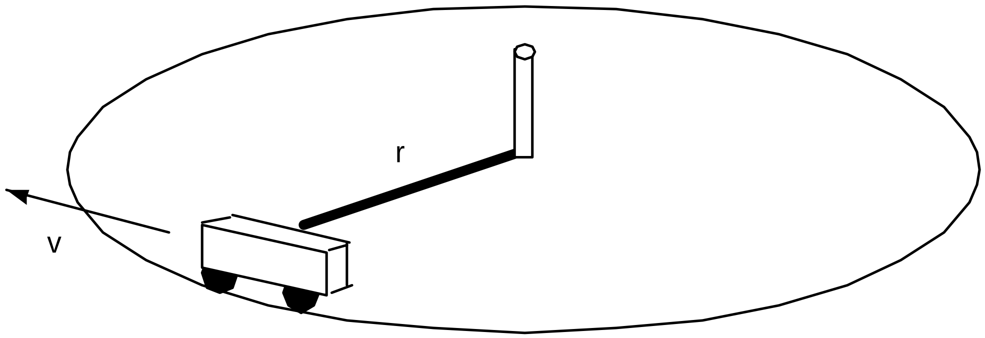
PID Theory and System Type

MEM 639 Real-Time Microcomputer Control 1

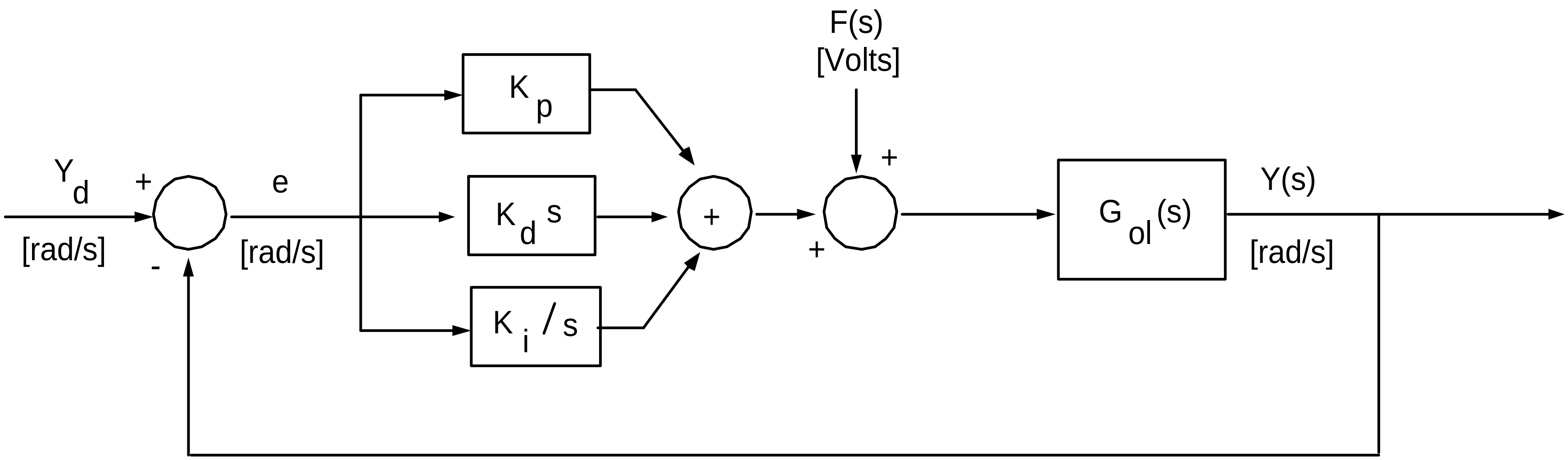
PID Control (Closed-Loop) of Motorized Tethered Cart



