# Multiple Resonators Coupling Josephson Junction Qubits as a Multi-Channel Bus


Z. E. Thrailkill, and R. C. Ramos

Low Temperature Lab, Department of Physics, Drexel University, Philadelphia, PA 19104 USA

(http://www.physics.drexel.edu/research/lowtemp/)


## Abstract


The Josephson junction phase qubit has been shown to be a viable candidate for quantum computation. In order to transfer and store quantum information, superconducting strips that act as resonators can be used. Resonators have already been shown to have long coherence times which make them ideal for this. We will explore the possibilities of using multiple resonators with different resonant frequencies to couple multiple qubits. Using resonators of different frequencies will allow for information to be stored and later retrieved, while allowing other processes to be executed using another resonator.

Two problems involved in scaling up superconducting qubit systems is how to couple large numbers of qubits together and how to store quantum information for later use. Using resonators is appealing due to their stability and long coherence times. Resonators don't need to be coupled, via wires, to the outside environment, making them more isolated than a Josephson junction qubit. The use of a resonator to hold quantum information and to couple qubits together has been experimentally shown.

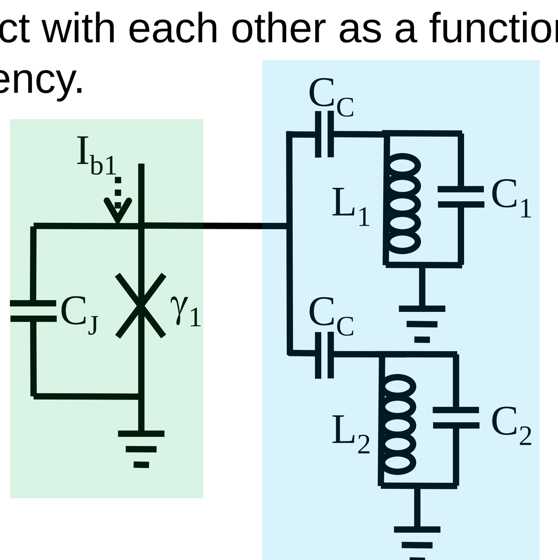

❖ **Goal :** To simulate different issues that need to be examined for an array of resonators to be used as a multi-channel bus.

- ✓ The frequency spacing of the array, so the resonators will be isolated from each other
- ✓ Dispersive coupling over the array, so the array will act as a bus for quantum information by simply tuning the qubits together
- ✓ Selectively coupling to only one resonator in the array, so the qubits can utilize the frequency band the array is in without coupling to the array

