Voltage Dividers

MEM 639 Real-Time Microcomputer Control 1
Digital-to-Analog Converters (DAC) – generate voltage using n-bit digital data

A. Voltage Dividers

Ohm’s Law, can prove that:

\[ V_A = \frac{R_A}{R_A + R_B} V_{in} \quad \text{and} \quad V_B = \frac{R_B}{R_A + R_B} V_{in} \]

NB: \( V_A + V_B = V_{in} \)

B. An N-bit DAC has N voltage dividers

Example: Class circuit:

\[ V_{out} = \frac{WORD - 128}{128} V_{ref} \]

Where \( V_{ref} = 5.0 \) V

<table>
<thead>
<tr>
<th>WORD</th>
<th>Voltage</th>
</tr>
</thead>
<tbody>
<tr>
<td>255</td>
<td>4.961 V</td>
</tr>
<tr>
<td>128</td>
<td>0.00 V</td>
</tr>
<tr>
<td>129</td>
<td>0.039 V</td>
</tr>
<tr>
<td>0</td>
<td>-5.00 V</td>
</tr>
</tbody>
</table>

Resolution = 40 mV
Op-Amps

MEM 639 Real-Time Microcomputer Control 1
Ideal Op-Amps: 2 rules and 5 properties

Rule 1: Voltage at “+” and “-” terminals are equal
Rule 2: No current at “+” and “-” terminals

- Input impedance \( R_d \approx \infty \)
- Output impedance \( R_a \approx 0 \)
- Open-loop gain is \( \infty \)
- Bandwidth is \( \infty \)
- \( v_a = 0 \) when \( v_1 = v_2 \)

Example: Derive the op-amp’s input-output relationship

Solution:
Step 1: Label currents and voltage points
Step 2: Apply Rules 1 and 2

With Rule 2, observe that \( i_i + i_f = 0 \)

Or, \( \frac{v_i - v_\alpha}{R_i} + \frac{v_o - v_\alpha}{R_f} = 0 \)

With Rule 1, observe that \( v_\alpha = 0 \)

Hence \( \frac{v_i}{R_i} = -\frac{v_o}{R_f} \) or \( v_o = -\frac{R_f}{R_i} v_i \)

The gain is \( -\frac{R_f}{R_i} \) and hence is called an inverting op-amp. In practice, one cannot have infinite gain.

In fact, even if the input and feedback resistors are small and large respectively, the gain will be clamped. Op-amps have rail voltages, often denoted as +V and –V. Instead of swinging to infinity, we have

If \( v_1 > v_2 \) then \( v_a = +V \)

If \( v_1 < v_2 \) then \( v_a = -V \)
Impedance – what’s the big deal?

Illustrative Problem: Design an inverting op-amp that has a gain of -10

Solution (non-unique):

Recall, we solved that: \( v_o = -\frac{R_f}{R_i} v_i \)

Choose \( R_f = 1000\Omega \) and \( R_i = 100\Omega \)

Naïve solution! Gain is -10 if no impedance!

Impedance and Resistance obstruct alternating (AC) and direct current (DC) respectively. Impedance is zero only at absolute zero temperature (molecules don’t move). Otherwise, everything in nature has impedance, including power supplies.

Re-examining the problem realistically:

Thus have by voltage divider equation \( v_i' = \frac{R_i v_i}{R_i + R_s} \) where \( R_s \approx 25\Omega \) typically.

Hence using a naive selection of resistors \( v_i' = \frac{100}{100 + 25} v_i = 0.8 v_i \)

You’re only getting 80% of the input voltage you think you’re putting in.

Rule of thumb: \( R_i > 10 \cdot R_s \) is a sloppy circuit
\( R_i > 100 \cdot R_s \) is reasonable.
Voltage Follower – what’s the big deal?

Illustrative Problem: Derive the input-output relationship for the following

Solution: Rule 1 says that the “+” and “-” voltages are equal.

Thus have \( v_o = v_i \)

Question: Why would anyone want a unity gain amplifier?

Answer: Because op-amps have near zero output impedance.

Thus even though the power supply generating \( v_i \) has impedance \( R_s \)

\( v_o \) will give \( v_i \) with little output impedance

As such, voltage followers are great and often used in instrumentation amplifiers