Within and Across Country Inequality in a Model of Trade and Endogenous Growth

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Abstract

This paper addresses the dynamics of income inequality, both within and across countries. In an endogenous growth model with North-South trade, the dynamics of income inequality depend on the ability of workers to adapt to new technologies, captured by the quality of education. For developing countries with low quality of education, Southern trade liberalization leads to: 1) an overall decline in effective human capital; 2) an inverted U-shape transition of income inequality, where within country inequality increases in the initial periods following a reduction in trade barriers; and 3) divergence in terms of average income in the short and long run. However, in cases where the South has a high quality of education, workers are better equipped to adapt to new technologies, and trade liberalization induces an U-shape dynamic transition of within country income inequality, where income inequality can decline in the transition. This paper highlights the critical role the quality of education plays in explaining the variations in the observed dynamics of income inequality in developing countries.

Keywords: Endogenous Growth, North-South Trade, Income Inequality, Quality of Education
JEL Classification: O15, O33

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1 Introduction

The distribution of income across individuals and across countries has long been an important issue in economics. In his seminal contribution, Kuznets (1955) suggests inequality within a country will rise in the early stages of development where investment in physical capital is the engine of growth, yet decline in latter stages as human capital becomes the primary growth mechanism. This paper links income inequality both within and across countries, and presents a mechanism that can explain the varied dynamics of inequality among countries and regions. Empirically, within country income inequality grew substantially in much of Latin America since the late 1970s, but fell (or remained relatively constant) within many East Asian countries during the same period.\(^1\) With regards to inequality across countries (as measured by the per capita GDP relative to the U.S.), on average Latin America and Africa diverged, while East Asia converged.\(^2\)

The theoretical literature addressing the dynamics of income inequality within countries often identifies skill-biased technical change and globalization as sources of changing inequality, but often lacks mechanisms to account for differences among countries. Galor and Tsiddon (1997), Greenwood and Yorukoglu (1997), Caselli (1999), Lloyd-Ellis (1999), Aghion et al. (2002) and Aghion (2002) focus on technological revolutions that give rise to an increase in demand for skilled workers, and thus, put upward pressure on their relative wage. Acemoglu (1998) argues a sharp (exogenous) increase in the supply of skilled workers raises the return to innovations targeted at skill-intensive sectors which leads to an increase in their relative wage. Galor and Moav (2000) introduce the idea that the rate of technological progress determines the relative demand, and reward, for skilled labor. While these papers emphasize the relationship between technology and inequality, this paper focuses on why the dynamics of inequality differ among developing countries.

The globalization argument (see Wood, 1994) stems from the Stopler-Samuelson theorem where the reduction in impediments to trade with skill-scarce countries increases the relative demand for skilled workers in the skill-abundant countries, and therefore raises the skill premium. However, the theory also suggests trade liberalization decreases the relative demand and rewards for skills in less developed countries, which is not consistent with empirical evidence. Dinopoulos and Segerstrom (1999, 2006), Sener (2001), Acemoglu (2003), Grieben (2005) and Zeira (2007) provide more unified models of technology and trade that avoid the pitfalls of the Stopler-Samuelson theorem.

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\(^1\)Uses inequality data from the University of Texas Inequality Project (available at http://utip.gov.utexas.edu/data.html).

\(^2\)Data on GDP per capita is taken from Penn World Tables (Heston et al., 2006).
Ripoll (2005) develops a general equilibrium model of trade and finds that initial conditions, such as the skilled-unskilled labor ratio, are important to the dynamics of income inequality following trade liberalization. The present model also accounts for income inequality across countries.

This paper suggests that the quality of education can explain the different patterns of inequality in developing countries. In a general equilibrium endogenous growth model with North-South trade in intermediate goods, the dynamics of inequality depend on the interaction of technological progress and the ability of workers to adapt to new technologies. The ability of the workers to adapt is a function of the quality of education, which characterizes the incentives to invest in human capital. Trade liberalization, then, is found to lead to an initial increase in income inequality, and divergence, if the quality of education in the South is low. If the quality of education is high, however, trade liberalization can reduce income inequality over the transition path.

The mechanism for the formation of human capital is motivated by three empirical observations. First, individual earnings increase with ability (Griliches and Mason, 1972; Murnane, Willett, and Levy, 1995). In this model, individuals are differentiated by ability in which higher innate ability reduces the cost of attaining secondary education. Second, technological progress increases the relative return to education (Ferguson, 1993; Bartel and Sicherman, 1999). This feature is captured in the model by assuming that the education premium is an increasing function of the rate of technological progress. A rise in the rate of technological progress increases the education premium for workers with secondary education, which increases the overall effective human capital. On the intensive margin, an increase in the rate of technological progress increases the productivity of skilled workers, while on the extensive margin, a rise in the rate of technological progress induces more unskilled workers to make the discrete decision to attain secondary education and become skilled. This combined effect is labeled as the productivity effect. The strength of the productivity effect is an increasing function of the quality of secondary education.

Third, Bartel and Sicherman (1998) show that an increase in the rate of technological progress increases the need to (re)train workers, especially low skill workers. To capture this effect, the efficiency units of unskilled labor supply are assumed to be decreasing in the rate of technological progress. Faster technological change, therefore, reduces the supply of effective human capital by eroding the productivity of unskilled workers, labeled the erosion effect. The severity of the erosion effect is mitigated by a higher quality of primary education.

Thus, faster rates of technological change induce competing effects on effective hu-
man capital. Faster technological growth raises the return to secondary education which serves to increase effective human capital on both the intensive and the extensive margins, while the loss of labor supply of unskilled workers reduces the level of human capital. The strength of each effect depends on the quality of secondary and primary education, respectively. Overall, effective human capital can rise or fall during periods of increased technological change depending on the quality of education. The model produces thresholds for the quality of education, above (below) which, an increase in the rate of technical progress enhances (reduces) the effective human capital. This threshold is found to be important in explaining the diverse patterns of income inequality.

The transitional dynamics of income inequality following trade liberalization also depend on the quality of education in the South. In cases in which the South has a high quality of education (above the threshold), trade liberalization induces a U-shape dynamic transition of income inequality between steady states. In this case inequality within the South will fall in the short run. In cases in which the South is below the threshold quality of education, inequality follows an inverted U-shape transition following a reduction of trade barriers, and rises in the short run. Preliminary empirical evidence supports these claims. Hall (2008), for example, find that a faster rate of technological progress increases inequality, unless the quality of education is sufficiently high. Furthermore, the model predicts greater divergence for countries with a low quality of education relative to those with a higher quality of education at the time of trade liberalization.

Allowing for heterogeneity in the quality of education across countries makes the model’s predictions consistent with the differential dynamics of inequality between East Asia and Latin America. In the long run, the model shows Southern-originating trade liberalization leads to: (a) a greater rise of income inequality within developing countries with relatively low quality of education; and (b) a greater convergence for countries with high quality of education relative to other developing countries with a lower quality of education. The model is also consistent with wage dynamics of developed countries. For example, the model predicts a growth in income inequality despite an increase in the supply of educated workers, the decline in the average wages of unskilled workers, along with an increase in inequality within skill cohorts. The key contribution of this paper is to present a mechanism that can account for a wide array of income inequality dynamics.

The remainder of the paper is organized as follows: Section 2 details the empirical motivation; Section 3 introduces the model; Section 4 solves for the steady-state and transitional dynamics; Section 5 presents and discusses the implications of Southern and
Northern led trade liberalization on the distribution of wages; and Section 6 concludes and discusses further empirical implications of the model.

2 Empirical Motivation

This paper relates to explaining: 1) the differences in the dynamics of within country inequality; and 2) the differences in the degree of convergence by developing countries. This section provides an overview of the observed paths of inequality, and in addition, highlights the notable differences in the quality of education among countries.