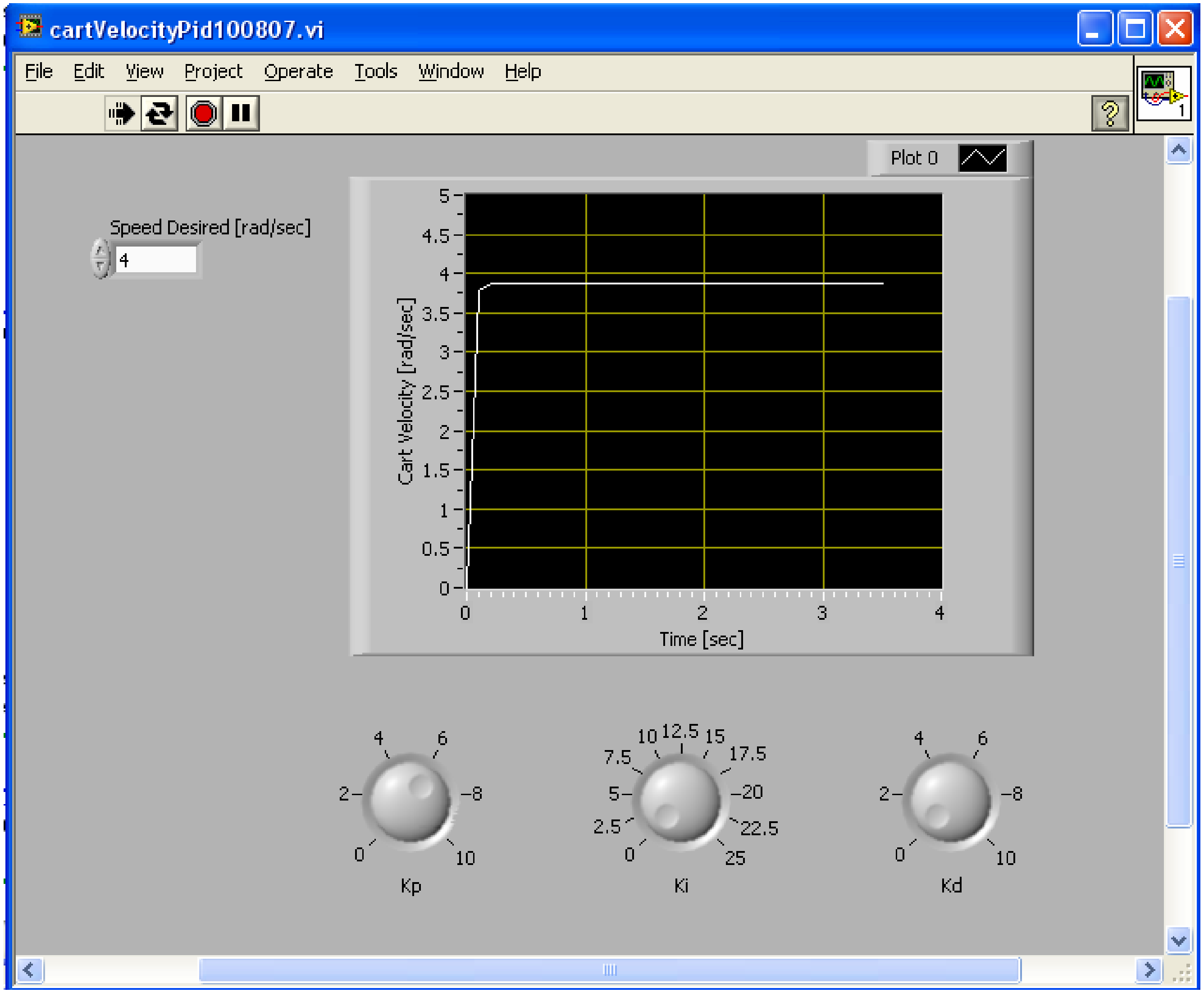
Goal: Want cart velocity to always be 3 rad/s

- Even if terrain has hills and valleys
- Even if cart weight changes

PID is the most common form of closed-loop control:

