FINAL PROJECT

Engineering Reliability
MEM 361

Data Sampling

By

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Problem:

An entrepreneur plans to sell 10000 products over a period of time at a price of $u per product. It costs $r to repair a product. [$u and $r are specified for each team]. The entrepreneur needs to determine the warranty period that will ensure at least a 10% profit. A series of tests of the products (sampling) to estimate the Time-Of-Failure (TOF) yields the data (units of time may be {hr, day, months, years}) given below. [This is different for each team]. Assume that the products fail only once.

Draw a histogram of TOF and recommend an appropriate pdf for TOF.

Using “mean-ranking –approach” establish/determine, for TOF
a. normal pdf,
b. long-normal pdf
c. weibull pdf

Determine in each of the three cases above
a. mean
b. mode
c. skewness

Determine the warranty period, based on each pdf.
### Data:

Sample: TOF(days), x: F(xi): Z(i):

<table>
<thead>
<tr>
<th>Sample</th>
<th>TOF(days)</th>
<th>x:</th>
<th>F(xi)</th>
<th>Z(i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>58.523</td>
<td>-447.697</td>
<td>0.048</td>
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<tr>
<td>2</td>
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<td>44.223</td>
<td>-294.080</td>
<td>0.095</td>
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<td>-294.080</td>
<td>0.143</td>
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<td>4</td>
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<td>0.190</td>
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<td>-180.362</td>
<td>0.238</td>
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<td>6</td>
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<td>7.023</td>
<td>-18.610</td>
<td>0.286</td>
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<td>7.023</td>
<td>-18.610</td>
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<td>7.023</td>
<td>-18.610</td>
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</table>

| Mean   | 14.65      | 5.918  | 0.811  | 35.028 |

**Histogram**

The histogram shows the distribution of TOF values across the samples.
**The Normal PDF:**

From the example in Chapter 3, (in the book), the method of solving for the Normal Distribution Function was shown. “x” is the amount of time that it takes the product to fail in order to make us a 10 percent profit.

The Z(i) vs. TOF graph shown below was used to solve for the mean and deviation. The sample data was compared to the actual data, and the values were almost the same. This tells shows that the samples used will give good approximation of how many products should fail in order to make a profit (10%).

In the graph below, the Z(i) values were calculated by interpolating the F(xi) values in Appendix III-A. The best fit line equation on this graph will give us the true values of the mean and standard deviation. From the Z(i) vs. TOF graph, the mean was determined (14.6494). The mode is equal to 30. (Highest value of xi). The standard deviation is equals to 7.30458. (Difference between Z(1) and Z(0)).
Z(i) vs. TOF

\[ y = 0.1369x - 2.0055 \]

F(xi) vs. TOF

\[ y = 0.0433x - 0.1348 \]
Log-Normal PDF:

The graphs were determined in the same manner as the Normal Distribution Plot, but the natural log of the TOF was used for the x axis.
**Weibull Distribution:**

Weibull Distribution:

![Graph of F(xi) vs. ln(xi)](image)

![Graph of Y vs. ln(xi)](image)

Maple Calculations for determining the Weibull Distribution Function

```maple
> y := 2.566*x - 7.2121;

\( y := 2.566 \xi - 7.2121 \)
> solve(y=0, xi);  
> xio=2.810639127;  
> fx:=(m/phi)*(x/phi)^(m-1)*exp(-(x/phi)^m);  
> solve(ln(phi)=xio,phi);  
> phi:=exp(xio);  
> phi:=16.6205;  
> m:=2.566;  
> fx:=(m/phi)*(x/phi)^(m-1)*exp(-(x/phi)^m);  
> plot(fx,x=-5..50);
Warranty Period:

In order to find the 10 percent profit, the calculation below was carried out.

\[(10,000 \times 50 \times 0.10) = 50000\]. Where 10,000 is the total products and \((50\times 0.10)\) is the cost of each product. The calculation of how many parts needed to fail in order to make a 10% profit is: \(10000(50) - (X \times 100) = 50,000\). Solved for X, and got \(X = 4,500\) products. Therefore, 4,500 products need to fail in order to make a 10% profit. From Appendix III-A, the z value was determined: \(z = -0.15\). Was calculated using the equation for calculating z (shown on the last page).

For the normal distribution function: \(x = (-0.15 \times 5.918403501) + 14.65 = 13.76\) days.

For the log-normal distribution function \(x = 12.49\) days. For the weibull distribution
function we used maple: \( x = 16.62 \) days. From this, a conclusion was reached that the warranty should be for 14.29 days. This will make sure that a 10% profit is always maintained, and hopefully help in gaining the trust of customers.

In conclusion, the Weibull Distribution function turned out to be the closest function to the histogram shown above, in the sense that they are both slightly off-centered.

**Equations:**

\[
\mu = \sum x_i \left( \frac{1}{N} \right),
\]

\[
(\sigma_s)^2 = \left( \frac{1}{N} \right) \sum (x_i - \mu_s)^2 \text{ used to calculate the variance}
\]

\[
(sk)_s = \left[ \frac{1}{N\sigma^3} \right] \sum (x_i - \mu_s)^3 \text{ used to calculate the skewness}
\]

\( F(x_i) \) was calculated from the following equation: \( F(i) = \frac{i}{N+1}, i = 1,20 \)

\( N = \) the total number of samples (20)

The normal distribution was calculated from the following equation: (equation 3.7)

\[
f(x) = \left( \frac{1}{b\sqrt{2\pi}} \right) e^{\left[ \frac{1}{2} \left( \frac{x-a}{b} \right)^2 \right]}
\]

The warranty was calculated from the following equation: \( z = \frac{(\ln(x) - \ln(x_0))}{\omega_0} \)

Used the equations \( z = \frac{(x - \mu)}{\sigma} \) and \( z = \frac{(\ln(x) - \ln(x_0))}{\omega_0} \) for the normal distribution and log-normal distributions respectively.