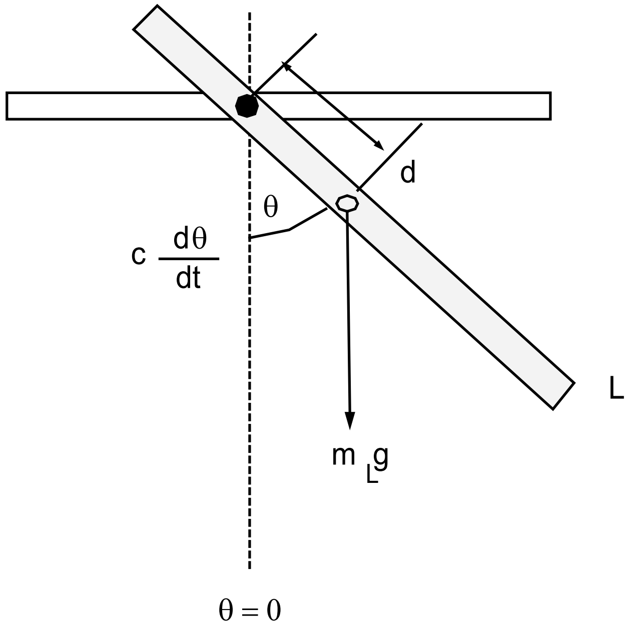
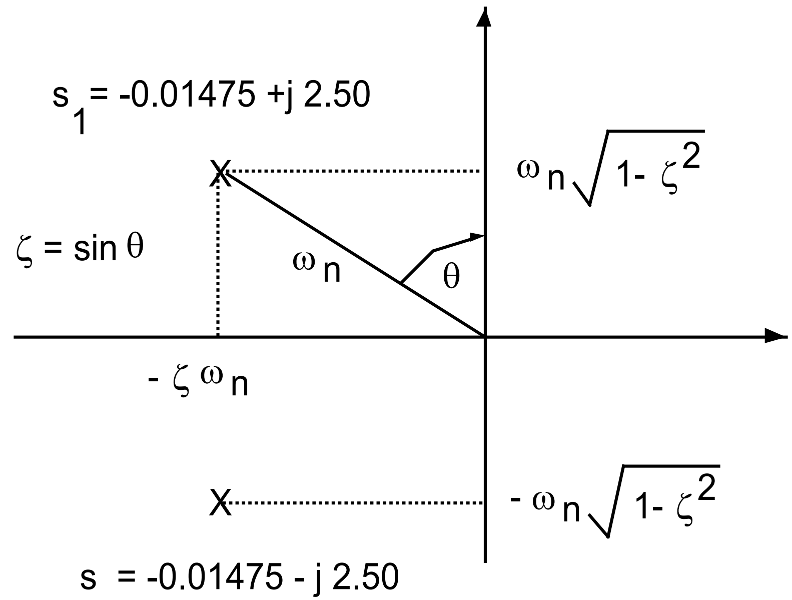
MEM 351 – Dynamic Systems Lab

Control Design 1: Pole-placement

Recap: Pole locations connote system stability





Designer:

Place poles in desired locations to yield desired response (damping, settling time etc)

2nd order damped system $\ddot{\theta} + 2\zeta\omega_n\dot{\theta} + \omega_n^2\theta = 0$

Yields complex roots, $s_{1,2} = -\zeta\omega_n \pm j \cdot \omega_n\sqrt{1-\zeta^2}$