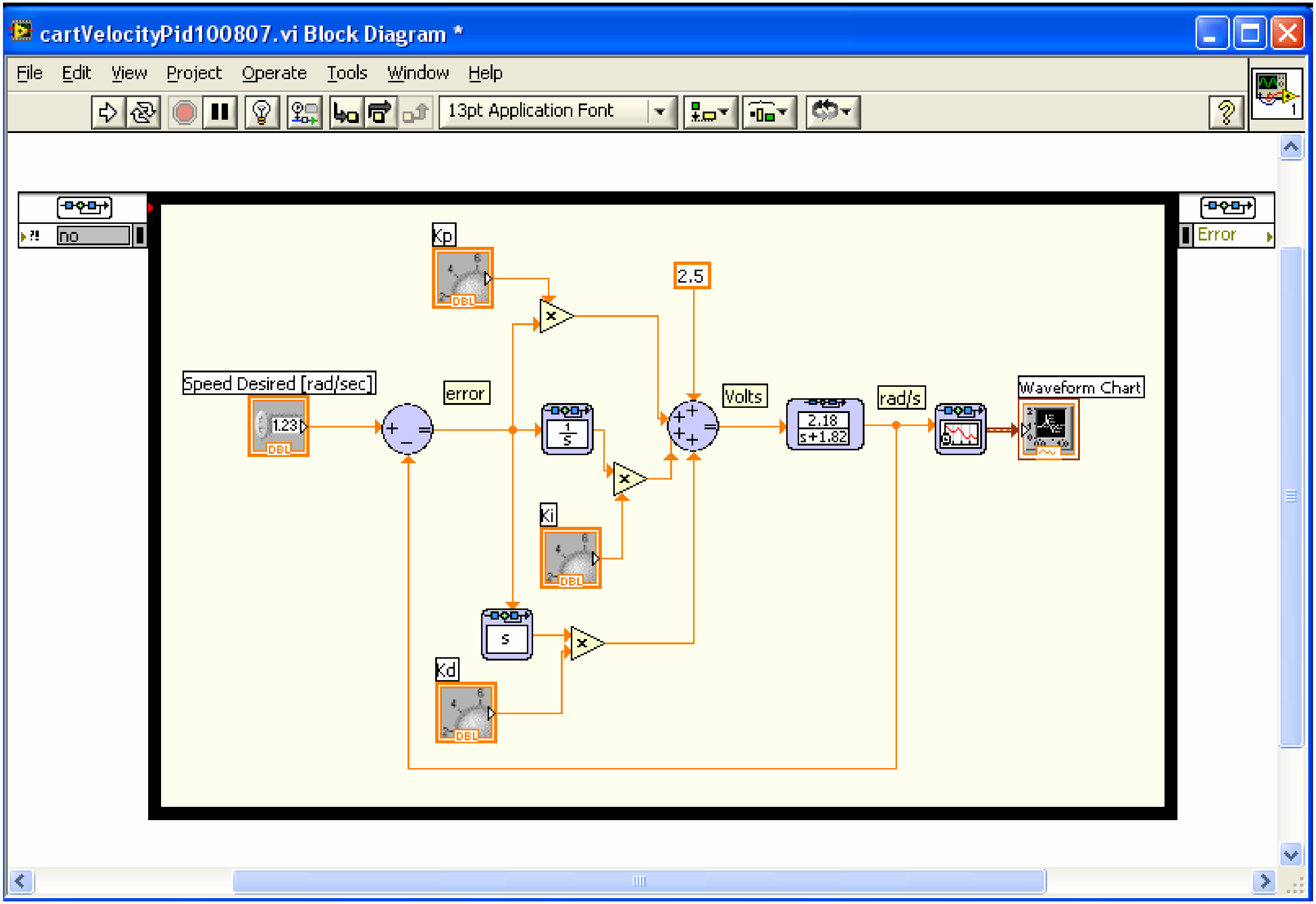


Why?

