Math 121 Final Exam Review Problems, Part I  
Sections 4 and 5 (Schmutz)

1. Which of the following equations has a solution in the interval [0, 2]?
   - \(x^4 + 4x^3 + 6x^2 + 4x + 2 = 0\)
   - \(2x^3 - 21x^2 + 72x + 1 = 0\)
   - \(2x^4 - (2 - x)^4 = 0\)

2. Evaluate the following limits
   (a) \(\lim_{x \to \infty} \frac{10x^2 + 7e^{-x}}{2x^2 + e^{-x}} = \)
   (b) \(\lim_{x \to \infty} \frac{10x^2 + 7e^x}{2x^2 + e^x} = \)
   (c) \(\lim_{x \to \infty} \frac{10x^2 + 7e^{-x}}{2x^2 + e^{-x}} = \)
   (d) \(\lim_{x \to \infty} \frac{10x^2 + 7e^x}{2x^2 + e^x} = \)
   (e) \(\lim_{x \to 2} (x^2 - 4) = \)
   (f) \(\lim_{x \to 2} \frac{(x^4 - 16)}{(x^2 - 4)} = \)
   (g) \(\lim_{x \to \infty} \frac{10x^2 + 3x^4 + x}{5x^2 + 201x + 121} = \)
   (h) \(\lim_{x \to -2} \frac{x^{121} - 2^{121}}{x - 2} = \)
   (i) \(\lim_{x \to \pi} \frac{\cot(x)}{x - \pi} = \)

3. Find the equation of the line that is tangent to the curve \(y = e^{x^2 - x}\) at \(x = 1\).

4. Find the derivatives of each of the following functions:
   (a) \(y = e^x + \cos(x)\)
   (b) \(y = e^{x^2} + \cos(x^2)\)
   (c) \(y = e^x \sin(x)\)
   (d) \(y = e^{x^3} + \sin(x^3 + x)\)
   (e) \(y = \tan(x) \ln(x)\)
   (f) \(y = \tan(x^2) \ln(x^2)\)

5. For each of the following functions, evaluate the derivative at \(x = 0\), i.e. calculate \(f'(0)\).
   (a) \(f(x) = \ln(1 + \sin(2x))\)
   (b) \(f(x) = e^{x^3 + 2x}\)
6. Find the derivatives of each of the following functions:

(a) \( y = \frac{x^2 + x}{x^3 + 4x} \)

(b) \( y = \frac{\sin^2(x) + \sin(x)}{\sin^3(x) + 3\sin(x)} \)

7. Find the equation of the line that is tangent to the curve \( x^2 + x^4 = y^2 - y \) at the point \((1, 2)\).

8. Cylindrical bins are to hold 100 liters, have a closed bottom and an open top. What choice of radius and height will require the least material?

TBC