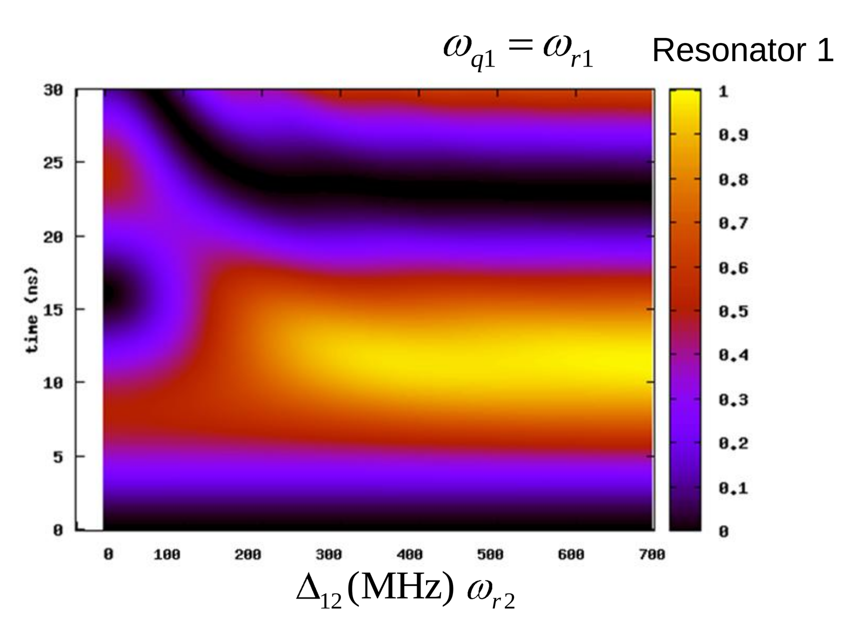
## Discussion of Results

### *Frequency Spacing of Resonator Array to Isolate the Resonators From Each Other*

In order to build an array of multiple resonators we must examine how they interact with each other as a function of frequency.