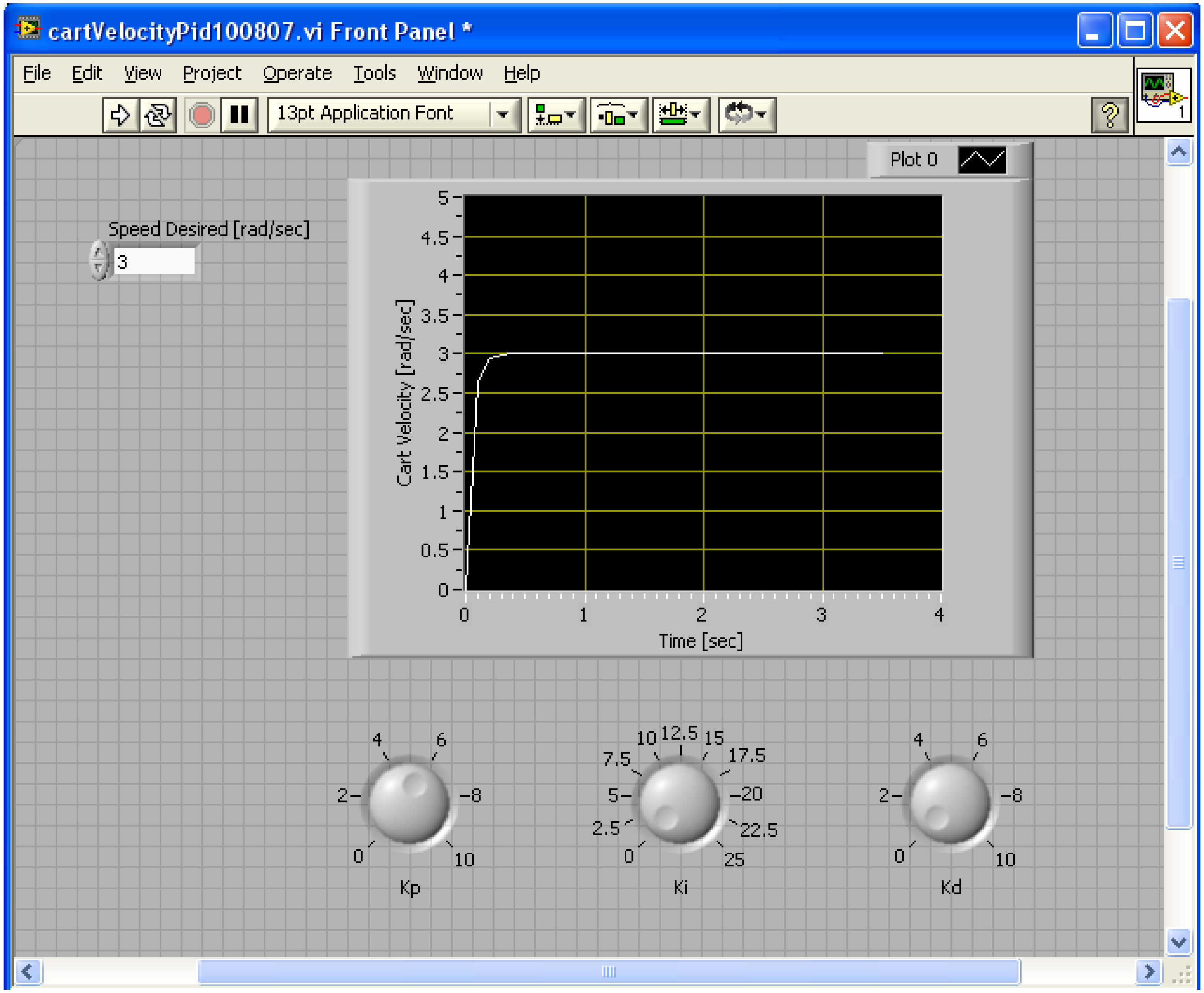


Answer: One can tune for desired performance without full knowledge of dynamics