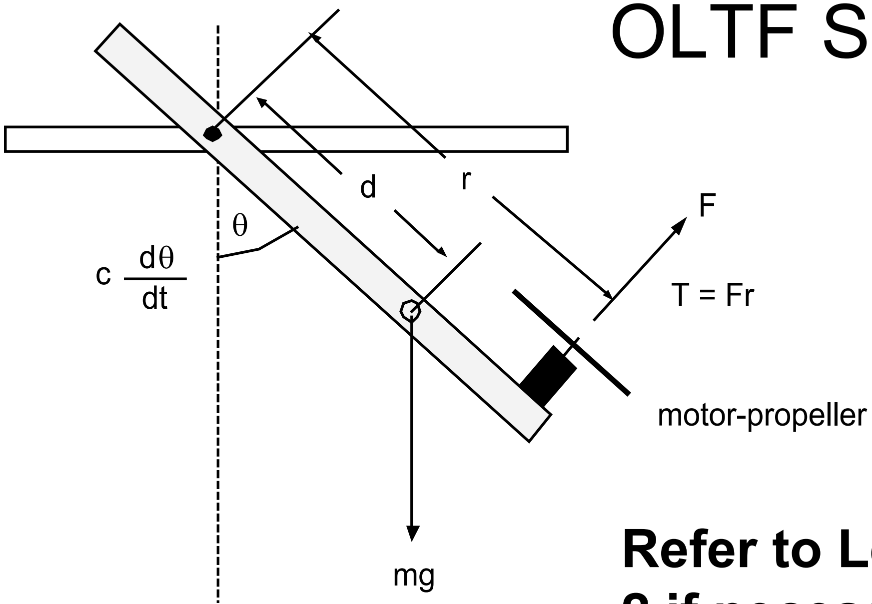
Refer to Lecture 2 if necessary

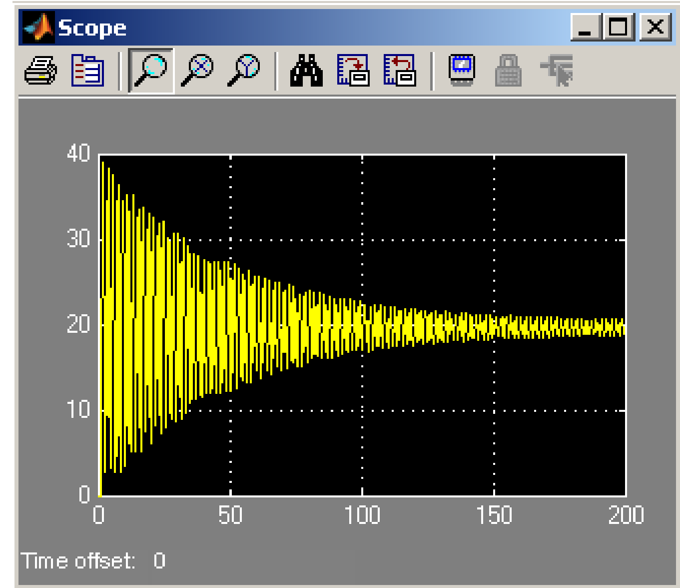
Time domain solution

$$\theta_c(t) = e^{-\zeta\omega_n t}\left\{A_1\cos\left(\omega_n t\sqrt{1-\zeta^2}\right) + A_2\sin\left(\omega_n t\sqrt{1-\zeta^2}\right)\right\} \quad (1)$$