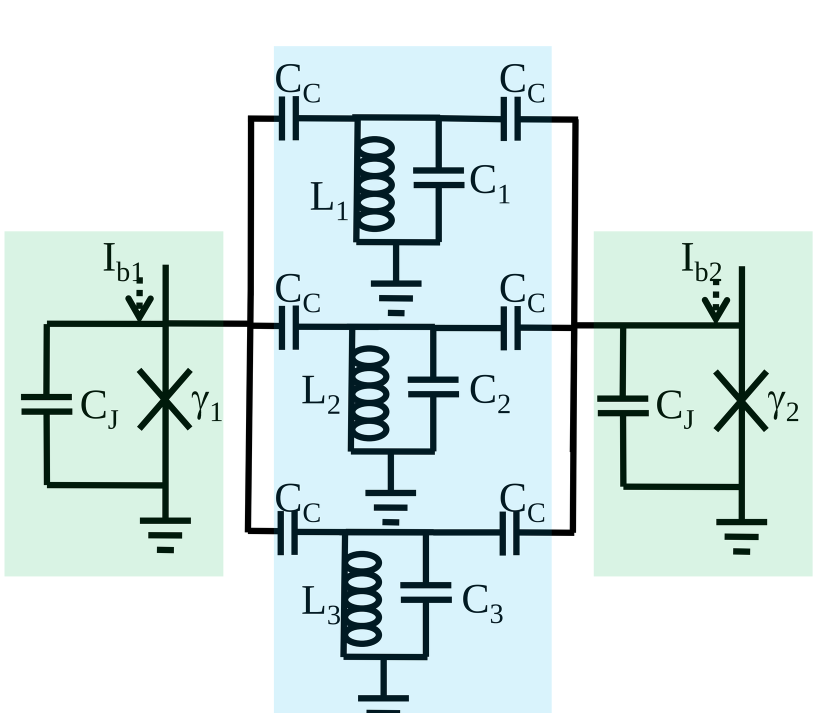


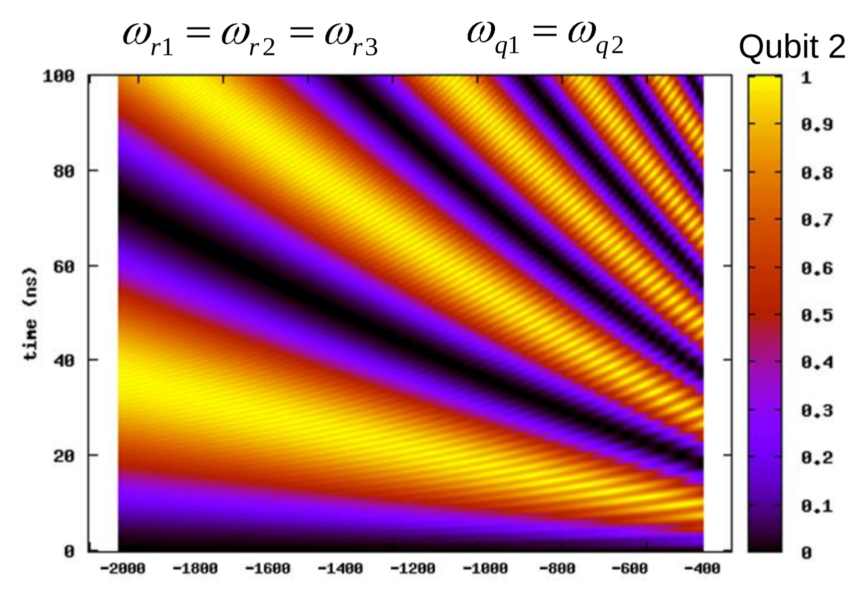

- One qubit coupled to two resonators.
- Coupling strengths are about 110 MHz.
- Qubit 1 is initialized in the excited state.




- A plot of excitation probability in the first resonator as a function of time and the detuning of resonator 2.
- The qubit and resonator 1 are in resonance while resonator 2 is held at different detuning values.
- Near zero detuning half the excitation is taken by the second resonator.

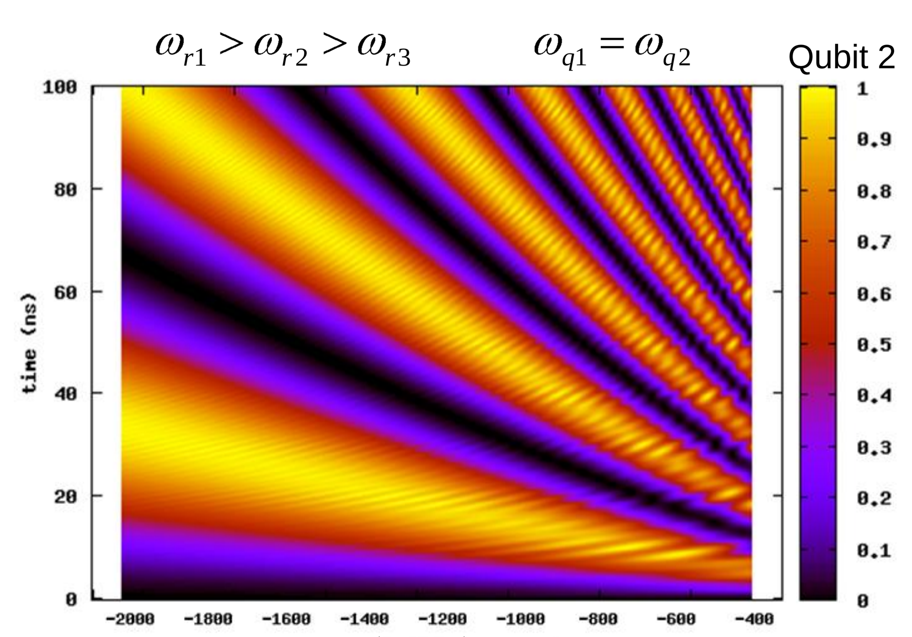
### *Dispersive Coupling over Resonator Array For Variable Inter –Qubit Coupling*




- Two qubits coupled together via three resonators in parallel.
- Coupling strengths are about 110 MHz.
- Qubit 1 is initialized in the excited state.

- The two qubits are in resonance while the resonators are held at the same frequency.
- The plot is of the excited state probability in qubit 2 as a function of time and the two qubits detuning from the resonators.
- As the detuning gets smaller the effective coupling gets stronger.
- At low detuning the small ripples seen are a result of the assumption $|\Delta_{ij}| \gg g_{ij}$ breaking down.




- The two qubits are in resonance while the resonators are held at different frequencies.
- The plot is of the excited state probability in qubit 2 as a function of time and the two qubits detuning from the resonators.
- The ripples are more irregular due to the resonators being held at different frequencies.