PID Control Block Diagram Simulation



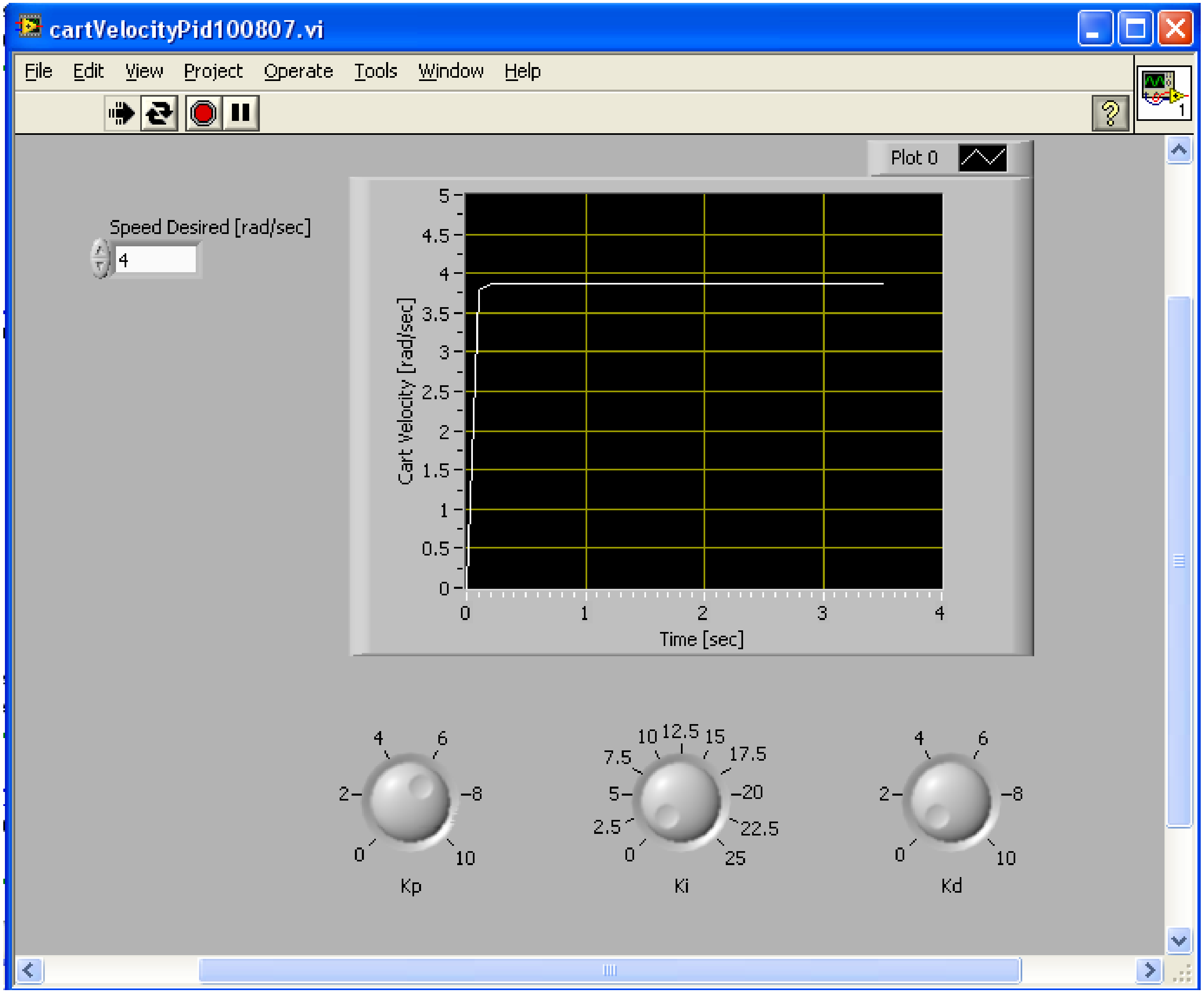
Lab: PID Simulation for Motorized Tethered Cart – Closed-loop Velocity Control