2.1 Within Country Income Inequality

The empirical literature on within-country income inequality is extensive. By most measures, inequality within the United States and other industrialized countries rose from the 1970s until 2000 (Wood, 1994; Machin, 1996; Autor et al., 2005). However, among developing countries there is no such clear pattern. Latin America, for example, experienced a growth in income inequality, while inequality within many East Asian countries declined. Hanson and Harrison (1999) show the skilled/unskilled wage gap grew in Mexico during the 1980s. Robbins (1996) and Wood (1997) find inequality also grew in Hong Kong, Argentina, Chile, Colombia, Costa Rica, Uruguay and Mexico, but fell in Korea, Taiwan, Singapore and Malaysia during the same period. Das (2002), similarly, finds rising income inequality in Mexico and Chile, and falling inequality within Philippines, Singapore and Taiwan. Michaely et al. (1991) show a rise and then fall of inequality within Singapore and Sri Lanka. Zhu and Trefler (2001) find that out of 29 developing countries, inequality is rising in 16 countries and falling (or remaining constant) in 13 countries.

To highlight the observed differences in income inequality, Figure 2.1 documents the average dynamics of income inequality within Latin American and East Asian countries using available data provided by the University of Texas Inequality Project. Figure 2.1 shows the increase of income inequality in Latin America, while income inequality within the East Asian economies declined.

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3 However, this finding is challenged in Duryea and Szekely (2000) and Behrman, Birdsell and Szekely (2000) where they show inequality fell within Brazil, Mexico, Venezuela, Argentina and Bolivia, and was constant in Chile and Costa Rica during the 1980s and 1990s.

4 The THEIL Index was first introduced in Theil (1967), and captures inequality across a wide range of countries and for extended periods of time.
2.2 Across Country Income Inequality

When addressing inequality across countries, one approach is to view countries as a unit of measure. Under this assumption, the empirical convergence literature established divergence in GDP per capita, most notably due to the poor economic performance of many Latin American and African countries. Specifically, the growth rates of poor countries have been lower than the growth rates of rich countries, and the dispersion of income per capita across countries has increased over time.

This finding of divergence, however, is not robust when considering the individual as the unit of measure. The convergence of more populous regions, including China and India, drives a decline in global inequality across all individuals. Overall, recent empirical evidence suggests that after peaking around 1979 global inequality is declining (See Sala-i-Martin, 2006).

Abstracting from the convergence debate, the focus of this paper is why certain countries diverged and others converged in terms of average GDP per capita relative to that in the United States. Figure 2.2 uses data from the Penn World Tables 6.2 (Heston et al., 2006) to show the GDP per capita relative to the U.S. GDP per capita for a subset of African, Latin American and East Asian countries from 1960 to 2002.

\(^5\) $\beta$-divergence in Barro and Sala-i-Martin (1992) terms.
\(^6\) $\sigma$-divergence in Barro and Sala-i-Martin (1992). See the “twin peaks” literature following Quah (1993).
\(^7\) The group of countries were selected to capture the diverse patterns of relative GDP per capita among different regions.
Figure 2.2 shows East Asia, on average, converged substantially in terms of living standards, while both Latin America and Africa experienced divergence. Also, the convergence of East Asia and the divergence of Latin America and Africa accelerated during the 1980s. This decade is known for widespread trade liberalization among developing countries, which suggests that opening to trade and new technologies that flow into the country may be an important source of the dynamics of income inequality.

2.3 Quality of Education

The quality of education varies widely across countries. Standard measures for educational quality, including adult literacy rates, teacher-pupil ratios, or expenditures per student, are typically insignificant in cross-country growth studies, and are notoriously poor measures for the quality of education in the labor force. A potential reason is that these measures do not directly capture the cognitive ability of the labor force. Hanushek and Kimko (2000) address this issue and develop measures for the quality of the labor force derived from a number of international mathematics and science tests between the years 1965 through 1991. While test score data is available for only 38 countries, Hanushek and Kimko use consistent estimators to forecast labor force quality for a large number of countries based on country specific characteristics. In all, they produce quality measures for 90 countries across the development spectrum. Table 1 provides the quality indices for a sub sample of Latin American and Asian countries taken from Hanushek and Kimko (2000).
Table 1: Labor-Force Quality Data
Hanushek and Kimko (2000)

<table>
<thead>
<tr>
<th>Latin America</th>
<th>Asia</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bolivia</td>
<td>China</td>
</tr>
<tr>
<td>27.47</td>
<td>64.42</td>
</tr>
<tr>
<td>Brazil</td>
<td>Hong Kong</td>
</tr>
<tr>
<td>36.60</td>
<td>71.85</td>
</tr>
<tr>
<td>Chile</td>
<td>Indonesia</td>
</tr>
<tr>
<td>24.74</td>
<td>42.99</td>
</tr>
<tr>
<td>Colombia</td>
<td>Korea</td>
</tr>
<tr>
<td>37.87</td>
<td>58.55</td>
</tr>
<tr>
<td>Ecuador</td>
<td>Malaysia</td>
</tr>
<tr>
<td>38.99</td>
<td>54.29</td>
</tr>
<tr>
<td>Honduras</td>
<td>Philippines</td>
</tr>
<tr>
<td>28.59</td>
<td>33.54</td>
</tr>
<tr>
<td>Mexico</td>
<td>Singapore</td>
</tr>
<tr>
<td>37.24</td>
<td>72.13</td>
</tr>
<tr>
<td>Peru</td>
<td>Taiwan</td>
</tr>
<tr>
<td>41.18</td>
<td>56.32</td>
</tr>
<tr>
<td>Uruguay</td>
<td>Thailand</td>
</tr>
<tr>
<td>52.27</td>
<td>46.26</td>
</tr>
<tr>
<td>Venezuela</td>
<td></td>
</tr>
<tr>
<td>39.08</td>
<td></td>
</tr>
</tbody>
</table>

Average 36.40 | Average 55.58

The quality of the labor force is consistently higher in Asia relative to Latin America, and the measure performs well in cross-country growth regressions. Using the Hanushek and Kimko measure, the poorest quality of the labor force is 18.26 (Iran), while the highest is 72.13 (Singapore). The mean for the entire sample of countries is 51.28. Overall, this data supports the idea that the quality of education varies significantly and systematically across countries and regions.

3 The Model

The model features North-South trade with Schumpeterian endogenous growth through creative destruction. The North represents a developed country, while the South represents a less developed country. Innovations increase the quality, or productivity, of intermediate goods used in final goods production. State-of-the-art quality levels are only discovered through research and development (R&D) efforts in the North, but once an innovation occurs, the South undertakes R&D to imitate the Northern frontier technology.

Human capital is an input for final goods production. As introduced by Galor and Moav (2000), the rate at which new state-of-the-art technologies enter the production process determines, in part, the level of effective human capital in the economy. The effective human capital in the economy is a weighted sum of skilled and unskilled workers whose productivity is determined by the quality of the education they receive and the rate of technological progress.
3.1 Final Goods Sector

The final goods production function includes a conventional quality ladder model, à la Grossman and Helpman (1991), Aghion and Howitt (1992), and Barro and Sala-i-Martin (1997, 2004). In this setup technology is embedded within the productivity, or quality, of the intermediate goods used in producing a final good. Denote the productivity of a given intermediate good industry $j$ to be $q^k$, where $q$ is the incremental rise in productivity per innovation, and $k$ is the number of innovations. Assume a continuum of intermediate goods, $j \in [0, 1]$.

The aggregate final goods production function in each region $m \in [N \text{ (North)}, S \text{ (South)}]$ takes the Cobb-Douglas functional form,

$$Y_m = A_m H_m^{1-\alpha} \int_0^1 \left( q^{kN_j} x_{mkj} \right)^{\alpha} dj$$

where $\alpha$ is the share of capital in production, $A_m$ is the total factor productivity parameter in the final goods sector, $x_{mkj}$ is the physical quantity of intermediate good $j$ with quality level $k$, and $q^{kN_j} x_{mkj}$ is the quality adjusted input for the intermediate good from industry $j$. The inclusion of $N$ indicates that in each industry, only the highest quality of intermediate good will be used in final goods production, and by definition, is discovered only through Northern innovative activity. $H_m$ represents the effective human capital of each country. Embedded within $H$ are the contributions of both skilled and unskilled workers, whose supply and productivity are both endogenous and determined in equilibrium. For simplicity, there is no population growth and workers are immobile across countries.

