delta \theta = \pi/4 \), and (c) \( \Delta \theta = \pi/2 \). First-order foci are separated and phase shifted relative to each other by the designed amounts. Specifically, the phase difference at the focal plane indeed equals the designed bias retardation in (a), (b), and (c), respectively.
then processed for quantitative phase retrieval. and the pupil function of the zone-plate doublet \( P_{ZPD}(x, y) \) can therefore be expressed as (see the Appendix),

\[
P_{ZPD}(x, y) = \frac{1}{\pi} \cos(\Delta s \gamma x + \Delta \theta) P(x, y).
\]

Note that the constant phase term \( \exp(-j\gamma(\Delta s)^2/4) \) in Eq. (4) has no effect on the resultant image intensity, and \( P(x, y) \) is the circular pupil function, which is 1 inside the lens aperture and 0 elsewhere [26]. For comparison, pupil function of a visible-light DIC microscope \( P_{DIC}(x, y) \) is given by [21]

\[
P_{DIC}(x, y) = -\cos \left( 2\pi \frac{\Delta s x}{\lambda f_1} + \Delta \theta \right) P(x, y).
\]

Since \( \gamma = \pi/|\Delta r(D - \Delta r)| = \pi/\lambda f_1 \), the pupil function of the zone-plate doublet is identical in form to that of its visible-light counterpart. Phase-retrieval algorithms used by visible-light DIC microscopy, such as phase-shifting interferometry, can therefore be directly applied to zone-plate doublet-based microscopes.

Performance of the zone-plate doublet as a DIC objective is evaluated numerically. DIC images are simulated by convolving the geometric image of the object with the point-spread function of the zone-plate doublet. Assuming that the illumination is coherent and the imaging system satisfies the space invariant condition [26,27], the point-spread function of the zone-plate doublet is obtained by Fourier transforming the pupil function given in Eq. (5). Note that this assumption on illumination coherence is achievable with current synchrotron-based x-ray microscopes [28] and that the resolving power, thus spatial resolution, of a microscope is determined by the width of its point-spread function. The effect of partially coherent illumination is therefore minimal in this case [29].

Phase-shifting interferometry requires four different DIC images to be obtained with specific bias retardations at \( \Delta \theta = 0, \pi/4, \pi/2, \) and \( 3\pi/4 \), respectively (Fig. 3). Intensity distributions thus generated are denoted as \( I_1(u, v), I_2(u, v), I_3(u, v) \), and \( I_4(u, v) \), respectively, where \( (u, v) \) are the coordinates of the image plane. A complex object, which has different absorption and phase patterns, is used here as the sample to

\[\text{FIG. 3. (Color online) Schematic of ZPD-based x-ray DIC microscope for quantitative imaging. Four ZPDs with } \Delta \theta = 0, \pi/4, \pi/2, \text{ and } 3\pi/4 \text{ are used as imaging objectives in a typical x-ray microscope setup. Four corresponding DIC images are obtained and then processed for quantitative phase retrieval.}\]
be imaged [Figs. 4(a) and 4(b)]. The use of such a complex object allows one to examine whether zone-plate doublet-based x-ray DIC microscopy can, indeed, perform quantitative phase and absorption imaging. The image intensity distribution of a complex object obtained with $\Delta \theta = \pi/4$, i.e., $I_2(u,v)$, is shown as an example in Fig. 4(c). The effects of both absorption and phase on the resultant DIC image are clearly observed. Lateral shear is fixed at $\Delta s = 2 \Delta r = 40 \text{ nm}$ in all cases. The object’s dimensions are $10 \mu m \times 10 \mu m$ over $512 \times 512$ pixels, and the image is $25 \text{ mm} \times 25 \text{ mm}$ in $512 \times 512$ pixels.

The differential phase contrast $\Delta \Phi(u,v)$ is given by [22]

$$\Delta \Phi(u,v) = \tan^{-1} \left[ \frac{I_2(u,v) - I_1(u,v)}{I_1(u,v) - I_3(u,v)} \right],$$

(6)

and the object’s phase distribution $\Phi(u,v)$ can be obtained by integrating $\Delta \Phi(u,v)$ in the direction parallel to the lateral shear. Object absorption can be approximated by $\rho_1(u,v) = I_1(u,v) + I_2(u,v) = I_2(u,v) + I_3(u,v)$ [22]. Note that $I_2(u,v)$ is the average of the two laterally shifted object absorption patterns as seen on the image plane, i.e., $I_2(u,v) = |A_0(u - M \Delta s/2,v)|^2 + |A_0(u + M \Delta s/2,v)|^2$, where $A_0$ is the absorption pattern of the complex object and $M$ is the magnification of the microscope. One can streamline the nanofabrication process by permuting three zone-plate phase-shift values, i.e., $\pi$ and $\pm \pi/2$, for the four desired amounts of bias retardation since it is the phase difference between the two interfering branches that determines the resultant DIC image. Figures 4(d) and 4(e) show the retrieved differential phase contrast $\Delta \Phi(u,v)$ and absorption $\rho_1(u,v)$ images, respectively. The phase distribution of the complex object $\Phi(u,v)$ is obtained by integrating $\Delta \Phi(u,v)$ [Fig. 4(f)]. Both absorption and phase distributions of the complex object are successfully reconstructed with fidelity.

