Berry's Phase in the Josephson Phase Qubit

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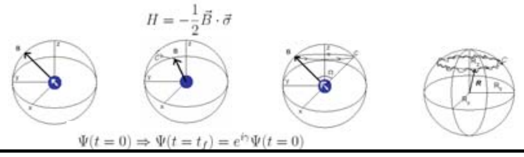
Abstract

Berry's phase often appears in quantum two-level systems with a degeneracy. An example of such a system is a spin-1/2 particle in a magnetic field. As the magnetic field is slowly evolved through a closed path, the particle has been shown to acquire an additional phase called Berry's phase, in addition to the usual dynamical phase. This phase has been found in two-level quantum systems intrinsic to many superconducting qubits and has particularly been calculated for the charge, flux and Josephson phase qubit. Here, we present an alternative derivation of the **geometric Berry's phase** in a Josephson phase qubit. We also calculate the **complete Berry's phase** from the expression $\gamma = i \int \langle\psi|\partial R\psi\rangle \cdot dR$ evaluated over a closed loop in frequency parameter space. Based on a comparison of these results, we examine the possibility of using a single phase qubit for topological quantum computing.

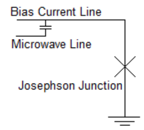
Introduction

It is difficult to decouple superconducting qubits from the environment [1]. To overcome this obstacle, features of these quantum systems that are insensitive to noise, such as their topological phases, are being studied. One such topological phase is Berry's phase, which is acquired through adiabatic variations of the Hamiltonian through a closed path. The classic system where this phase appears is a spin-1/2 particle in a slowly-rotating magnetic field [2].

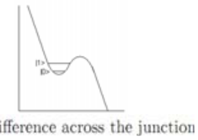


Many qubits can be thought of as two-level systems analogous to the spin-1/2 particle. Berry's phase has already been calculated in flux qubits [3], measured in charge qubits [4] and explored in the phase qubit [5]. We present an alternative derivation of this last result and explore the possibility of using this phase for topological quantum computing.

The phase qubit consists of a current-biased Josephson junction [1,6,7]. Its dynamics is described by a washboard potential as a function of the phase difference across the junction. For a bias current close to the critical current, the Hamiltonian describing the system can be approximated as:



$$H = -\frac{\hbar\omega_p}{2}\sigma_z \tag{1}$$



Since this Hamiltonian is time-varying, it is usually convenient to perform a change of basis to obtain:

$$H = -\frac{\hbar\omega_p}{2}\sigma_z - \frac{E_J l}{\sqrt{2}}\left(\frac{I(t)}{I_0} - \frac{I_B}{I_0}\right)\sigma_x \tag{2}$$

Discussion and Results

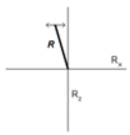
We can recast the Hamiltonian (2) in a parameter space as:

$$H = -\tfrac{1}{2}\vec{R}\cdot\vec{\sigma} \tag{3}$$

where $\vec{R} = E_J l\sqrt{2}\left(\frac{I(t)}{I_0} - \frac{I_B}{I_0}\right)\hat{x} + \hbar\omega_p\hat{z}$

This phase space vector R oscillates about a point that is just off the x-axis.

We rotate the parameter space about the y-axis so that the vector is oscillating about the z-axis.



Now, we consider applying the signal

$$I(t) = \Delta I\cos(\omega t + \phi) + I_B + \Delta I$$

where the amplitude of the signal is ΔI, the frequency is ω, and the phase is ϕ. The frequency of the signal is not slow enough for us to discuss the paths it traces in parameter space in relation to Berry's phase. Thus, we apply the rotating frame approximation [9]

$$H_{eff} = \exp\left(\frac{i\omega t}{2}\sigma_z\right)H'\exp\left(-\frac{i\omega t}{2}\sigma_z\right) + \frac{\hbar\omega}{2}\sigma_z \tag{4}$$

where H' is (2) after rotation. The result of this approximation is

$$H_{eff} = \left(\frac{\hbar\omega}{2} - \frac{\hbar\omega_p}{2}\right)\sigma_z - \frac{E_J l\Delta I}{2\sqrt{2}I_0}\sin\phi\sigma_y + \frac{E_J l\Delta I}{2\sqrt{2}I_0}\cos\phi\sigma_x \tag{5}$$

We see that all of the ωt terms have been removed. This is necessary as in the stationary frame, the adiabatic behavior of ϕ which is a parameter adiabatically varied to produce the Berry's phase, is overpowered by the much more rapidly varying ωt.