Small real roots, means long settling time

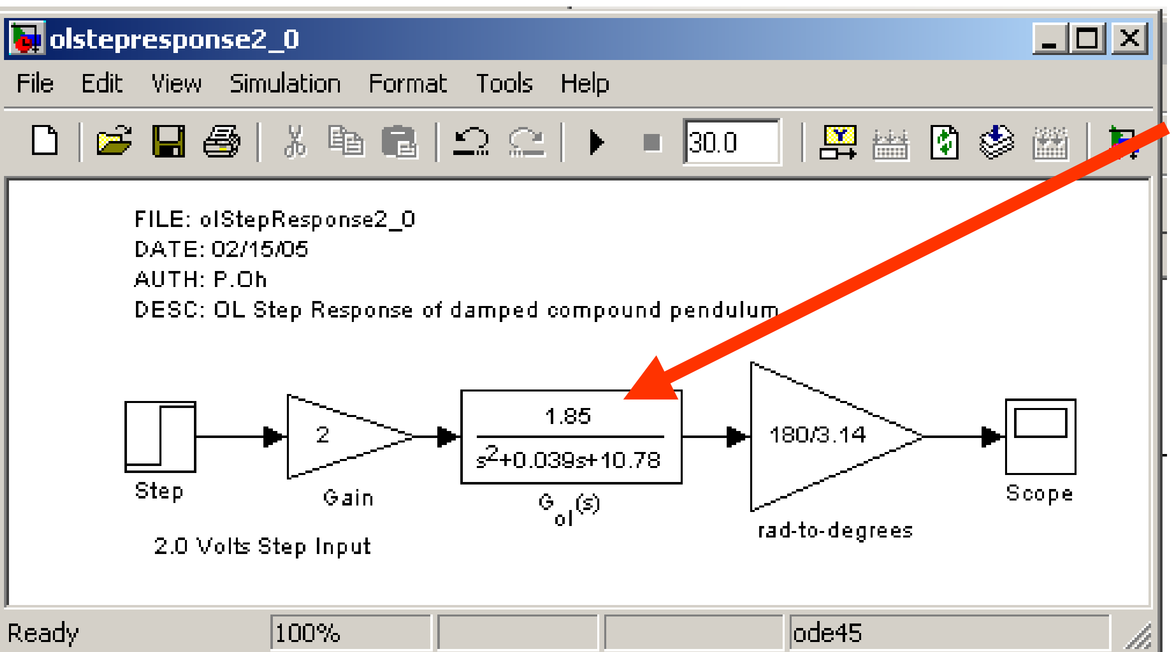
OLTF Simulations





Refer to Lecture 2 if necessary

Simulink

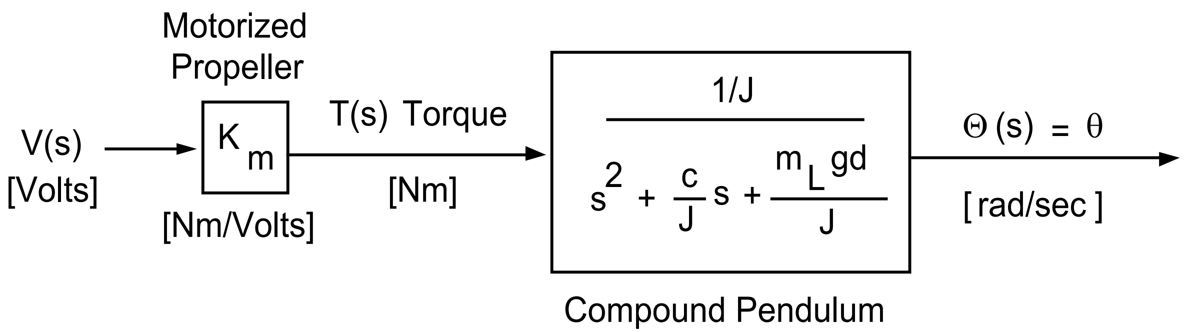


Roots of the denominator (i.e. the poles) are:

$$\begin{aligned} s_1 &= -0.0019 + j3.28 \\ s_2 &= -0.0019 - j3.28 \end{aligned} \qquad (2)$$

Small real root will yield long settling times

Recall: State Space Realization (Lecture 3)



Given $\ddot{\theta} + \frac{c}{J}\dot{\theta} + \frac{m_L gd}{J}\theta = \frac{T}{J}$ then $\ddot{\theta} + \frac{c}{J}\dot{\theta} + \frac{m_L gd}{J}\theta = \frac{K_m}{J}V$ (3)

Suppose define two state variables:

$$x_1 = \theta \quad \text{and} \quad x_2 = \dot{\theta}$$

One can re-write (3) as

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{c}{J}x_2 - \frac{m_L gd}{J}x_1 + \frac{K_m}{J}V \qquad (4)$$

State Space Realization continued…

State space form given by matrices:

$$\dot{\vec{x}} = F\vec{x} + G\vec{u}$$
$$\vec{y} = H\vec{x} + J\vec{u} \qquad (5)$$

Hence, re-expressing

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{c}{J}x_2 - \frac{m_L g d}{J}x_1 + \frac{K_m}{J}V$$

Gives

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{m_L g d}{J} & -\frac{c}{J} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ K_m / J \end{bmatrix} u \qquad (6)$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0 = x_1 = \theta$$

The characteristic equation from state space (6) is defined as:

$$\alpha(s) = \det(sI - F) = 0 \qquad (7)$$

$$\alpha(s) = \det\left[\begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ -\dfrac{m_L g d}{J} & -\dfrac{c}{J} \end{pmatrix}\right]$$

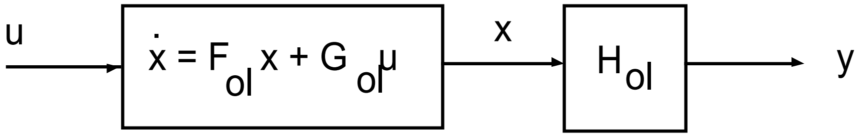
$$= \det\left[\begin{pmatrix} s & -1 \\ 10.77 & s + 0.039 \end{pmatrix}\right] = s^2 + 0.039s + 10.77 \qquad (8)$$