## Theory

In order to simulate these qubit resonator systems, one must find the Hamiltonian for a general configuration of circuit elements:

$$H = \sum_i \frac{\hbar\omega_{qi}}{2}\hat{\sigma}_+^i\hat{\sigma}_-^i + \sum_j \hbar\omega_{rj}\hat{a}_j^\dagger\hat{a}_j + \sum_{i,j}\hbar g_{ij}(\hat{a}_j^\dagger\hat{\sigma}_-^i + \hat{a}_j\hat{\sigma}_+^i)$$

Qubit term, Resonator term, Coupling term

$$\omega_r = 1\Big/\sqrt{LC}$$

Here, $\omega_{qi}$ is the frequency of qubit *i* corresponding to the energy difference between its ground and excited state, $\omega_{rj}$ is the frequency of resonator *j*, and $g_{ij}$ is the coupling strength between qubit *i* and resonator *j*.

When there are two qubits in resonance and coupled together through a resonator, and the resonator is sufficiently detuned from the qubits, then dispersive coupling can occur. In order to see this, we make a unitary transformation of the Hamiltonian: $UHU^\dagger = H_{eff}$

$$U = \exp\left[\sum \frac{g_{ij}}{\Delta_{ij}}(\hat{a}_j^\dagger\hat{\sigma}_-^i + \hat{a}_j\hat{\sigma}_+^i)\right]$$

In the dispersive limit, $|\Delta_{ij}| = |\omega_{qi} - \omega_{rj}| \gg g_{ij}$

The Hamiltonian becomes:

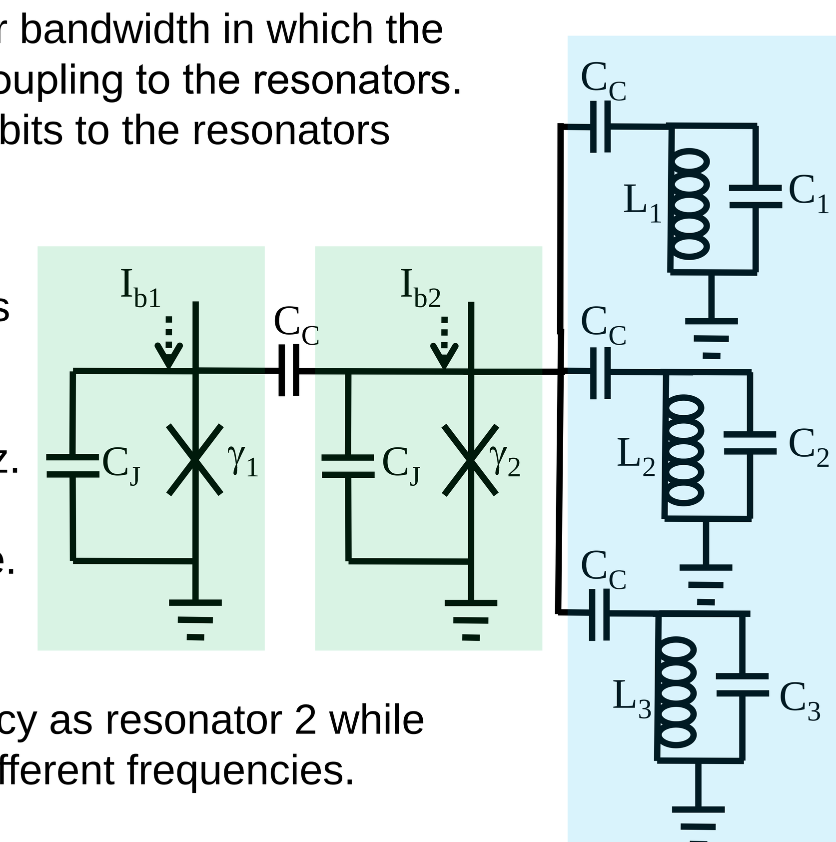
$$H_{eff} = \sum_i \frac{\hbar\omega_{qi}}{2}\hat{\sigma}_+^i\hat{\sigma}_-^i + \sum_j \hbar\Big(\omega_{rj} + \sum_i \frac{g_{ij}^2}{\Delta_{ij}}\hat{\sigma}_+^i\hat{\sigma}_-^i\Big)\hat{a}_j^\dagger\hat{a}_j + \sum_{i>k}\hbar\frac{g_{ij}g_{kj}(\Delta_{ij}+\Delta_{kj})}{2\Delta_{ij}\Delta_{kj}}(\hat{\sigma}_-^i\hat{\sigma}_+^k + \hat{\sigma}_+^i\hat{\sigma}_-^k)$$