**IV. CONCLUSION**

The zone-plate doublet introduced here as a DIC objective is capable of quantitative phase imaging on existing x-ray microscopes. This high-resolution microscopic phase-imaging capability complements x-ray Talbot interferometry, which images larger fields of view with nonmagnifying one-dimensional gratings [3,4,30,31]. In addition, DIC imaging with phase-shifting interferometry places a less stringent requirement on illumination coherence when comparing with ptychography, which needs sufficiently high signal-to-noise ratio and, consequently, extended exposure times for its iterative reconstruction process [5,6]. We therefore anticipate prevailing use of this zone-plate doublet objective to provide insights for studies that require high-resolution quantitative x-ray phase information.

**APPENDIX: PUPIL FUNCTION OF ZONE-PLATE DOUBLET**

The two component zone plates $Z_1$ and $Z_2$ are given by [24]

$$Z_{1,2}(x,y) = \frac{i}{2} \{1 + \text{sgn} \left[ \cos \left( \gamma r_{1,2}^2 \pm \Delta \theta \right) \right] \}. \hspace{1cm} (A1)$$

Equivalently, one can write

$$Z_1(x,y) = \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \frac{\sin(m\pi/2)}{m\pi} \exp \left\{ -jm \left( \gamma r_1^2 + \Delta \theta \right) \right\}, \hspace{1cm} (A2)$$

and

$$Z_2(x,y) = \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \frac{\sin(n\pi/2)}{n\pi} \exp \left\{ -jn \left( \gamma r_2^2 - \Delta \theta \right) \right\}. \hspace{1cm} (A3)$$

where $r_{1,2} = \sqrt{(x \pm \Delta s/2)^2 + y^2, \pm \Delta \theta}$ are the phase shifts of the two zone plates, $\Delta s$ is the lateral shear, $\gamma = \pi/[(\Delta r + \Delta x)], \Delta r$ is the outermost zone width, and $D$ is the diameter of the zone plate, respectively. The zone-plate doublet $Z_D(x,y)$ can now be expressed as

$$Z_D(x,y) \overset{\Delta}{=} Z_1Z_2 = \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \frac{\sin(m\pi/2)}{m\pi} \frac{\sin(n\pi/2)}{n\pi} \exp \left\{ -jm\left( \gamma r_1^2 + \Delta \theta \right) + n(\gamma r_2^2 - \Delta \theta) \right\}$$

$$\times e^{-j(m+n)\gamma(x^2+y^2)} + e^{-j(m-n)\Delta s y x + \Delta \theta)}$$

$$\times e^{-j(m+n)\gamma(x^2+y^2)/4}. \hspace{1cm} (A4)$$

Examining the zeroth $(m+n = 0)$ and first $(m+n = 1)$ orders, i.e., the transmission term and the first-order foci, Eq. (A4) can be approximated by

$$Z_D(x,y) \approx \frac{1}{4} \sum_{k=1}^{+\infty} \frac{1}{(2k-1)^2 \pi^2} \left[ e^{-j2(2k-1)\gamma^2(x^2+y^2)/4} + e^{j2(2k-1)\Delta s y x + \Delta \theta)} \right]$$

$$+ \frac{1}{2\pi} e^{-j\gamma(x^2+y^2)} e^{-j\gamma(\Delta s)^2/4} \sum_{m+n=1}^{m+n=0} \left[ e^{-j(\Delta s y x + \Delta \theta)} + e^{j(\Delta s y x + \Delta \theta)} \right]. \hspace{1cm} (A5)$$

Using the identity [32]

$$\sum_{k=1}^{+\infty} \frac{\cos(2k-1)y}{(2k-1)^2} = \frac{\pi}{4} \left( \frac{\pi}{2} - |y| \right).$$

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Eq. (A5) can be further simplified as

\[
Z_D(x, y) \approx \frac{1}{4} + \frac{2}{\pi^2} \sum_{k=1}^{+\infty} \frac{1}{(2k-1)^2} \cos[2(2k-1)(\Delta s y x + \Delta \theta)] + \frac{1}{\pi} e^{-j\gamma(x^2+y^2)/2}e^{-j\gamma(\Delta s y x + \Delta \theta)}/4 \cos(\Delta s y x + \Delta \theta)
\]

The zeroth-order term in Eq. (A6), i.e., \(1/2 - (1/\pi)|\Delta s y x + \Delta \theta|\), has an extent equal to the size of the zone-plate doublet and therefore has negligible effects on image intensity. Also, note that the constant phase term \(\exp(-j\gamma(\Delta s y x + \Delta \theta))/4\) in Eq. (A6) has no effect on the resultant image. The pupil function of the zone-plate doublet \(P_{ZPD}(x, y)\) is therefore

\[
P_{ZPD}(x, y) = \frac{1}{\pi} \cos(y \Delta s x + \Delta \theta) P(x, y),
\]

where \(P(x, y)\) is a circular pupil function given by [26],

\[
P(x, y) = \begin{cases} 
1 & \text{inside the lens aperture}, \\
0 & \text{elsewhere}.
\end{cases}
\]