Homework – Filters

1. Sketch and derive the differential equation for a low-pass filter. Show that it takes 5 time constants for the system to reach 99% of steady-state (10 points)

2. Derive the Laplace transform for a low-pass filter (5 points)

3. Sketch and derive the differential equation for a high-pass filter (10 points)

4. Derive the Laplace transform for a high-pass filter (5 points)

5. Mathematical show that the low-pass filter performs integration (5 points)

6. Mathematical show that the high-pass filter performs derivatives (5 points)

7. Given a low-pass filter with $R = 47$ kilo-Ohms, $C = 0.1$micro-Farads and a +5 Volt input voltage. Assuming zero initial conditions, what is the steady-state voltage across the cap? At what time does this voltage reach 63.3% of steady-state voltage? (10 points)

8. Read up on lead-lag controllers. What is the general form (in Laplace domain) for such controllers? Based on the "grandmother explanation" given in class, show which components of the general form perform lead (and hence differentiation or high-pass filtering) and which perform lag (and hence integration or low-pass filtering). (10 points)