

Course Information for Math 221, Sec 2, W 2019

Required Textbooks

- Oscar Levin, [Discrete Mathematics: an Open Introduction, 2'nd Edition](#)
- Joseph Silverman, [A Friendly Introduction to Number Theory, 4'th edition.](#) (Chapters 5,6)
- William Stein, [Elementary Number Theory: Primes, Congruences, and Secrets](#)

Possibly Useful Supplements

- Peter Cameron, [A Course on Number Theory](#)
- Ted Sundstrom, [Mathematical Reasoning: Writing and Proof .](#)

Prerequisites

Most students have taken either Math 220 or CS270 and will already be familiar with elementary logic (including predicates and quantifiers) and the following proof techniques:

- Proof by Contradiction
- Contraposition
- Mathematical Induction

Therefore I will not spend a lot of class time explaining these methods, and I will use them whenever it is convenient. Some students may need to independently read sections 2.5 and 3.2 of Levin, plus chapters 2 and 4 of of Sundstrom. To help fill in the gaps, I will distribute some notes via Blackboard Learn.

Learning Goals:

After taking this course you should be better able to...

- Use discrete structures (e.g. sets, graphs, etc) to model applied problems.
- Recognize which kind of counting (e.g. permutations vs. combinations, counting with or without repetition, equivalence classes vs. representatives, etc.) is appropriate for an applied enumeration problem.

- Use techniques such as inclusion-exclusion, bijections, and recurrence relations to solve enumerative problems.
- Deduce facts about recursively defined sequences and structures using induction and related techniques.
- Understand and apply the Extended Euclidean Algorithm
- Use modular arithmetic and solve simple congruences.
- Use continued fractions to find rational approximations for real numbers.

Contact Information:

- **INSTRUCTOR:**
Eric Schmutz, Eric.Jonathan.Schmutz@drexel.edu, 264 Korman,
- **OFFICE HOURS:** MWF 11-12.
You are welcome to stop by anytime, with or without an appointment. However I can only promise to be available if you make an appointment or come during the scheduled office hours.
- **COURSE INFO**
Assignments, an updated version of the syllabus, selected readings, and other important documents will be posted on Blackboard Learn. Some course information (including last-minute changes in quiz and exam dates, rescheduled office hours, etc) may be communicated by email. It is therefore important to monitor your Drexel email.

Course Calendar: The calendar below gives an indication of the timing, but it is subject to revision during the quarter. A detailed checklist of topics follows as an appendix.

| Week | Topic | Reading | Events |
|---------------|--|----------------|---------------|
| W1: Jan.7-13 | Set Theory | L. Ch 0 | |
| W2: Jan.14-20 | Counting | L.Ch.1 | Q1 |
| W3: Jan.21-27 | Counting, cont | L.Ch 1, cont. | Mon. Holiday |
| W4: J.28-F.3 | Graphs | | HW 1 Friday |
| W5: Feb.4-10 | Equivalence Relations | Notes | Exam 1 |
| W6: Feb.11-17 | Recurrence relations. Well ordering, M.C.M. | Notes | HW 2 |
| W7: Feb.18-24 | The Euclidean Algorithm | Silverman Ch.5 | Q2 |
| W8: F.25-M.3 | Congruences | Silverman Ch.6 | Q3 |
| W9: Mar.4-10 | Continued Fracts Rational Approx. | Stein Ch 5 | Exam 2 |
| W10:Mar.11-17 | Special Topic | T.B.A. | HW3 |
| W11:Mar.18-23 | | | Final Exam |

Grades: Here is a tentative grading scheme. (I expect to use this weighting, but reserve the right to make changes.)

| Component | Weight |
|--------------------------|---------------|
| Submitted HW and Quizzes | 30% |
| Exam 1 | 20% |
| Exam 2 | 20% |
| Comprehensive Final Exam | 30% |

The phrase “Submitted HW ”refers to three graded problem sets that are not from the textbooks. Although the recommended textbook problems are not graded, some similar problems will appear on the exams and quizzes.

