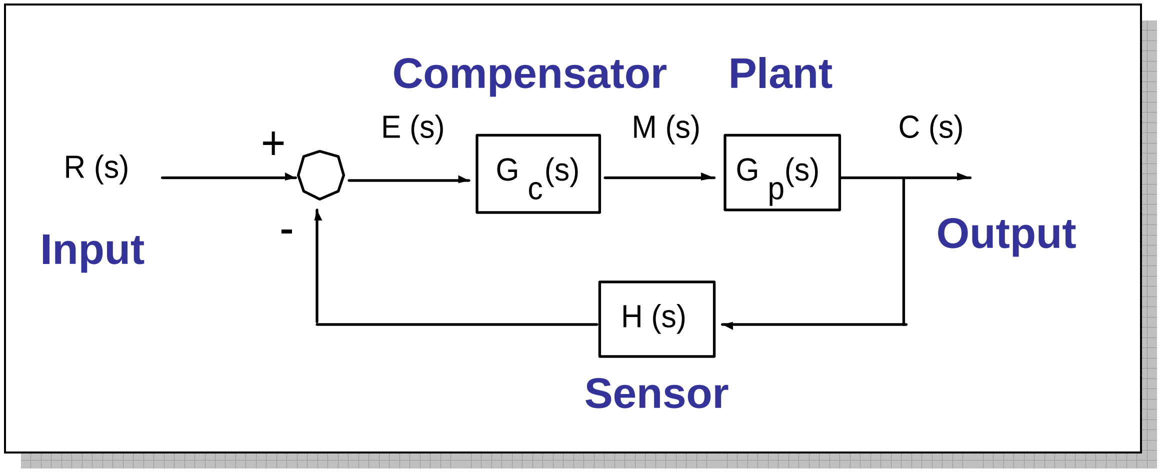
Block Diagrams and Transfer Functions

MEM 639 Real-Time Microcomputer Control 1

Block Diagrams

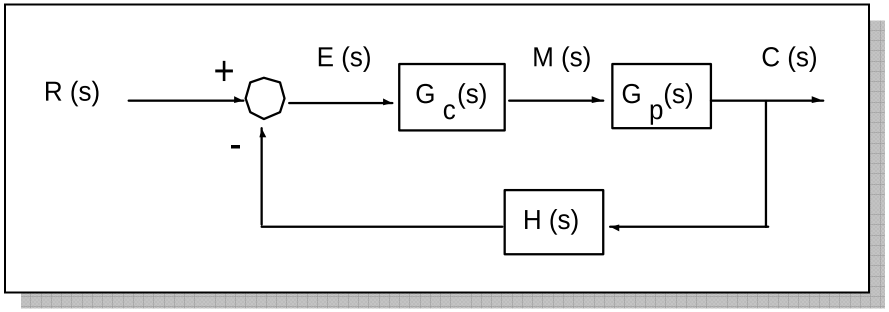


Typical Block Diagram

Pictorially provides

- Input-output relationships
- Signal flow
- Functional role
- Blueprint for Design

Block Diagrams: Can Modularize!



