Hands-on Lab

Frequency Response

Previous labs and lectures investigated a system’s step response. One could observe time-domain characteristics such as the time constant (rise time) and thus predict steady-state properties. By contrast, the frequency-domain provides additional information that is not immediately apparent from the time-domain. Hence concepts on attenuation and phase lag are thus presented.

Preamble: Summary of Step Response and Modeling

Recall that for the NXT DC motor, the open-loop transfer function (OLTF) was given by

\[ G_{OL} = \frac{b}{s+a} \]

with the rise-time \( \tau = \frac{1}{a} \)

Recall that programs that commanded the NXT DC motor to run at 75% power led to Figure A

From Figure A, one observed the steady-state velocity is \( \omega_{ss} = 70.5 \) RPM and deduced that the rise time \( \tau = 0.12 \) sec. Consequently one derived the OLTF as:

\[ G_{OL} = \frac{7.83}{s + 8.33} \]  (1)

Figure B depicted a Simulink simulation of (1) to a motor command of 75%. Given its similarity to Figure 1A, increases one’s confidence in the OLTF.

Taking the inverse Laplace of (1) or solving the first-order differential equation, one derived that the motor velocity is given by:

\[ \omega(t) = \omega_{ss} \left(1 - e^{-\frac{t}{\tau}}\right) \]  (2)

What is not immediately obvious from (2) is how the motor responds to sinusoidal signals. This is important because one may recall from Fourier theory that any signal is simply a weighted sum of sinusoids. Thus, by understanding how (1) responds to a sine wave, one can predict response to any signal.

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Concept 1: Bode Plot

Step 1: Launch Matlab and type the following script, saved as nxtOltf1_0.m

```matlab
% FILE: nxtOltf1_0.m
% DATE: 10/27/12
% AUTH: P.Oh
% DESC: Matlab script for NXT Motor OLTF

OltfNum = [7.83];
OltfDen = [1, 8.33];
GOltf = tf(OltfNum, OltfDen);
h=bodeplot(GOltf)
p=getoptions(h);
setoptions(h,p);
p.FreqUnits = 'Hz';
setoptions(h,p);
```

Step 2: Under File – Set Path – Add Folder and navigate to the folder where you save the above program, then click Save and Close. In the command window, type nxtOltf1_0. The following Bode magnitude and phase plots should be displayed as given in Figure 1-1.

![Bode Plot](image)

Figure 1-1: The Bode magnitude (top) and phase (bottom) plots for the OLTF given by (1).

Exercise 1: In the Matlab Figure (Figure 1-1), click Tools – Data Cursors to display data values on the curves.

1-1 What is the magnitude (in dB) and phase (in degrees) for the system at 1.32 Hz?

1-2 What is the magnitude (in dB) and phase (in degrees) for the system at 0.1 Hz and 2.0 Hz?

1-3 Replace (1) with your own NXT DC motor’s open-loop transfer function. Rewrite nxtOltf1_0.m with your own values. What is frequency when the magnitude is -3 dB and phase is -45 degrees?

1-4 From 1-3, write down the magnitudes and phase angles for your NXT DC motor at 0.1 Hz and 2.0 Hz.
Concept 2: Simulink Frequency Response: Attenuation and Phase

Step 1: Launch Simulink and create the following model. Note the transfer function uses (1).

![Simulink model for Equation (1)](image)

Step 2: Set the sine wave control an amplitude of 25, bias of 75 and frequency in rad/s (i.e. $\omega = 2\pi f = 2\pi 1.32 = 8.29$ rad/s). Hit OK. Set the simulation time to 2 seconds. Hit run.

![Simulink output scope](image)

Figure 2-1: A 1.32 Hz (or 8.29 rad/s) input sine wave gives a baseline of 75% motor command. The simulated output has the same frequency, but an attenuated output and phase lag.

Exercise 2: Using Simulink

2-1 Modify the sine input for 0.1 Hz and display the output scope. Is the amplitude peak-to-peak value smaller or larger than Figure 2-1 (right)? Do they peaks of the output lag the peaks of the input? If so, by approximately how many seconds? Is this shorter or longer than in Figure 2-1 (right)?