The inverse demand for an intermediate good $x$ from industry $j$ is

$$P_j = \alpha \tilde{P}_m H_m^{1-\alpha} x_{mkj}^{\alpha-1} q^{kN_j} \alpha$$

where $P_j$ is the price of the intermediate good from industry $j$, and $\tilde{P}_m$ is the price of the (nontradeable) final good. The price of the Northern final good, $\tilde{P}_N$, is the numeraire, $\tilde{P}_N \equiv 1$. Therefore, $\tilde{P}_S$ is defined as the relative price of the final good in the South. As later sections discuss, $\tilde{P}_S$ is endogenous, and adjusts in every period to balance trade.

3.2 Human Capital

Individuals choose between working as skilled or unskilled based on their expected income, thus the supply of each type of labor is endogenous. Workers are differentiated by their innate, cognitive ability which is reflected in their individual cost of education.
A uniformly distributed continuum of individuals $i$ in each region is indexed by ability $a_{mi}$. Each worker is endowed with one unit of labor at every point in time. To become skilled, the individual devotes a fraction of their labor endowment to the acquisition of secondary education. The cost of education is decreasing in ability, thus for individual $i$ in country $m$, the cost of education is the fraction $1 - a_{mi}$. Individual $i$ supplies, as a skilled worker,

\[ h_{mi} = a_{mi} \]  

(3)

efficiency units of labor to final goods production.

Choosing to work as unskilled avoids the costs of acquiring secondary education, but more time must be spent adapting to new technologies. Individual $i$ supplies, as an unskilled worker,

\[ l_{mi} = 1 - \frac{1}{\delta_{m}^l} (1 - a_{mi}) p_I \]  

(4)

efficiency units of labor, where $0 < \frac{1}{\delta_{m}^l} (1 - a_{mi}) p_I < 1$ determines the time cost required for worker $i$ to adapt to new technologies entering at the probability $p_I$. $\delta_{m}^l$, the quality of primary education, reduces the amount of time needed to adapt to new technologies. Equation (4) illustrates the erosion effect of an increase of $p_I$ on the effective labor supply of unskilled workers. Faster rate of innovation reduces the supply of unskilled labor, but a higher quality of primary education reduces the severity of this effect.

The aggregate effective human capital $H$ is given by a weighted sum of the endogenous total supplies of skilled and unskilled workers. The effective human capital takes the form of,

\[ H_m = \left( \beta_m + \delta_{m}^h p_I \right) h_m + \beta_m l_m, \]  

(5)

where $0 < p_I < 1$ and $\beta_m, \delta_{m}^h > 0$. The interaction between the quality of secondary education and the rate of technological progress determines, in part, the relative return to acquiring secondary education. Equation (5) illustrates the productivity effect of an increase in the rate of technological progress on the effective human capital on the intensive margin. An increase in $p_I$ increases the productivity of workers already with a secondary education, and the effect is increasing in the quality of secondary education, $\delta_{m}^h$. Skilled workers adapt quickly and benefit, in terms of productivity, from utilizing new technologies in final goods production.

Using equations (1) and (5), skilled ($s$) and unskilled ($u$) wages in country $m$ are

\[ w_m^s \equiv \omega_m \left( \beta_m + \delta_{m}^h p_I \right) \] and \[ w_m^u \equiv \omega_m \beta_m, \]  

(6)

Introducing imperfect substitution between skilled and unskilled workers is possible, but the qualitative results would carry through and the analysis would be less transparent.
where $\omega_m \equiv (1 - \alpha) \frac{\hat{P}_m Y_m}{H_m}$. Income is the wage rate times the efficiency units of labor supplied. The individual with ability $a_{mi}$ earns income $I^s_{mi}$ working as a skilled worker, or $I^u_{mi}$ if unskilled:

Skilled $\Rightarrow I^s_{mi} = w^s_m h_{mi} = w^s_m a_{mi}$

Unskilled $\Rightarrow I^u_{mi} = w^u_m l_{mi} = w^u_m \left( 1 - \frac{1}{\delta_m} (1 - a_{mi}) p_I \right)$.

Figure 3.2 plots the skilled and unskilled incomes across individuals for two rates of innovation ($p^0_I < p^1_I$). The dark line shows the incomes for each individual.

From Figure 3.2, there exists a threshold ability level where that individual is indifferent between becoming skilled or unskilled, $a^*_m$:

$$a^*_m = \frac{1 - \frac{1}{\delta_m} p_I}{1 - \frac{1}{\delta_m} p_I + \frac{\delta^h_m}{\beta_m} p_I} \equiv a^*_m (p_I).$$

Any worker with ability $1 > a_{mi} > a^*_m$ will choose to become skilled, while the rest, $0 < a_{mi} < a^*_m$, choose to remain unskilled. The threshold level of ability decreases in the rate of technical progress, $p_I$, implying an increase the rate of innovation will induce more workers to acquire an education,

$$\frac{\partial a^*_m}{\partial p_I} = -\frac{\delta^h_m / \beta_m}{\left( 1 - \frac{1}{\delta_m} p_I + \frac{\delta^h_m}{\beta_m} p_I \right)^2} < 0.$$
This reflects the productivity effect on the extensive margin, in which an increase in the rate of innovation increases the mass of workers choosing to acquire secondary education, which in turn, raises the effective human capital. With more workers acquiring additional education, the more productive the overall labor force.

Figure 3.2 summarizes the ways an increase in the rate technological progress alters the income and productivity of each type of worker: the productivity effect (on the intensive and extensive margins) and the erosion effect. First, faster rates of innovation increase the productivity of educated workers on the intensive margin. This is reflected in the increase of income for skilled workers. Second, an increase in the rate of innovation increases the time required for unskilled workers to adapt to the new technologies (the erosion effect), reducing the income and contributions of unskilled workers. Third, there is an endogenous response of more workers acquiring secondary education (the productivity effect on the extensive margin). This is reflected in the decrease of the share of workers without secondary education, \( a^*_m \). The total effect of an increase in the rate of innovation growth on effective human capital is ambiguous and is a function of the quality of primary and secondary education. Specifically, an increase in the quality of secondary education increases the magnitude of the productivity effect, while an increase in the quality of primary education reduces the magnitude of the erosion effect.

Aggregating across individuals yields

\[
\begin{align*}
    h_m &= \int a_m^1 a_m di = \frac{1}{2} \left[ 1 - (a_m)^2 \right] \equiv h_m(p_I) \\
    l_m &= \int^a_m 1 - \frac{1}{\delta_m} (1 - a_m) p_I di = \left( 1 - \frac{1}{\delta_m} p_I \right) a_m^* + \frac{1}{2} \frac{1}{\delta_m} p_I (a_m^*)^2 \equiv l_m(p_I).
\end{align*}
\]  

(9)

The effective human capital level is, therefore,

\[
H_m = \left( \beta_m + \delta_m^h p_I \right) h_m(p_I) + \beta_m l_m(p_I) \equiv H_m(p_I),
\]  

(10)

where

\[
\frac{\partial H_m}{\partial p_I} = \frac{1}{2} \delta_m^h \left( 1 - (a_m^*)^2 \right) - \beta_m \left( 1 - \frac{1}{2} a_m^* \right) a_m^*.
\]

(11)

The partial derivative \( \partial H / \partial p_I \) is positive or negative depending on the relative magnitude of the productivity effect and the erosion effect as determined by the quality of secondary and primary education respectively. For a sufficiently high \( \delta^h \) or \( \delta^l \), an increase in the rate of innovation will increase the effective stock of human capital, while for a sufficiently low \( \delta^h \) or \( \delta^l \), an increase in the rate of innovation will decrease the effective stock of human capital. For the discussion that follows, a country with edu-
carnation parameters such that $\partial H/\partial p_I < 0$ has low quality of education, while a country with parameters such that $\partial H/\partial p_I > 0$ has high educational quality. Equation (11) implies that an increase in the rate of technological progress will increase the effective human capital for countries with a high quality of education, but decrease if the quality of education is low.

