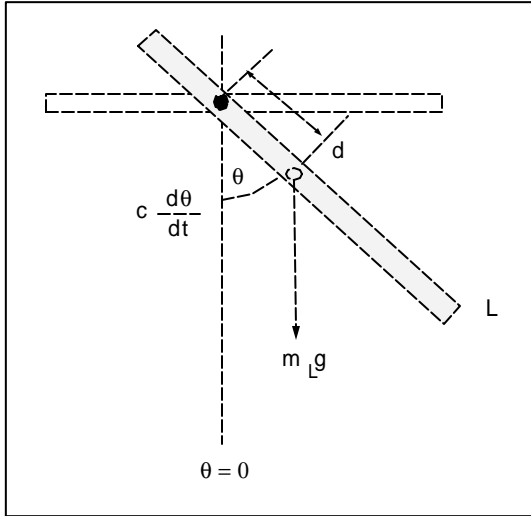
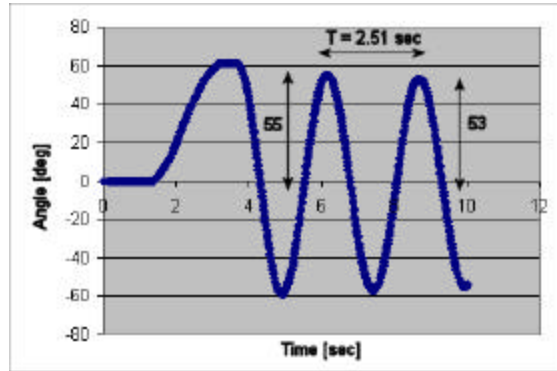


Homework – Representations, Poles and Zeros

Below is a free-body diagram of the damped compound pendulum, dimensions and experimental data plotting the time response from free fall.



L	Bar length	0.495	m
d	Pivot to CG distance	0.023	m
m_L	Mass of pendulum	0.43	kg

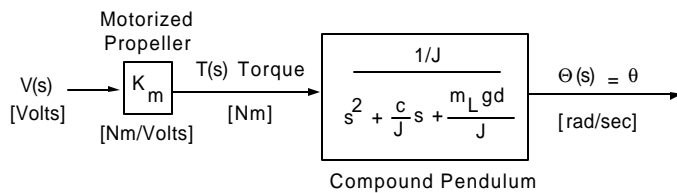


- If c is the viscous damping coefficient, show that the equation of motion for the damped compound pendulum sketch above is given the equation below (10 points)

$$\ddot{q} + \frac{c}{J}\dot{q} + \frac{m_L g d}{L}q = 0$$

- Given the following block diagram and values, show that the open-loop transfer function is given by (5 points)

$$\frac{\Theta(s)}{V(s)} = \frac{1.89}{s^2 + 0.039s + 10.77} = G_{ol}(s)$$



J	= 0.0090	$kg \cdot m^2$
c	= 0.00035	Nms / rad
m_L	= 0.43	kg
d	= 0.023	m
g	= 9.81	m / s^2
K_m	= 0.017	Nm / V

- Show that the time constant $T_c = 0.01475 \text{ sec}$ and settling time $t_s = 271.2 \text{ sec}$ for the damped compound pendulum given that $z = 0.0059$ and natural frequency $\omega_n = 2.50 \text{ rad} / s$ (5 points)
- Show that the complex poles $s_{1,2} = -2.4 \pm j2.4$ will yield $\omega_n = 3.39 \text{ rad} / s$ and phase angle $q = 0.785 \text{ rad}$ (5 points)