From (5), we can again define a parameter space vector

$$\vec{R} = \frac{E_J l\Delta I}{\sqrt{2}I_0}\cos\phi\hat{x} - \frac{E_J l\Delta I}{\sqrt{2}I_0}\sin\phi\hat{y} + (\hbar\omega - \hbar\omega_p)\hat{z} \tag{6}$$

which will trace out a closed path as ϕ is adiabatically varied from 0 to π. The solid angle enclosed by this path will be

$$\Omega = 2\pi\left(1 - \frac{\hbar\omega - \hbar\omega_p}{\sqrt{(\hbar\omega - \hbar\omega_p)^2 + \left(\frac{E_J l\Delta I}{\sqrt{2}I_0}\right)^2}}\right) \tag{7}$$

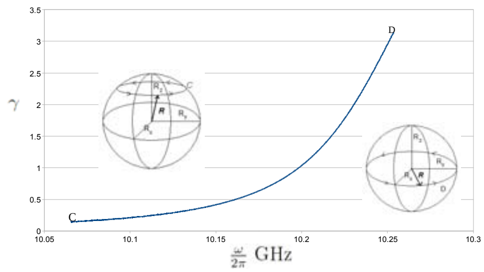
From Berry [3], the Berry's phase is proportional to the solid angle enclosed by path with a constant of proportionality which can be defined as

$$\nabla_{\vec{R}}H = -\frac{1}{2}\vec{\sigma} \tag{8}$$

Thus, from (7) and (8), we see that the Berry's phase is

$$\gamma = \pi\left(1 - \frac{\hbar\omega - \hbar\omega_p}{\sqrt{(\hbar\omega - \hbar\omega_p)^2 + \left(\frac{E_J l\Delta I}{\sqrt{2}I_0}\right)^2}}\right) \tag{9}$$

Thus, we find that Berry's phase is dependent upon the frequency of the applied signal in (9). This differs from the charge and flux qubit cases where the amplitude of the applied signal determined the Berry's phase produced [4,5].



We also calculate the complete Berry's phase using, from [2]:

$$\gamma = i\oint \langle\psi|\nabla_{\mathbf{R}}\psi\rangle \cdot d\mathbf{R} \tag{10}$$

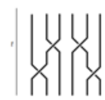
$$|0\rangle = \begin{pmatrix} \frac{(z-E_0)(\cos\phi - i\sin\phi)}{\sqrt{2r^2+2z^2-2zE_0}} \\ \frac{r}{\sqrt{2r^2+2z^2-2zE_0}} \end{pmatrix} \quad E_0 = +\frac{1}{2}\sqrt{z^2+r^2}$$

For the ground state, we find that both geometric and complete Berry's phase are identical. For the phase qubit to be a candidate for topological quantum computation, these two terms must be different, due to an extra topological term [8]. This term is necessary for the anyonic behavior needed for toplogical quantum computing.

In topological quantum computing, states are represented by anyons which have quantum statistics that are neither bosons or fermions [8].

$$|\psi(x_2, x_1)\rangle \longrightarrow |\psi(x_1, x_2)\rangle e^{\alpha}$$

Braiding these particles allows us to manipulate information. By examining systems where phase qubits are entangled in complex ways, it may be possible to produce a system where we can easily produce and manipulate anyons.



Summary and Future Work

- We have derived the geometric Berry's phase of a phase qubit.
- We have calculated the complete Berry's phase which is identical to the geometric Berry's phase. This means that individual phase qubits do not exhibit anyonic behavior and cannot, by themselves, be used for toplogical quantum computing.
- We are currently extending our studies to multiple phase qubits as well as qubits coupled to harmonic oscillators.

References

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