2-2 Repeat 2-1 but use a 2.0 Hz signal.

2-3 Replace Figure 1-1 with your own NXT DC motor's open-loop transfer function. Run your simulation with the frequency obtained from Exercise 1-3 and generate your own version of Figure 2-1.

2-4 Repeat 2-1 and 2-2 but use your own NXT DC motor's open-loop transfer function.
Concept 3: Data Acquisition Experiment

Figure 1-1 is a Matlab-generated Bode plot. It tells us that the system’s output will mimic the input with little change in steady-state amplitude and frequency for frequencies lower than 0.1 Hz. This is evidenced by (1) the flat horizontal part of the Bode magnitude plot (thus reflecting a constant gain); and (2) the almost 0-degree values for the Bode phase plot.

As the frequency increases, we see a significant change at the system’s rise-time. Recall that $\tau = 0.12$ sec and hence $\omega = \frac{1}{0.12} = 8.33$ rad/sec (or $f = 1.32$ Hz). Called the cutoff frequency, the magnitude is -3 dB and phase is -45 degrees. In other words, the output amplitude reduces to about 70.7% of steady-state and lags the input by 45 degrees. For inputs greater than the cutoff frequency, the output will continue to attenuate and the lag will increase.

The Simulink simulated graph (Figure 2-1) confirms that the output indeed attenuates as the frequency goes from 0.1 to 1.32 to 2.0 Hz. One also sees that the output lag increases too.

This last concept uses NxC timer and file-saving capabilities to acquire data and experimentally verify the motor’s cutoff frequency, attenuation and phase characteristics.

Step 1: Write an NxC program (recall Week 04 Concept 4 on Sine Waves) to generate a 75% motor command with a peak-to-peak amplitude of 25 and your cutoff frequency (i.e. answer to Exercise 1-3). Capture both the motor command and motor velocity for about 4 cycles worth of data. For example, if the cutoff frequency $f = 1.32$ Hz, then $\pi f = 0.76$, so collect data for $4\pi f = 3.04$ seconds.

Plot your output data and annotate similarly to Figure 3-1:

![Figure 3-1](image)

**Figure 3-1:** A 1.32 Hz sinusoidal input (top curve) commands the NXT DC motor. The motor velocity was acquired (bottom). One observes the output lags the input by 0.25 seconds. One also observes the peak-to-peak amplitude of the output $77 - 44 = 30$ is attenuated.
Step 2: Compare your answers from Exercise 1-3 to the values you observe in your own version of Figure 3-1.

In Figure 3-1 the output peaks at $\tau \approx 0.28$ seconds. Hence must have $\omega \tau + \phi = \frac{\pi}{2}$ when $\tau = 0.28$.
For our input frequency of $\omega = 2\pi f = 2\pi \cdot 1.32 = 8.29 \text{ rad/s}$:

$$\phi = \frac{\pi}{2} - \omega \tau = 1.57 - 8.29 \cdot 0.28 = 1.57 - 2.32 = -0.75 \text{ rad}$$

Or

$$\phi = -0.75 \frac{180}{\pi} = -43.0 \text{ deg}$$

One notices that $\phi = -43.0$ degrees is very close to the Bode phase plot (Figure 1-1) simulated value of -45 degrees.

Also from Figure 3-1, one sees the output’s peak-to-peak amplitude is equal to: $77 - 47 = 30$.
Hence:

$$dB = 20 \log \frac{30}{50} = 20 \log 0.6 = -4.4$$

(4)

The Bode magnitude plot (Figure 1-1) has a simulated value of -3 dB which is quite close to (4).

Exercise 3: Modify your NX program and acquire data to obtain Excel plots for the following:

3-1 Using a 0.1 Hz input frequency, what is the phase angle (degrees) and magnitude (dB)? How do these values compare to your answers in Exercise 1-4?

3-2 Using a 2.0 Hz input frequency, what is the phase angle (degrees) and magnitude (dB)? How do these values compare to your answers in Exercise 1-4?