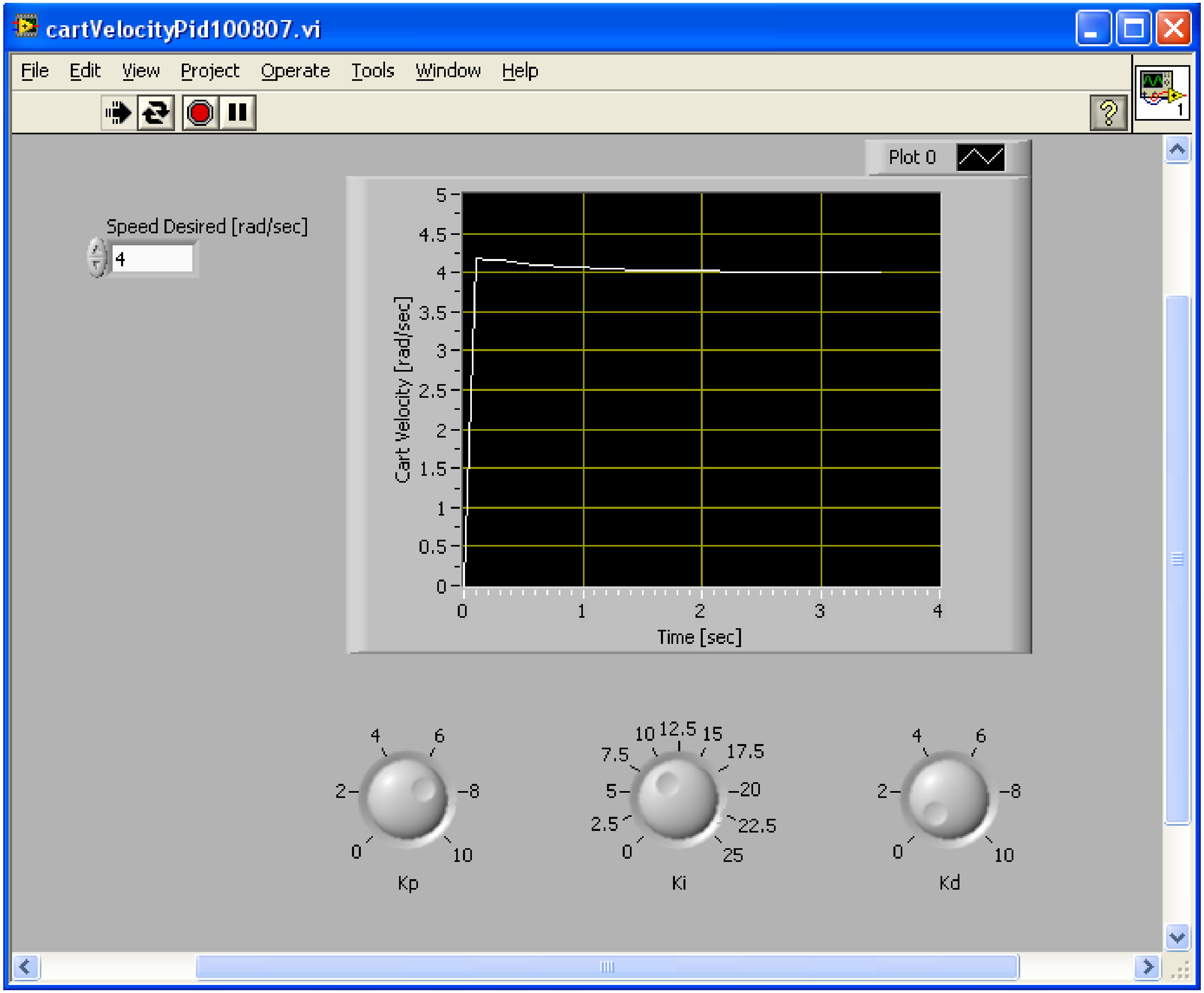
Implement simulation

Observe:

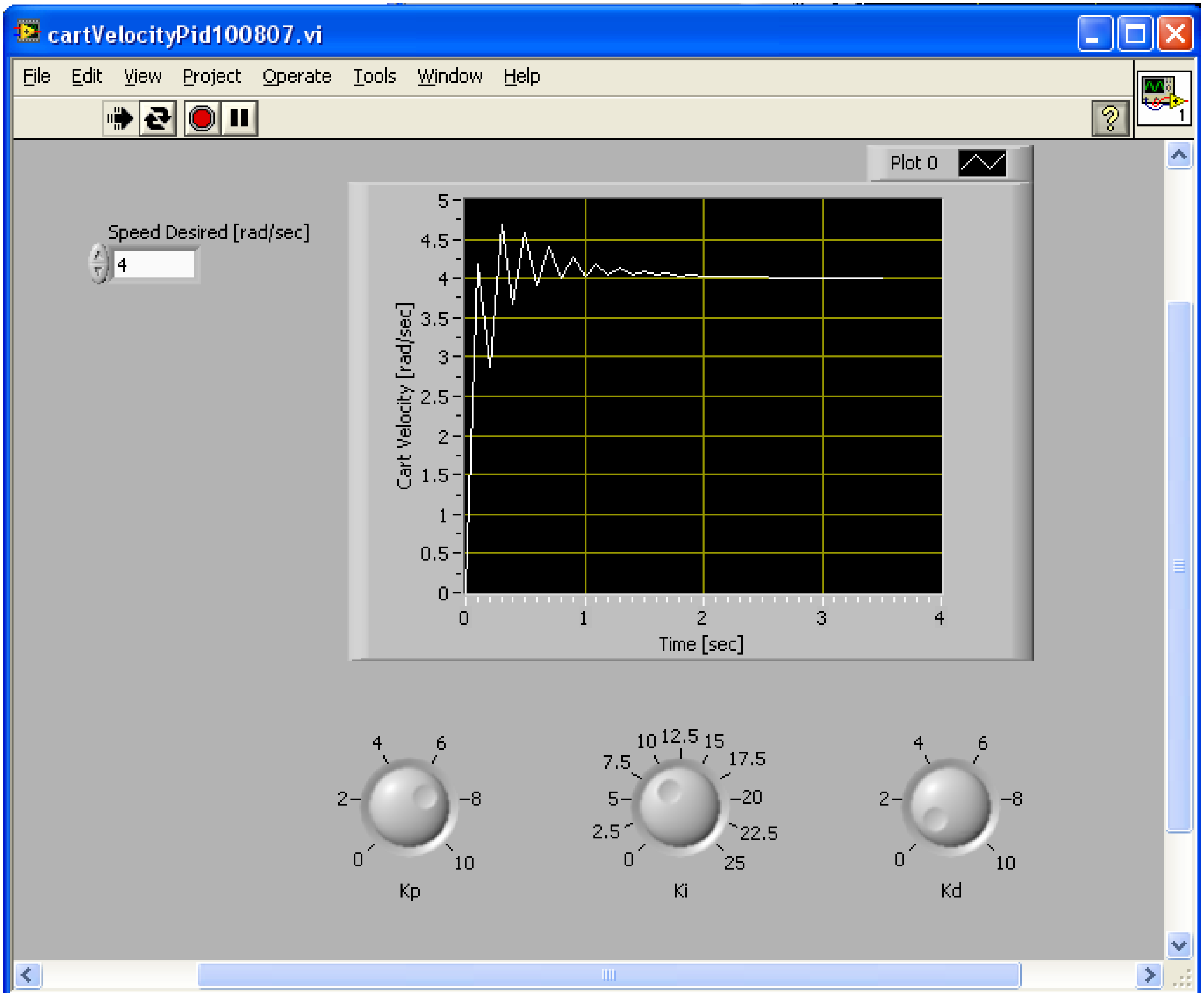
- Faster time constant
- Response vs. stability
- Steady-state error
- When goes unstable