### 3.3 Income Inequality

This section derives the relationship between technical innovation, the quality of education, and the measures of wage inequality. The analysis focuses on two types of income inequality: 1) within-country income inequality, and 2) across-country income inequality. The measures of inequality are both derived directly from the incomes of skilled and unskilled labor in both the North and South.

Within country inequality is naturally defined to be ratio of the average income of skilled workers to that of unskilled workers. Exploiting the linearity of incomes with respect to ability, average incomes for skilled, $\tilde{I}_m^s$, and unskilled, $\tilde{I}_m^u$, workers are

$$\tilde{I}_m^s = \frac{I_m^s(a_m^*) + I_m^s(1)}{2} = \frac{1}{2} w_m^s m (1 + a_m^*)$$

$$\tilde{I}_m^u = \frac{I_m^u(a_m^*) + I_m^u(0)}{2} = \frac{1}{2} w_m^u m \left(2 \left(1 - \frac{1}{\delta_m} p_I \right) + \frac{1}{\delta_m} p_I a_m^* \right).$$

Using the average incomes for the two types of labor, within-country inequality for both the North and South is expressed as

$$\Omega_m^{s/u} = \frac{\tilde{I}_m^s}{\tilde{I}_m^u} = \left[1 + \frac{\delta_m^h}{\beta_m} p_I \right] \left[1 + a_m^* (p_I) \right] \left[2 \left(1 - \frac{1}{\delta_m} p_I \right) + \frac{1}{\delta_m} a_m^* (p_I) p_I \right],$$

where $a_m^* (p_I)$ is the function defined in equation (8). Each set of bracket refers to the effects the rates of innovation has on the effective level of human capital. The term in the first set of brackets captures the productivity effect on the intensive margin for the skilled workers in the economy. Higher rates of innovation increase the relative wages of skilled workers. Within the second set of brackets is the effects of a change in the mass of workers with a secondary education. Higher $p_I$ will increase the relative supply of skilled workers (a decrease in $a^*$) which puts downward pressure on the relative wages of skilled workers. Finally, the third bracket corresponds to the loss of efficiency units of unskilled workers due to higher adaptation costs associated with a greater rate of technical progress. This effect puts upward pressure on the relative wages. Again, the effects of $p_I$ on within-country inequality are ambiguous, and a function of the relative magnitude of these effects.
Inequality across-countries is measured as the average income of all Northern workers relative to the average income of Southern workers. Thus,

\[
\Omega_{N/S} = \frac{\int_0^1 I_{Ni} di}{\int_0^1 I_{Si} di} = \frac{(a_N^*) \bar{I}_N + (1 - a_N^*) \bar{I}_N}{(a_S^*) \bar{I}_S + (1 - a_S^*) \bar{I}_S} = \frac{Y_N}{\bar{P}_S Y_S},
\]

where \(Y_N\) and \(\bar{P}_S Y_S\) are the values of the aggregate final good produced in the North and South, respectively. Notice the level of the aggregate quality of intermediate goods has no effect on the relative income of the Northern workers. An increase in the value of the Southern final good relative to the Northern output implies a reduction in across-country income inequality.

### 3.4 Intermediate Goods Sector

The intermediate goods sector follows a two-stage process: 1) the process of research and development; and 2) monopolistic competition given the stage one R&D results. The first stage is the allocation of resources into the research and development (R&D) of new technologies. The North, by assumption, is technologically more advanced than the South, and alone, possesses the capability to invent a new state-of-the-art technology. If there is R&D success in the North in a given industry \(j\), then the quality of that intermediate good rises by a constant size, \(q\), from \(q^{kj}\) to the new quality level \(q^{kj+1}\),

where \(k\) is the number of innovations. Since Northern firms have the ability to create new technologies, they must innovate in order to expand the world’s technology frontier. R&D in the South, on-the-other-hand, imitates frontier technologies. Successful imitation increases the domestic technology in the South.

In the second stage, the successful firms set prices and realize the rents from innovation or imitation. Within intermediate industries, competing firms holding different quality grades of a substitutable intermediate good engage in Bertrand price competition. Under the condition that \(q < 1/\alpha\), firms follow a limit price strategy.\(^{10}\) Following Grossman and Helpman (1991), limit pricing drives out lower quality grades within a given industry.

Intermediate firms are immobile across countries, but the location of production for intermediate \(j\) depends on the stage one R&D results. Following a successful innovation in industry \(j\), the Northern firm serves as the *global* source for that good. The firm

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\(^9\)Within industries, different quality grades are perfect substitutes, however, weighted by their respective grades.

\(^{10}\)If this inequality does not hold, successful innovators charge monopoly prices and successful imitators engage in limit pricing strategies.
supplies the domestic final goods market and the foreign final goods market through exporting. The Northern firm holds the market until either a competing Northern firm makes a further innovation, or the quality grade is successfully imitated by a Southern competitor. Following successful imitation in industry \( j \), the Southern firm serves, again, as the global source by exporting to the North. Assuming a continuum of industries with a mass of one, \( n_{NN} \) is defined as the share of industries served by a firm located in the North whose competitor with the next highest quality grade is a fellow Northern firm, \( n_{NS} \) to be the share of industries served by a firm located in the North whose competitor with the next highest grade is located in the South, and \( n_S \) are the share of industries with production in the South.\(^{11}\)

**Stage 2: Expected Profits**

Expected profits are determined by the type of competition faced by each type of firm. Specifically, firms engage in limit pricing strategies which are determined by the type of competition faced. Bertrand price competition results in prices set low enough to drive the closest competitor (the firm with the next highest quality grade) out of the market. Northern firms facing Northern competition choose the lowest price at which the previous innovator, or closet competitor, could sell before earning negative profits. Since new innovations are \( q > 1 \) units more productive than the next best good (in this case held by a competing Northern firm), the innovating firm charges a price \( q \) times the marginal cost of their rival in order to completely capture the market for that industry. Northern firms facing Northern competition charge a domestic price \( P_{NN} = (q - \epsilon)MC_N = q \),\(^{12}\) where \( \epsilon \) is an arbitrarily small positive amount. This is sufficient to capture the entire market for that industry. In the export market, similar intuition implies \( P^*_NN = q(1 + \tau_{XS}) \), where \( \tau_{XS} \) is the tariff imposed by the South.

For a Northern firm facing Southern competition, the innovating firm charges a price \( q - \epsilon \) over the marginal cost of the Southern competitor. In the Northern market, the marginal cost for a Southern firm is \( MC_S (1 + \tau_{XN}) \), where \( \tau_{XN} \) is the tariff imposed by the North. Thus, Northern firms facing Southern competition charge the domestic price, \( P_{NS} = qMC_S (1 + \tau_{XN}) \). In the Southern market, the marginal cost for a Southern firm is simply \( MC_S \), so the export price is \( P^*_NS = qMC_S \).

\(^{11}\)Intermediate producers in the South only face competition from the North. Bertrand price competition drives prices down to marginal costs, thus fellow intermediate firms in the South have no incentive to devote resources to imitate a good that has already been imitated.

\(^{12}\)The marginal cost of intermediate producers is equal to the price of the final good in that region. In the North, the price of the final good and therefore the marginal cost for intermediate firms in the North is unity, while the price of the Southern final good and marginal costs for Southern intermediate producers adjust to balance trade, \( P_S = MC_S < 1 \).
Finally, Southern firms always face Northern competition, and capture the global market by charging a price $1 - \epsilon$ times the marginal cost of the Northern firm. In the Northern market, the marginal cost for a Northern firm is $MC_N = 1$, and the price is $P^*_N = 1$. In the South, Southern firms price at $P_S = 1 + \tau_{XS}$. Table 2 summarizes the limit pricing schedule and the marginal cost of production for each type of firm.

<table>
<thead>
<tr>
<th>Table 2: Limit Pricing Schedule and Marginal Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{NN} = q$</td>
</tr>
<tr>
<td>$P^*<em>N = q(1 + \tau</em>{XS})$</td>
</tr>
<tr>
<td>$P_{NS} = qMC_S (1 + \tau_{XN})$</td>
</tr>
<tr>
<td>$P^*_N = qMC_S$</td>
</tr>
<tr>
<td>$P_S = 1 + \tau_{XS}$</td>
</tr>
<tr>
<td>$P^*_S = 1$</td>
</tr>
</tbody>
</table>

The pricing strategies for intermediate firms reveals the role of barriers to trade in the model. Trade liberalization alters the limit prices for the three types of firms, and thus, changes demand, expected profits and ultimately the incentives for innovation and imitation.

