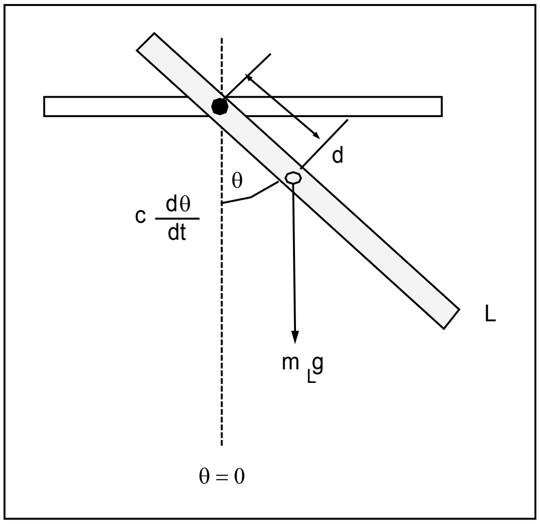
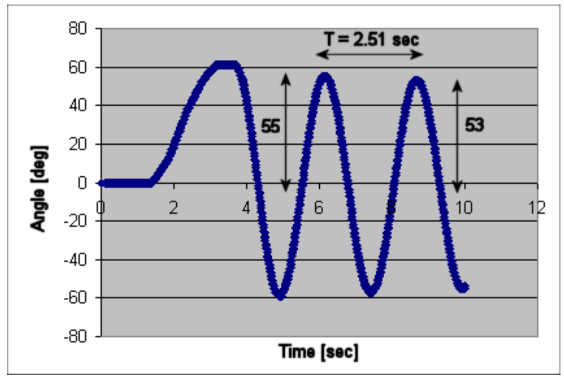
Homework: Modeling and System Identification

Below is a free-body diagram of the damped compound pendulum, dimensions and experimental data plotting the time response from free fall.



L	Bar length	0.495	m
d	Pivot to CG distance	0.023	m
m_L	Mass of pendulum	0.43	kg



1. Prove the moment of inertia J about the pivot point is 0.0090 $kg \cdot m^2$. Assume a slender rigid bar and use the parallel axis theorem (5 points)

2. Use the experimental data above to show that $\zeta = 0.0059$ and $\omega_n = 2.50$ rad/sec. (5 points)

3. The equation for the experimental data is given by $\ddot{\theta} + 2\varsigma\omega_n\dot{\theta} + \omega_n^2\theta = 0$. Compare this to equation of motion $\ddot{\theta} + \frac{c}{J}\dot{\theta} + \frac{m_L gd}{J}\theta = 0$ to show that $c = 0.00035 \frac{N \cdot m \cdot s}{rad}$ and $\omega_n = 3.28 \frac{rad}{s}$ (5 points)

4. Include an Excel plot of the data you acquired in Lab (where you used the encoder to capture angle data). Label consecutive peaks and period. Show calculations for ζ and ω_n (20 points)

5. Given the following differential equation $\ddot{\theta} + \frac{c}{J}\dot{\theta} + \frac{m_L gd}{J}\theta = \frac{K_m}{J}V$

   Use state variables $x_1 = \theta$ and $x_2 = \dot{\theta}$ to show that the state space realization is (5 points)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{m_L gd}{J} & -\frac{c}{J} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ K_m / J \end{bmatrix} u$$

$$\vec{y} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u$$