Proportional only: faster time response, but always steady-state error!



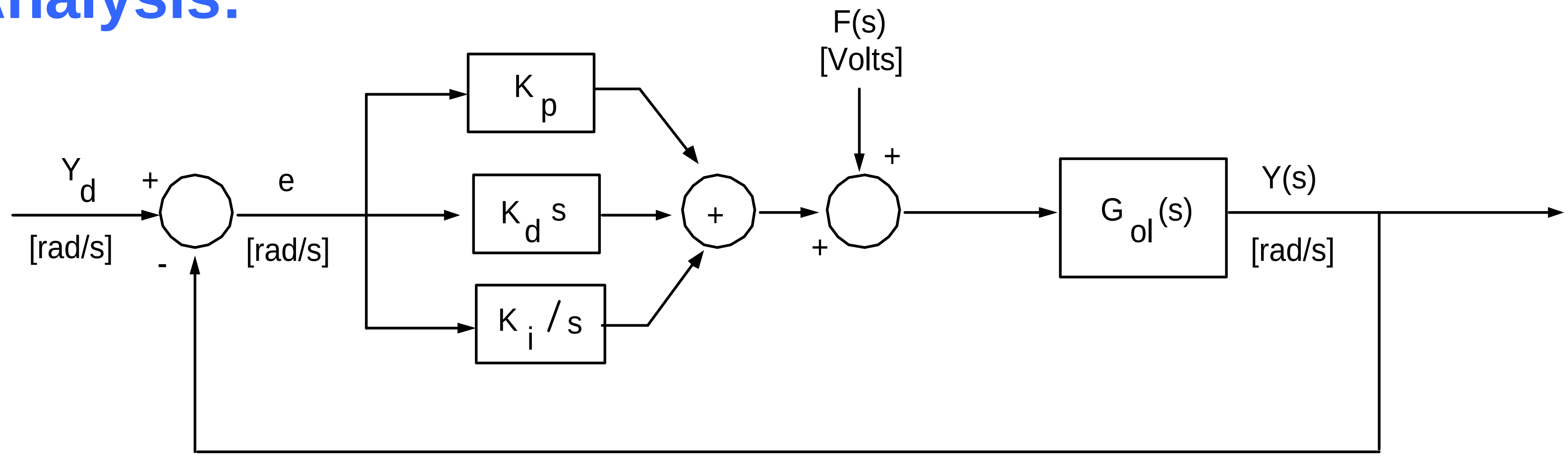
Adding integral control eliminates steady-state error (but overshoot)



Adding derivative yields more overshoot – and possibly unstable

However, tuning can be very tedious. Some knowledge of system type aids in tuning, make performance expectations realistic and avoid instability.