From the inverse demand function from equation, for a given limit price $P_j$ and final goods price $\tilde{P}_m$, the demand for intermediate goods in industry $j$ from region $m$ is

$$x_{mj} = H_m \left( A_m \alpha q^{k_{Nj} \alpha} \frac{\tilde{P}_m}{P_j} \right)^{1/(1-\alpha)}.$$  \hspace{1cm} (14)

Successful innovators and imitators earn the sum of domestic and export profits. Let $X_j$ and $X^*_j$ be the total domestic and export demand, respectively, for a firm in industry $j$. In general, profits are then

$$\pi_j = \left( P_j - MC_j \right) X_j + \left( P^*_j - MC^*_j \right) X^*_j$$

Export Profits

where $P_j$, $P^*_j$, $MC_j$ and $MC^*_j$ are given in Table 2 for $j \in [NN, NS, S]$. I assume trade barriers are sufficiently low that export profits are always positive.

The average expected profits in the North and South are summarized as

$$\pi_N \equiv \bar{\pi}_N (\tau_{XS}, \tau_{XN}, p_I) Q_N,$$

where $\frac{\partial \bar{\pi}_N}{\partial \tau_{XS}} < 0$, and $\frac{\partial \bar{\pi}_N}{\partial \tau_{XN}} < 0$ \hspace{1cm} (15)

and

$$\pi_S \equiv \bar{\pi}_S (\tau_{XS}, \tau_{XN}, p_I) Q_N,$$

where $\frac{\partial \bar{\pi}_S}{\partial \tau_{XS}} > 0$, and $\frac{\partial \bar{\pi}_N}{\partial \tau_{XN}} < 0$, \hspace{1cm} (16)

where $\bar{\pi}_N$ and $\bar{\pi}_S$ are the quality adjusted expected profits for firms in the North and South, respectively, and $Q_N = \int_0^1 q^{k_{Nj} \alpha/(1-\alpha)} dj$ is the average quality level on the frontier.
Equations (15) and (16) are expressed as a function of the barriers to trade, \( \tau_{XS} \) and \( \tau_{XN} \), and are also a function of \( p_I \) embedded within the effective human capital in the North and South. See the appendix for additional details.

**Stage 1: Research & Development**

Intermediate firms decide the amount of resources to devote to R&D based on the expected present value of profits for successful research. This, in turn, depends on the probabilities of innovation \( (I) \) and imitation \( (C \) for copying). \( p_{Ikj} \) and \( p_{Ckj} \) denote the instantaneous probability of innovation and imitation, respectively, for industry \( j \) with current quality level \( k \). The probabilities of research success per unit of time follow a Poisson process depending on the resources devoted to R&D and the quality level in that industry. For the aggregate economy, it is sufficient to characterize the rates of technical progress by looking at the *average* quality. \( Q_N \) is the average frontier technology, and \( Q_S \) is average quality of Southern intermediates.

In the North, the instantaneous probability of innovation is given by

\[
p_I = \phi_I f (Q_N) Z_N
\]

where \( \phi_I \) is the productivity parameter for the Northern R&D sector, \( Z_N \) are the average resources devoted to innovative R&D in the North and \( f (Q_N) \) is a function which captures the effect of the current technology on the probability of innovation. For simplicity, the function \( f \) is defined as

\[
f (Q_N) = Q_N^{-1}.
\]

This specification captures the idea that new innovations become increasingly complex and, thus, innovation becomes more costly as the average quality level rises. The increasing complexity outweighs any learning-to-learn effects in which innovations become easier to discover over time.\(^{13}\) However, the easiest innovations are discovered first, and thus, it becomes increasingly difficult to innovate over time.

In the South, the instantaneous probability of imitation for the aggregate economy is

\[
p_C = \phi_C g (Q_S, Q_N) Z_S
\]

where \( \phi_C \) is the productivity parameter for the Southern R&D sector, \( Z_S \) are the average resources devoted to R&D and \( g (Q_S, Q_N) \) captures three effects of the current techno-

\(^{13}\)For more details on learning-to-learn effects refer to Connolly (2003) and Connolly and Valderrama (2007).
logical environment on the probability of imitation. The first effect is the positive effect of $Q_S$ reflecting learning-to-learn in the South. Higher $Q_S$ implies greater experience with the imitative process, thus reducing the costs of imitation as $Q_S$ increases. The second effect is the increasing difficulty in imitating good of a higher quality. As the frontier expands, innovations are increasingly complex, and costs of imitation increase. Again, by assumption, the increasing difficulty effect dominates the learning-to-learn effect. Finally, imitation becomes increasingly costly as the aggregate quality in the South catches up to the Northern aggregate quality. The relative quality of the South is defined as $\hat{Q} \equiv Q_S/Q_N$. Intuitively, as the South approaches the North in terms of quality, the pool of potential imitations shrinks, with only the most complex innovations left available for imitation. Thus, the costs of imitation increase as $\hat{Q}$ increases. Considering these three effects, the functional form of $g$ is defined as

$$g (Q_S, Q_N) = Q_S Q_N^{-2} \hat{Q}^{-\sigma} = \hat{Q}^{1-\sigma} Q_N^{-1}$$

where $\sigma > 1$ represents how quickly the imitation rates fall as the South approaches the aggregate quality level in the North. The inclusion of $\hat{Q}$ guarantees smooth transitional dynamics. Furthermore, if the South completely catches up to the North in terms of average quality, or $Q_S = Q_N$, the function $g$ is equivalent to the function $f$.

### 3.5 Consumers

Consumers live in either the North or the South and are immobile across countries. Consumer $i$ makes consumption and savings decisions to maximize the present value of lifetime utility. There is no trade in final goods and so the consumers only have access to domestically produced final goods. The general consumer problem is

$$\max_{C_{mi}, b_{mi}} \int_0^\infty u (C_{mi}) e^{-\rho t} dt$$

subject to

$$\dot{b}_{mi} = I_{mi} + r_m b_{mi} - \bar{P}_m C_{mi}.$$  \hspace{1cm} (20)$$

where $C_{mi}$ is the consumption of individual $i$ in region $m$, $r_m$ is the endogenously determined country specific interest rate, $I_{mi}$ is the income for individual $i$ in region $m$, and $b_{mi}$ is the net assets for person $i$ in region $m$. Using a constant elasticity of substitution utility function:

$$u (C_{mi}) = \frac{C_{mi}^{1-\theta} - 1}{1 - \theta},$$
the usual expressions for consumption growth are:

\[ \frac{\dot{C}_N}{C_N} = \frac{1}{\theta} (r_N - \rho) \quad (21) \]

\[ \frac{\dot{C}_S}{C_S} = \frac{1}{\theta} \left( r_S - \frac{\hat{P}_S}{\hat{P}_S} - \rho \right) \quad (22) \]

where \(1/\theta\) is the constant elasticity of substitution for all consumers in both regions. The growth rates of consumption are independent of the individual income level and are equal across all individuals within the region.

4 The Steady-State and Transitional Dynamics

4.1 The Steady-State

The steady-state is defined by a system of five dynamic equations, given two aggregate resource constraints and the balanced trade condition. In steady-state, the relative aggregate quality level of the South must be constant, or \(\dot{\hat{Q}}/\hat{Q} = 0\); the distribution of intermediate firms must be constant, or \(\dot{n}_{NN} = \dot{n}_{NS} = \dot{n}_S = 0\); and the growth of consumption must equal the growth rate of technology, or \(\dot{C}_N/C_N = \dot{C}_S/C_S = \dot{Q}_N/Q_N\).