Policies:

For submitted homework assignments, you may work alone or collaborate with another student. In the latter case, put both names on the paper, and just submit one set of solutions. During exams and quizzes, no collaboration is permitted. During exams and quizzes, all electronic devices (including cell phones) must be powered down and out of view. I normally do not give makeup exams and quizzes. Only in exceptional and well-documented circumstances can I excuse you from a missed exam. Talk to me beforehand if at all possible.

Additional Resources and Information:

I adhere to all University policies, including those regarding accommodation of disabilities and those regarding cheating. For more resources and information, see the [Student Handbook](#)

Disclaimer:

This document is provisional. Although I do not anticipate major changes, I reserve the right to modify the course during the term.

Appendix: Checklist of Topics Covered¹

Part I. Elementary Set Theory (Levin Ch.0+class notes)

- Sets and Multisets versus Sequences
- Defining sets using predicates and set builder notation
- Set Operations: $\cap, \cup, /$, Powerset, Cartesian Product
- Cardinality
- Graphs: Standardly labelled graphs and digraphs.
- Functions
 - Functions as relations in $D \times T$
($D = \text{domain}, T = \text{codomain } T$)
 - Injective, surjective and bijective functions.
 - The Pigeonhole Principle
 - Schröder-Bernstein theorem

Part II. Combinatorics(Levin Ch 1+class notes)

- Enumeration via bijections
- “Counting Table”:no.ways to pick k elements from of $[n]$ is . . .

| | No Reprs | w. Reprs |
|----------------------------------|----------------|---|
| Ordered | $(n)_k$ | n^k |
| Unordered (sets or multisets) | $\binom{n}{k}$ | $\binom{n+k-1}{n-1} = \binom{n+k-1}{k}$ |

- Inclusion-Exclusion
- Binomial and Multinomial Theorems
 - Generating Functions
 - Applications to Lattice walks

¹This list may be updated during the quarter

- Combinatorial Proofs of Identities
- **Graphs** (Notes and HW)
 - Vertex neighborhoods and degrees.
 - Paths and connected components.
 - Subgraphs
 - The chromatic polynomial

Part III. Misc. (TBA)

- Equivalence Relations
 - Set Partitions
 - Reflexive, Symmetric, and Transitive Binary Relations
 - Graph Isomorphism
 - Congruences
 - Well-defined functions on set partitions.
- Recurrence relations, Min.Counterexample Method
 - Sum and product notations. Arithmetic and geometric series.
 - Well ordered sets and proofs using the “Minimum Counterexample Method” (M.C.M.)
 - Finding a recurrence satisfied by a sequence of structures.
 - Using the M.C.M. to prove facts about a recursively defined sequence.

IV. Elementary Number Theory (Silverman Ch 5,6 + class notes)

- The Euclidean Algorithm
 - Prime factorization, g.c.d. and l.c.m of factored integers.
 - The Extended Euclidean Algorithm
 - $\text{g.c.d.}(a, b) = d \Leftrightarrow \{ax + by : x, y \in \mathbb{Z}\} = \{dm : m \in \mathbb{Z}\}$.
 - Solving $ax \equiv 1 \pmod n$.
 - Chinese Remainder Theorem
- Continued Fractions (Stein Ch.5.)

Suggested Textbook Problems

Levin, Chapter 0(Elementary Set Theory)

- Page 17: 7,9,10
- Page 28:1,2,3,4,8,12, 14,15
- Page 37: 1,4,5,11,14,16

Levin, Chapter 1 (Counting)

- Page 51: 4,11,12,13
- Page 61: 1-13
- Page 69: 2,3,8,13
- Page 81: 1,2,4,5,6,8,10
- Page 105: 1-18.

Levin Chapter 2(Sequences)

- Page 117: 5
- Page 127: 1,7,11,12

Silverman Ch 5,6(g.c.d.'s)

- Page 34: 1,3
- Page 43: 2,5,6

Levin, Chapter 5.2 (Number Theory)

- Page 267:1-9.

Continued Fractions (TBA)