Start from output end first

$$C(s) = MG_p \;\; = EG_cG_p \; = (R - HC)G_cG_p$$

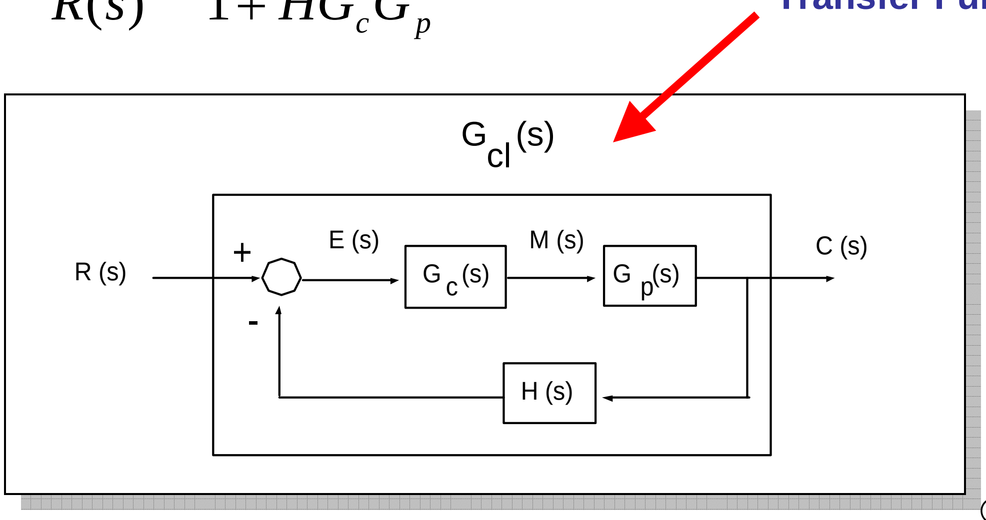
Get into TF form

$$C(s)(1 + HG_cG_p) = RG_cG_p$$

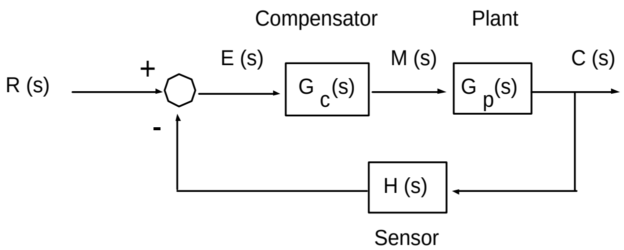
$\frac{\text{Output}}{\text{Input}}$

$$\frac{C(s)}{R(s)} = \frac{G_cG_p}{1 + HG_cG_p} \;\; = G_{cl}(s)$$

CLTF: Closed-Loop Transfer Function



Illustrative Problem: Given the following



$$G_c(s) = 2$$

$$G_p(s) = \frac{3s+8}{s^2+2s+2}$$

$$H(s) = 1$$

A. Write the differential equation of the plant that relates $c(t)$ and $m(t)$

$$G_p(s) = \frac{3s+8}{s^2+2s+2} = \frac{C(s)}{M(s)} \quad \text{then} \quad (s^2+2s+2)C(s) = (3s+8)M(s)$$

Consequently $\ddot{c}(t) + 2\dot{c}(t) + 2c(t) = 3\dot{m}(t) + 8m(t)$

B. Write the differential equation of the closed-loop system relating $c(t)$ and $r(t)$

Can show that the CLTF is given by $\dfrac{C(s)}{R(s)} = \dfrac{G_cG_p}{1+HG_cG_p}$

Consequently

$$\frac{C(s)}{R(s)} = \frac{2\dfrac{3s+8}{s^2+2s+2}}{1+1\cdot 2\cdot\dfrac{3s+8}{s^2+2s+2}} = \frac{6s+16}{s^2+2s+2+(6s+16)}$$

Hence $\left\{s^2+8s+18\right\}C(s) = \left\{6s+16\right\}R(s)$ or $\ddot{c}+8\dot{c}+18c = 6\dot{r}+16r$

C. The transfer function pole term $(s+a)$ yields a time constant $\tau = 1/a$

Find the time constants for both the open-loop and closed-loop systems

The OLTF is: $$\frac{C(s)}{R(s)} = G_c G_p = 2\frac{3s+8}{s^2+2s+2} = \frac{6s+16}{(s+1+i)(s+1-i)}$$

Taking the real-part of the complex root, then $a = 1$ and hence $\tau = 1/a = 1$ sec

The CLTF is: $$\frac{C(s)}{R(s)} = \frac{6s+18}{s^2+8s+18} = \frac{6s+18}{(s+4+0.707i)(s+4-0.707i)}$$

Taking the real-part of the complex root, then $a = 4$ and hence $\tau = 1/4 = 0.25$ sec