Aggregate Resource Constraint and Balanced Trade

Substituting for the prices and demand of intermediate goods, aggregate output for the North and South, respectively, is:

\[ Y_N = H_N \Lambda_N \left( \frac{q}{\alpha} \right) \left[ n_{NN} + n_{NS} \left( \frac{1}{\hat{P}_S(1 + \tau_{XX})} \right)^{\frac{\alpha}{1-\alpha}} + n_S q^{\frac{\alpha}{1-\alpha}} \right] Q_N \quad (23) \]

\[ Y_S = H_S \Lambda_S \left( \frac{q}{\alpha} \right) \left( \frac{\hat{P}_S}{1 + \tau_{X_S}} \right)^{\frac{\alpha}{1-\alpha}} \left[ n_{NN} + n_{NS} \left( \frac{1 + \tau_{X_S}}{\hat{P}_S} \right)^{\frac{\alpha}{1-\alpha}} + n_S q^{\frac{\alpha}{1-\alpha}} \right] Q_N \quad (24) \]

where \(\Lambda_N = A_N^{1/(1-\alpha)} \left( \frac{a}{q} \right)^{1/(1-\alpha)}\) and \(\Lambda_S = A_S^{1/(1-\alpha)} \left( \frac{a}{q} \right)^{1/(1-\alpha)}\). The aggregate output in each region is determined, in part, indirectly by the probabilities of innovation and imitation. As with the profit equations, \(p_I\) and \(p_C\) are embedded within the effective human capital, the balanced trade determination of \(\hat{P}_S\), and the distribution of firms.

\[ ^{14}\text{We assume trade is balanced at all times by the endogenous adjustment of the relative price of the Southern final good, } \hat{P}_S. \]
The two aggregate resource constraints reflect that the final goods are used for domestic consumption, R&D, or transformed into intermediate goods:

\[
\begin{align*}
Y_N &= C_N + X_{NN} + X_{NN}^* + X_{NS} + X_{NS}^* + Z_N \\
Y_S &= C_S + X_S + X_S^* + Z_S
\end{align*}
\]

(25)

where \(X_{NN} + X_{NN}^*\) is the total intermediate output for Northern firms facing Northern competition supplied to both the domestic and foreign markets. \(X_{NS} + X_{NS}^*\) is the total intermediate supply from Northern firms facing Southern competition, and \(X_S + X_S^*\) is the total supply of intermediate goods from Southern producers. Tariff revenues are used by the government for no gain in utility or income for the individuals.\(^{15}\)

Using the two resource constraints, I summarize the expressions for \(Z_N\) and \(Z_S\) as (see the appendix for details)

\[
Z_N \equiv Z_N (\tau_{XS}, \tau_{XN}, p_I) Q_N
\]

(26)

and

\[
Z_S \equiv \tilde{Z}_S (\tau_{XS}, \tau_{XN}, p_I) Q_N,
\]

(27)

where \(\tilde{Z}_N\) and \(\tilde{Z}_S\) are the quality adjusted expenditures on R&D. Using equations (17), (18), (26) and (27) the steady-state probabilities of innovation or imitation in the average industry at a given moment are implicitly solved by

\[
\begin{align*}
p_I &= \phi_I Z_N (\tau_{XS}, \tau_{XN}, p_I) \\
p_C &= \phi_C \dot{Q}^{1-\sigma} \tilde{Z}_S (\tau_{XS}, \tau_{XN}, p_I).
\end{align*}
\]

(28)

(29)

The relative price of the Southern final good, \(\tilde{P}_S\), will adjust to balance trade at all times. The trade balance equates the value of Northern intermediate good exports and the value of intermediates produced in the South and exported to the North,

\[
\text{TB} = \frac{\text{Value of N. Exports}}{\text{Value of S. Exports}} = 0.
\]

(30)

Equation (30) implicitly solves for the relative prices of the Southern final goods, \(\tilde{P}_S\). See the appendix for additional details.

\(^{15}\)A lump sum transfer of tariff revenues does not change the dynamics of income inequality.
Relative Quality Level of the South

Using the definition of \( \hat{Q} = Q_S/Q_N \), the relative average quality of the South evolves according to

\[
\frac{\dot{\hat{Q}}}{\hat{Q}} = \left( q^{\frac{1}{\alpha}} - 1 \right) (p_C - p_I). \tag{31}
\]

In steady-state the evolution of the relative Southern quality level is constant. Therefore the steady-state probability of innovation is exactly that of the probability of imitation, or, \( p_I = p_C \).

Distribution of Intermediate Firms

Consider, first, the fraction of industries characterized by Northern firms facing Northern competition, \( n_{NN} \). The share of this type of firm will fall if a technology level is copied, as the production in that industry shifts to the South. On the other hand, the share of Northern firms facing Northern competition increases with an innovation over a Northern firm previously facing Southern competition, provided that technology is not imitated. These dynamics are captured in equation (32).

The share of Northern firms facing Southern competition increases through innovation in the industries where production is currently in the South (\( n_S \)), but will fall through imitation or further innovation, as detailed in equation (33). Finally, the share of firms located in the South increases through imitation in any industry where production is currently in the North (\( n_{NN} + n_{NS} \)) and falls with successful innovation, as captured in equation (34).

\[
\begin{align*}
\dot{n}_{NN} & = p_I (1 - p_C) n_{NS} - p_C n_{NN} \tag{32} \\
\dot{n}_{NS} & = p_I (p_C n_{NN} + n_S) - [(1 - p_I) p_C + p_I (1 - p_C)] n_{NS} \tag{33} \\
\dot{n}_S & = (1 - p_I) p_C (n_{NN} + n_{NS}) - p_I n_S \tag{34}
\end{align*}
\]

In steady-state, \( \dot{n}_{NN} = \dot{n}_{NS} = \dot{n}_S = 0 \). Solving the system of equations, the steady-state share of each type firm is

\[
\begin{align*}
n_{NN} & = \frac{p_I^2 (1 - p_C)}{(p_I + p_C - p_I p_C)^2} \\
n_{NS} & = \frac{p_I p_C}{(p_I + p_C - p_I p_C)^2} \\
n_S & = \frac{p_C (1 - p_I)}{p_I + p_C - p_I p_C}
\end{align*}
\]

Using the steady-state result of the equalization of \( p_I \) and \( p_C \), the steady-state shares
of each type of firm (setting \( p = p_C = p_I \)) is

\[
\begin{align*}
n_{NN} &= \frac{1-p}{(2-p)^2} \\
n_{NS} &= \frac{1}{(2-p)^2} \\
n_S &= \frac{1-p}{2-p}.
\end{align*}
\]

A steady-state increase in the probability of innovation and imitation yields an increase in the share of Northern firms facing Southern competition, or in other words, an increase in global competition. Furthermore, the share of Northern firms facing Northern competition and the share of firms located in the South both decrease.

**Consumption and Technological Growth**

From equations (21) and (22), the growth rates of consumption depend on the country-specific interest rate and the evolution of the relative price of the Southern final good, \( \dot{P}_S \), as determined by the balanced trade condition. To determine \( r_N \) and \( r_S \), two free-entry conditions imply that firms will devote resources to research until the expected value of R&D success equals the R&D costs for the average industry. The Northern and Southern free entry conditions, respectively are

\[
\begin{align*}
p_I\pi_N \int_{t}^{\infty} e^{-\int_{t}^{s} [r_N(v) + p_C(v) + p_I(v) - p_C(v)p_I(v)] dv} ds &= Z_N \\
p_C\pi_S \int_{t}^{\infty} e^{-\int_{t}^{s} [r_S(v) + p_I(v)] dv} ds &= Z_S
\end{align*}
\]

The expected value of innovation is the probability of R&D success times the average profits discounted by the interest rate and the probability of rival innovation and Southern imitation. The Southern profits are discounted only by the interest rate and the probability Northern innovation. Differentiating both sides of the free entry conditions using Leibniz’s rule yields the interest rates in both countries:

\[
\begin{align*}
r_N &= \frac{p_I\pi_N}{Z_N} + \frac{\dot{Z}_N}{Z_N} - \frac{\dot{p}_I}{p_I} - \frac{\dot{\pi}_N}{\pi_N} - p_C - p_I + pCP_I \\
r_S &= \frac{p_C\pi_S}{Z_S} + \frac{\dot{Z}_S}{Z_S} - \frac{\dot{p}_C}{p_C} - \frac{\dot{\pi}_S}{\pi_S} - p_I
\end{align*}
\]

The interest rates determine, in the long run, the rate of growth for output, consumption, and research expenditures in both countries.