Which is the same as the denominator given in (2)

$$\frac{\Theta(s)}{V(s)} = \frac{1.89}{s^2 + 0.039s + 10.77} = G_{ol}(s)$$

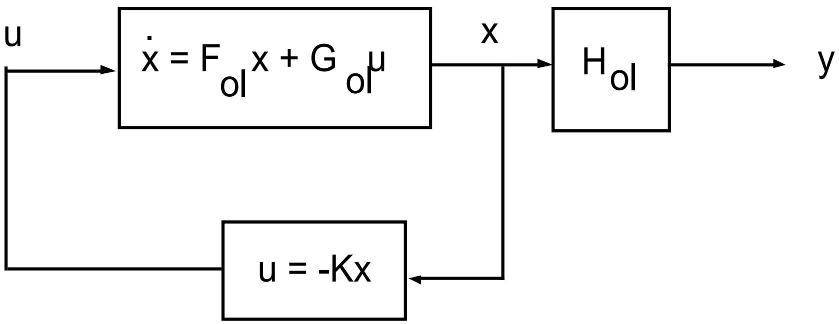
Controller Design 1: Pole Placement

Open-Loop System



$$\dot{\vec{x}} = F_{ol}\vec{x} + G_{ol}\vec{u}$$
$$\vec{y} = H_{ol}\vec{x} + J_{ol}\vec{u} \quad (9)$$

Feedback System



$$\vec{u} = -K\vec{x} \quad (10)$$

$$\dot{\vec{x}} = F_{ol}\vec{x} - G_{ol}K\vec{x} = (F_{ol} - G_{ol}K)\vec{x}$$

Characteristic equation for (10) given by

$$\alpha(s) = \det(F_{ol} - G_{ol}K) = 0 \quad (11)$$

Now suppose one has a desired characteristic equation α_c and hence desired pole locations:

$$\alpha_c = (s - s_1)(s - s_2)\cdots(s - s_n) = 0 \qquad (12)$$

One then matches the coefficients in (12) with those of (11) to yield values for gains

$$K = [k_1 \ldots k_n]$$

Example: Pole Placement

Suppose one wants a settling time of $t_s = 1.67$ sec
and a damping ratio $\zeta = 0.707$

This results in poles $s_{1,2} = -2.4 \pm j2.4$
for the damped compound pendulum

Calculate gains k_1 and k_2

Solution: Substituting desired poles in (12) yields

$$\alpha_c = (s + 2.4 + j2.4)(s + 2.4 - j2.4) = s^2 + 4.8s + 2 \cdot 2.4^2$$

$$\alpha_c = s^2 + 4.8s + 11.52 = s^2 + \alpha_1 s + \alpha_0$$

The coefficients must be equated to (7)