Analysis:



Input-Output Relationship given by:

$$Y(s) = G_{\mathrm{ol}}\left\{F(s) + \left(K_p + \frac{K_i}{s} + K_d s\right)\left(Y_d - Y\right)\right\} \qquad (1)$$

Can reduce to show that (1) becomes:

$$Y(s) = \frac{G_{\text{ol}} s}{s + G_{\text{ol}}\left(K_p s + K_i + K_d s^2\right)} F + \frac{G_{\text{ol}}\left(K_p s + K_i + K_d s^2\right)}{s + G_{\text{ol}}\left(K_p s + K_i + K_d s^2\right)} Y_d \qquad (2)$$

Case Study 1: Proportional only control $Y_d = \frac{A}{s}$ So (2) becomes:

$$Y(s) = \frac{G_{\text{ol}} s}{s + G_{\text{ol}}\left(K_p s\right)} F + \frac{G_{\text{ol}}\left(K_p s\right)}{s + G_{\text{ol}}\left(K_p s\right)} \cdot \frac{A}{s} \qquad (3)$$

Final Value Theorem states that:

$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} s \cdot Y(s)$$

Thus steady-state part of (3) becomes:

$$y_{\text{ss}} = \lim_{t \to \infty} y(t) = \lim_{s \to 0} s \cdot Y(s) = \lim_{s \to 0} \frac{G_{\text{ol}}\left(K_p s\right)}{s + G_{\text{ol}}\left(K_p s\right)} A \qquad (4)$$

Given that cart velocity has an open-loop transfer function of the following form:

$$G_{\mathrm{ol}}(s) = \frac{b}{s+a} \tag{5}$$

Substitution of (5) into (4) yields

$$y_{\mathrm{ss}} = \lim_{s\to 0} \frac{\dfrac{b}{s+a}\left(K_p s\right)}{s + \dfrac{b}{s+a}\left(K_p s\right)} A = \lim_{s\to 0} \frac{bK_p s}{s(s+a)+bK_p s} A$$

Applying L'Hopital to calculate the limit yields:

$$y_{\mathrm{ss}} = \lim_{s\to 0} \frac{bK_p}{2s+a+bK_p} A = \frac{bK_p}{a+bK_p} A$$

Note: If K_p is very large, then $y_{ss} \approx A$

Will always have steady-state error

Case Study 2: Proportional + Integral control $Y_d = \dfrac{A}{s}$

So (2) becomes:

$$Y(s) = \frac{G_{\text{ol}} s}{s + G_{\text{ol}}\left(K_p s + K_i\right)} F + \frac{G_{\text{ol}}\left(K_p s + K_i\right)}{s + G_{\text{ol}}\left(K_p s + K_i\right)} \cdot \frac{A}{s}$$

Apply Final Value Theorem:

$$y_{\text{ss}} = \lim_{t \to \infty} y(t) = \lim_{s \to 0} s \cdot Y(s) = \frac{G_{\text{ol}}\left(K_p s + K_i\right)}{s + G_{\text{ol}}\left(K_p s + K_i\right)} A \qquad (6)$$

Substituting the OLTF (2) into (6) yields:

$$y_{\text{ss}} = \lim_{s \to 0} \frac{\frac{b}{s+a}\left(K_p s + K_i\right)}{s + \frac{b}{s+a}\left(K_p s + K_i\right)} A = \lim_{s \to 0} \frac{b\left(K_p s + K_i\right)}{s(s+a) + b\left(K_p s + K_i\right)} A = \frac{bK_i}{bK_i} A = A$$

Hence integral action ensures zero steady-state error

Systems like the motorized tethered cart are called Type 0 systems:

$$G_{\text{ol}}(s) = \frac{b}{s+a}$$

All transfer functions can be factored into the general form:

$$G_{ol}(s) = \frac{(\tau_a s+1)\cdots(\tau_m s+1)}{s^i(\tau_1 s+1)\cdots(\tau_n s+1)}$$

$i=0$	Type 0 System
$i=1$	Type 1 System
$i=2$	Type 2 System

General form any TF

A Type i system is the number of "free" integrators, i. The motorized tethered cart, for velocity control, is a Type 0 system

For Type 0 Systems:

- Will always have steady-state error (with proportional only control): see Case 1
- Integral action will eliminate steady-state error (see Case 2)
- Derivative action may increase transient response but cause instability

Homework:

- Show steady-state response for derivative action on Type 0 system
- Response to ramp input