The final dynamic expressions represent the conditions for balance growth. Let \( \chi_N \equiv C_N/Q_N \) and \( \chi_S \equiv C_S/Q_N \) denote the quality adjusted consumption. In steady
state, the rate of consumption growth equals the growth rate of the frontier technology level, or \( \frac{\dot{\chi}_N}{\chi_N} = \frac{\dot{\chi}_S}{\chi_S} = 0 \). The expressions for the North and South, respectively are

\[
\frac{\dot{\chi}_N}{\chi_N} = \frac{1}{\theta} (r_N - \rho) - \left( q^{\frac{\alpha}{1-\alpha}} - 1 \right) p_I \tag{37}
\]

\[
\frac{\dot{\chi}_S}{\chi_S} = \frac{1}{\theta} \left( r_S - \frac{\dot{P}_S}{P_S} - \rho \right) - \left( q^{\frac{\alpha}{1-\alpha}} - 1 \right) p_I. \tag{38}
\]

As a result, in steady-state the change in the relative price of the Southern final good, \( \frac{\dot{P}_S}{P_S} \), is zero, thus, the diffusion of technology from the North to the South is sufficient to equalize the interest rates in the North and South, or \( r_N = r_S \) in steady-state.

4.2 Transitional Dynamics

The dynamic system of five equations and five unknowns consist of the evolution of \( \hat{Q} \), defined by equation (31); two firm entry and exit conditions, defined by equations (33) and (34); and the consumption growth conditions in the North and South, defined by equations (37) and (38) respectively. Using three initial conditions for \( \hat{Q}, n_{NS}, \) and \( n_S \), the transitional dynamics of wage inequality in the North and South are fully characterized. The transition path is solved by log-linearizing the system of equations around the steady-state and using the reverse shooting methodology.

The model is solved using numerical simulation for reasonable parameter values.\(^{16}\) Parameter values are based on theoretical and empirical priors, and chosen such that they yield saddle path stability. Other parameter selections potentially lead to non-existent steady-states, or globally divergent transitional paths. The parameters are restricted as to yield saddle paths with all real eigenvectors and three negative eigenvalues in the transitional matrix. See Eicher and Turnovsky (2001) and Connolly and Valderrama (2007) for details. The benchmark parameter values are listed in Table 3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta )</td>
<td>3</td>
<td>inverse of constant elasticity of sub.</td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.02</td>
<td>subjective discount rate</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.3</td>
<td>capital share in final goods production</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>3.5</td>
<td>elasticity of ( p_C ) w.r.t. ( \hat{Q} )</td>
</tr>
<tr>
<td>( A_N )</td>
<td>2.25</td>
<td>Northern final goods productivity</td>
</tr>
<tr>
<td>( A_S )</td>
<td>2.75</td>
<td>Southern final goods productivity</td>
</tr>
<tr>
<td>( \phi_I )</td>
<td>0.15</td>
<td>Northern innovation productivity</td>
</tr>
<tr>
<td>( \phi_C )</td>
<td>0.075</td>
<td>Southern imitation productivity</td>
</tr>
<tr>
<td>( q )</td>
<td>1.5</td>
<td>constant size of quality improvements</td>
</tr>
</tbody>
</table>

The parameter values for the trade barriers, \( \tau_{XS} \) and \( \tau_{XN} \), and educational quality measures, \( \beta_m, \delta^h_m, \) and \( \delta^l_m \), are discussed in the next section.

\(^{16}\)Code is available upon request
5 Trade Liberalization

Assume the global economy is in a steady-state in which there are positive tariffs on intermediate imports, $\tau_{XS}$ and $\tau_{XN}$. Trade liberalization, simply, removes the barriers to trade, and I characterize the transition path to the new steady state.

The steady-state and transitional dynamics of the effective level of human capital, the relative supply of skilled labor, within- and across-country inequality are each endogenously determined by the rate of innovation. However, the induced effects of trade liberalization on innovation depend on the initial conditions of educational quality in both regions. Specifically, a change in the rate of innovation following a shock to the economy alters the effective contributions of human capital, thereby further affecting the rates of global innovation. The interaction between the rates of global innovation and the effective levels of human capital is, therefore, key to the analysis.

By assumption, the quality of education is sufficiently high in the North that an increase in $p_I$ will increase the effective human capital, $H_N$, or using equation (11), $\frac{\partial H_N}{\partial p_I} > 0$. The South is, in case 1, considered to have low quality of education, while in case 2, the South has high quality of education. The educational quality values are summarized in Table 4.

| Table 4: Educational Quality Values |
|-------------------------------|----------------|----------------|
| North | South (Case 1) | South (Case 2) |
| $\beta$ | 11 | 10 | 10 |
| $\delta^h$ | 27 | 17 | 25 |
| $\delta^l$ | 1 | 1 | 1 |

The difference between the two cases is the quality of secondary education in the South, although similar results carry through by changing the quality of primary education. In case 2 the quality parameters are set such that an increase in the probability of innovation increases the effective human capital. The increase in the quality of secondary education is by approximately fifty percent, which is similar to the differences in the quality of education summarized in Table 1 between Latin America and East Asia.

5.1 Southern Trade Liberalization

Case 1: South has low quality of education.

Southern trade liberalization is the removal of the tariff rate $\tau_{XS}$, and in this analysis $\tau_{XS}$ drops from $\tau_{XS} = 0.5 \Rightarrow 0.0$.\textsuperscript{17} The first case involves trade between a Northern

\textsuperscript{17}Trade liberalization of a lesser magnitude induces the same qualitative results.
region with a high quality of education and a Southern region with a low quality of education. In steady-state, Southern led trade liberalization increases the incentives to innovate which increases the rates of technological progress on the frontier. In the long run: 1) within-country inequality rises in both regions; 2) across-country inequality rises; 3) the supply of skilled workers in both regions increase; and 4) global growth rates rise.

Figures 5.1a document the evolution of the rates of innovation and imitation in the transition from the initial steady-state with barriers to trade to the open steady-state. Panels 1 thru 20, show the evolution of the other key variables. The variables $Y_m$, $X_m$, $Z_m$ and $C_m$ in Figure 5.1a are presented adjusting for aggregate quality. Each, however, grows at a rate equal to the growth rate of technologies on the frontier, $\dot{Q}_N/Q_N$. Additionally, each variable is presented as the percentage change from the initial steady state.
Figure 5.1a: Southern Led Trade Liberalization, $p_I$ and $p_C$
Beginning with the immediate effects, trade liberalization reduces the price of intermediate goods for the Southern final goods market in two ways, $P_{NN}^* = q (1 + \tau_{XS})$ and $P_S = 1 + \tau_{XS}$, thus increasing the demand from the South. Overall, average profits of intermediate good firms in the North increase, which leads to an increase in the resources allocated for innovation. The fall in $P_{NN}^*$ increases the demand for exports from Northern firms facing Northern competition, $X_{NN}^*$, which decreases the relative price of the Southern final good, $\tilde{P}_S$. The fall in $\tilde{P}_S$ increases $Y_N$ by equation (23), decreases $\bar{\pi}_N$ and decreases $X_{NN}^*$. The net total initial effect is an immediate jump in $Z_N$, and thereby, an increase in $p_I$.

In the South, trade liberalization reduces average profits for successful imitators, and through the balance trade condition, reduces aggregate output, $Y_S$ by equation (24). The net initial effect is a drop in the probability of imitation $p_C$.

The initial increase in the rate of innovation ($p_I$) and decrease in the rate of imitation ($p_C$) also have implications for the effective human capital in both regions, as well as for the distribution of firms. The effective human capital increases in the North, and decreases in the South, which, by equation (26), increases Northern aggregate output, and decreases in Southern demand for intermediates both of which further increases the resources allocated to innovation in the North. Therefore, $p_I$ continues to rise, further increasing effective human capital in the North and decreasing Southern human capital. As for the distribution of firms, the share of industries with production in the South, $n_S$ falls, while the shares of both types of Northern firms, $n_{NN}$ and $n_{NS}$ increase immediately following Southern trade liberalization.