Dispersive coupling term

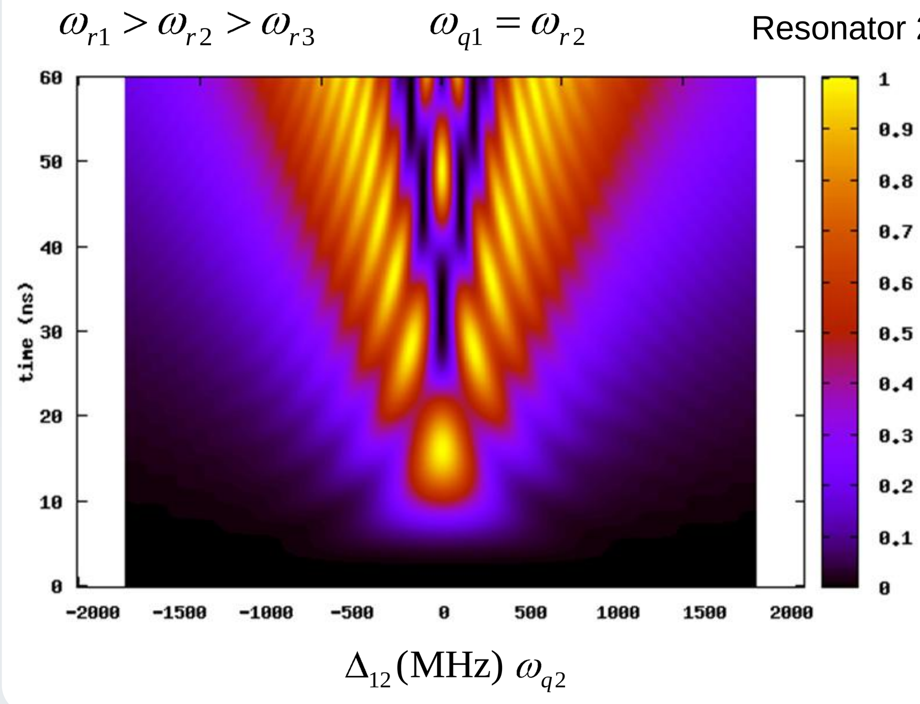
### *Selective coupling to Resonator Array*

One problem when using multiple resonators at different frequencies is that they take up a larger bandwidth in which the attached qubits can't operate without coupling to the resonators. This can be solved by coupling your qubits to the resonators through a single qubit.

- One being coupled to three resonators via qubit 2.
- Coupling strengths are about 110 MHz.
- Qubit 1 is initialized in the excited state.




- Qubit 1 is tuned to the same frequency as resonator 2 while the other two resonators are held at different frequencies.




- The plot is of the excited state population of resonator 2 as a function of time and the detuning of qubit 2 from qubit 1.
- This shows that the coupling to any given resonator can be turned on and off using the middle control qubit by making the coupling weak enough.

## Summary

- The spacing of the resonant frequencies of an array of resonators must be about 3.5 times the coupling strength in order to remain isolated from each other.
- At high detuning, the dispersive coupling over an array of resonators is mostly unaffected by having resonators at different frequencies. At lower detuning, the dispersive coupling becomes more erratic for arrays with resonators at different frequencies.
- Coupling qubits to a resonator array, via a control qubit, successfully allows coupling between individual resonators in the array to be turned on and off. This allows the qubits to be operated in the frequency rang of the array without coupling to the array.

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