Block Diagrams and Transfer Functions

MEM 639 Real-Time Microcomputer Control 1
Block Diagrams

Typical Block Diagram

- Pictorially provides
- Input-output relationships
- Signal flow
- Functional role
- Blueprint for Design

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Block Diagrams: Can Modularize!

Start from output end first

\[ C(s) = MG_p = EG_c G_p = (R - HC)G_c G_p \]

Get into TF form

\[ C(s)(1 + HG_c G_p) = RG_c G_p \]

\[ \frac{C(s)}{R(s)} = \frac{G_c G_p}{1 + HG_c G_p} = G_{cl}(s) \]

CLTF: Closed-Loop Transfer Function
**Illustrative Problem:** Given the following

![Block diagram of a control system with a compensator, plant, and sensor.]

\[ G_c(s) = 2 \]
\[ G_p(s) = \frac{3s + 8}{s^2 + 2s + 2} \]
\[ H(s) = 1 \]

A. Write the differential equation of the plant that relates \( c(t) \) and \( m(t) \)

\[ G_p(s) = \frac{3s + 8}{s^2 + 2s + 2} = \frac{C(s)}{M(s)} \]

then \( (s^2 + 2s + 2)C(s) = (3s + 8)M(s) \)

Consequently \( \ddot{c}(t) + 2\dot{c}(t) + 2c(t) = 3\dot{m}(t) + 8m(t) \)

B. Write the differential equation of the closed-loop system relating \( c(t) \) and \( r(t) \)

Can show that the CLTF is given by \( \frac{C(s)}{R(s)} = \frac{G_c G_p}{1 + H G_c G_p} \)

Consequently

\[ \frac{C(s)}{R(s)} = \frac{2 \cdot \frac{3s + 8}{s^2 + 2s + 2}}{1 + 1 \cdot 2 \cdot \frac{3s + 8}{s^2 + 2s + 2}} = \frac{6s + 16}{s^2 + 2s + 2 + (6s + 16)} \]

Hence \( \{s^2 + 8s + 18\}C(s) = \{6s + 16\}R(s) \) or \( \dddot{c} + 8\dddot{c} + 18\dot{c} = 6\ddot{r} + 16\dot{r} \)
C. The transfer function pole term \((s + a)\) yields a time constant \(\tau = \frac{1}{a}\)

Find the time constants for both the open-loop and closed-loop systems

The OLTF is: 
\[
\frac{C(s)}{R(s)} = G_c G_p = 2 \frac{3s + 8}{s^2 + 2s + 2} = \frac{6s + 16}{(s + 1 + i)(s + 1 - i)}
\]

Taking the real-part of the complex root, then \(a = 1\) and hence \(\tau = \frac{1}{a} = 1\) sec

The CLTF is: 
\[
\frac{C(s)}{R(s)} = \frac{6s + 18}{s^2 + 8s + 18} = \frac{6s + 18}{(s + 4 + 0.707i)(s + 4 - 0.707i)}
\]

Taking the real-part of the complex root, then \(a = 4\) and hence \(\tau = \frac{1}{4} = 0.25\) sec