Recall (7) from previous slide $\alpha(s) = \det(F_{ol} - G_{ol}K) = 0$

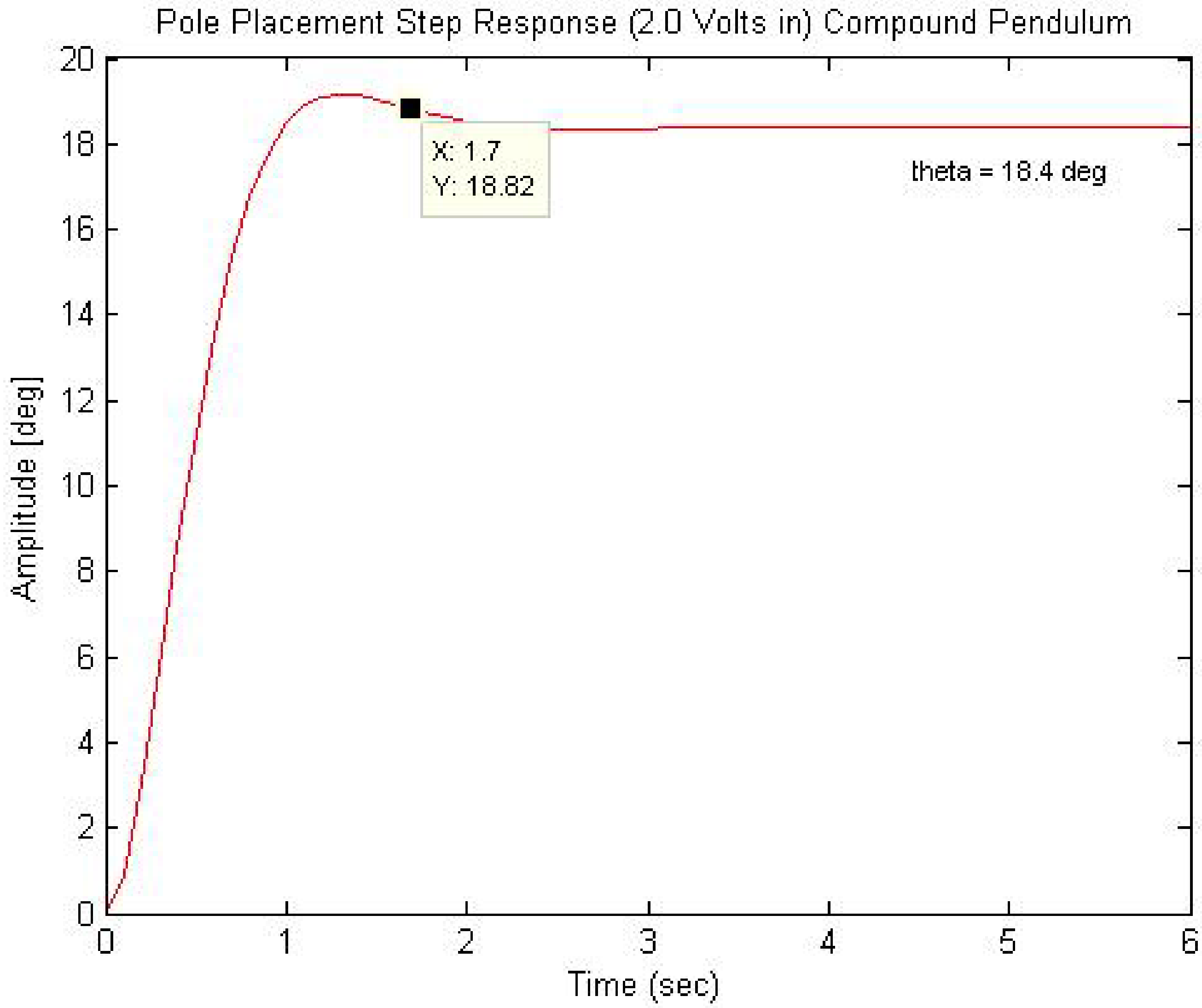
(7) becomes

$$\det\left\{\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \left(\begin{bmatrix} 0 & 1 \\ -\dfrac{m_L gd}{J} & -\dfrac{c}{J} \end{bmatrix} - \begin{bmatrix} 0 \\ K_m / J \end{bmatrix}\begin{bmatrix} k_1 & k_2 \end{bmatrix}\right)\right\}$$

$$\det\begin{bmatrix} s & -1 \\ \dfrac{m_L gd + K_m k_1}{J} & s + \dfrac{c + K_m k_2}{J} \end{bmatrix} = s^2 + \left(\frac{c + K_m k_2}{J}\right)s + \frac{m_L gd + K_m k_1}{J} = 0$$

Can show that

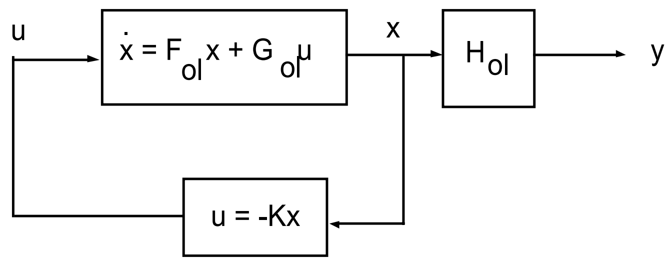
$$k_1 = \frac{\alpha_0 J - m_L gd}{K_m} \qquad k_2 = \frac{\alpha_1 J - c}{K_m}$$

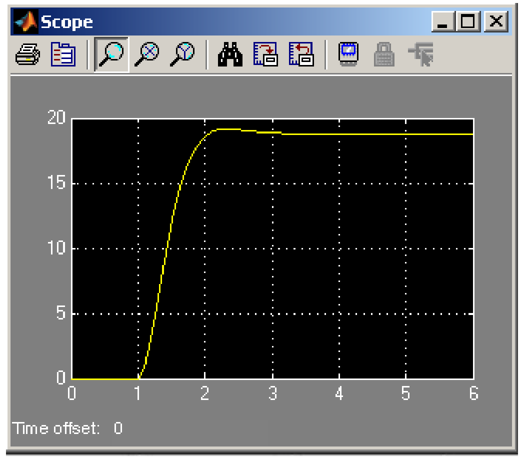


Slight overshoot and reduced settling time – inline with desired response

There are Many Controllers on the Market…

Pole Placement





Considerations

- Analytical or Ad hoc?
- Physically realizable?
- Linear Assumptions?
- Sensors?
- Computer S/W H/W?

PID