Immediately following trade liberalization, the initial jump in $p_I$ has important implications for income inequality within both regions. Specifically, as shown in panels 1 and 2 from Figure 5.1a, income inequality rises in both the North and South, but by a larger percentage in the North. More workers in both regions acquire secondary education, again by a greater percentage in the North. The reasoning for the higher percentage changes for North is the higher quality of secondary education magnifies the productivity effect of the initial rise in the rate of innovation.

The complete transitional path, however, is non-monotonic. Since $p_I$ is greater than $p_C$ in the transition, $\hat{Q}$ decreases. As $\hat{Q}$ falls, successful imitation becomes easier by equation 29, and the rate of imitation begins to converge to the rate of innovation. The share of firms in the South, along with the relative price of the Southern final good, also begin to rise. As a result $p_I$ begins to fall as the transitional path approaches the new long run steady state. After the initial rise in income inequality, inequality is falling in the latter stages of the transition. And in the long run, the rate of innovation exceeds its
initial level and long run inequality in both regions is higher. The dynamics of the rate of innovation are also reflected in the changes to human capital, within-country inequality, and the endogenous supply of skilled workers. The dynamics of global inequality are derived later in this section.

Case 2: South has high quality of education

Figures 5.1b document the same trade liberalization experiment, with the only difference being the quality of secondary education in the South is now sufficiently high and an increase in $p_I$ will increase the effective human capital $H_S$. In steady-state, Southern trade liberalization increases the incentives for innovation, leading to, again, an increase in within-country income inequality in the North and South. Also, the supply of skilled workers increase in steady-state.

The initial impact of Southern trade liberalization, where both trading partners have high education quality level, mirrors that in the first case. Trade liberalization reduces the price of intermediate goods, $P^*_{NN}$ and $P_S$, which increases the average profits in the North and decreases the average profits in the South. In the South however, the initial rise in the rate of innovation increases the effective human capital, $H_S$, which increases the aggregate final good output in the South. This jump in $Y_S$ leads to an initial increase in the resources devoted to imitation, and $p_C$ initially increases despite the downward pressure on the expected profits from successful imitation. The initial rise in $p_I$ exceeds that of $p_C$. The rise in output and the rise in profits in the North both increase the rates of innovation, however, the effects of the rise in output in the South on $p_C$ is dampened by the initial drop Southern average profits.
Figure 5.1b: Southern Led Trade Liberalization, $p_I$ and $p_C$
Panels 1 and 2 from Figure 5.1b show that income inequality (after an initial increase) drop below their initial levels in the transition. This implies that when the South has a high quality of education, trade liberalization leads to a short run decline in income inequality.

In the long run, however, the rates of innovation and imitation increase following trade liberalization, and it follows that income inequality also rises in the long run. However, the transitional dynamics differ substantially. In case 2, the transition of \( p_I \) and \( p_C \) is first decreasing and then increasing to the higher steady-state level. Again, the evolution is non-monotonic, but the increase in the quality of education in the South is sufficient to induce a transitional path that is the inverse of the case 1 transition.

**The Dynamics of Global Inequality**

In steady-state, Southern trade liberalization increases the relative wages of Northern workers, thereby increasing the degree of global inequality. In general the transitional path is first increasing, then decreasing to the post-trade liberalization steady state. Figure 5.1c documents the dynamics of global inequality (log of the Northern average wage relative to that in the South) under the cases in which the South is of high or low educational quality.

![Figure 5.1c: The dynamics of global inequality.](image)

In both cases there is an immediate rise in the Northern relative wages, but this initial increase is much more pronounced if the South has a low quality of education. The key difference is the manner in which the initial jump in \( p_I \) alters the effective human capital in the South. When the quality of education is high in the South, the increase in \( H_S \) increases Southern aggregate output which reduces the rise in the relative
wages of the North. Following the initial response to trade liberalization, in both cases the relative average wages in the North declines, but this convergence of the South is much sharper and quicker in the case when the quality of education in the South is high. At one point in the transition from Figure 5.1c Case 2, the South nearly returns to the pre-trade liberalization relative wages. In the long-run, trade liberalization, in both cases, reduces the relative average wages in the South.

The same experiments are performed for Northern trade liberalization, and the results can be found in the Appendix. Northern trade liberalization, however, decreases the average profitability for Northern intermediate firms in the long run and, thus, reduces the incentives to innovate, which lowers the rate of technological progress and income inequality within both regions. As with Southern trade liberalization, the transitional dynamics are non-monotonic. When both regions have a high quality of education the rate of innovation (hence inequality) first increases, then decreases. The dynamics of inequality when the South has a low quality of education is first increasing then decreasing. Hence, the direction of trade liberalization and the initial quality of education are both critical for the dynamics of income inequality.

6 Conclusion

The dynamics of wage inequality, both within and across countries, are dependent on the interaction between the rate of new technologies entering the production process and the effective human capital. In the long run, Southern trade liberalization increases the rates of technological progress, as well income inequality in both the North and South. However, the transitional path of the inequality within countries following trade liberalization is first increasing then decreasing, or an inverted U-shape in cases when the South has a low quality of education. The dynamics of inequality when both countries are of high educational quality is first decreasing then increasing, or is U-shaped. As such, the dynamics of within-country income inequality depend critically on the initial quality of education at the time of trade liberalization.

The key contribution of this paper is introducing a source of heterogeneity among developing countries that accounts for a wide array of income inequality dynamics. Broadly speaking, the results provide intuition as to why, following trade liberalization, developing countries in Latin America have experienced increasing income inequality. A high percentage of unskilled workers and a low quality of education have lead to pronounced growth in inequality as new technologies are diffused into the region through greater trade openness. On-the-other-hand, East Asian countries, with a higher quality of ed-
ucation and a greater stock of educated workers, were better equipped to adapt to new technologies. As a result, those countries did not experience the same growth in income inequality and were able to converge to the incomes in develop countries.

The results of the paper suggest several empirical implications linking the quality of education to the formation of effective human capital, within country income inequality and convergence following periods of trade liberalization. Specifically, the model predicts that developing countries with a higher quality of education experience: 1) an increase in effective human capital; 2) an initial decline in income inequality within the country; and 3) greater convergence relative to developing countries with a lower quality of education. In a preliminary paper, Hall (2008) provides empirical support for the relationship between inequality, technological change and the quality of education. To capture the change in new technologies through trade liberalization, Hall estimates the skill factor content of trade using a gravity equation for a wide range of developed and developing countries. Using the Hanushek and Kimko (2000) measure for the quality of education, the estimates show that a greater skill factor content of trade increases income inequality more for those countries with a lower quality of education. This preliminary research is robust to a number of empirical specifications and lends support to one of the fundamental claims of this paper that emphasize the critical role of education and technological change in determining the dynamics of income inequality.

7 References


8 Appendix

8.1 Detailed Equations

Northern and Southern Average Profits

This appendix provides more details for equations (15) and (16). Firms located in the North whose closest competitor is a Northern firm obtain a flow sum of domestic and export profits,

\[
\pi_{NN} = (q - 1) H_N \Lambda_N Q_N + (q - 1)(1 + \tau_{XS})(\frac{\tilde{P}}{1 + \tau_{XS}})^{\frac{1}{1 - \alpha}} H_S \Lambda_S Q_N, 
\]

where \( \Lambda_N = A_N^{1/(1-\alpha)} \left( \frac{a}{q} \right)^{(1/(1-\alpha))} \), and \( Q_N = \int_0^1 q^{k_N \alpha/(1-\alpha)} dj \) is the average quality level on the frontier. Northern firms facing Southern competition, likewise, earn a sum of import and export profits

\[
\pi_{NS} = (q \tilde{P} (1 + \tau_{XN}) - 1) \left( \frac{1}{\tilde{P}(1 + \tau_{XN})} \right)^{\frac{1}{1 - \alpha}} H_N \Lambda_N Q_N + (q \tilde{P} - (1 + \tau_{XS})) H_S \Lambda_S Q_N
\]

where the size of the incremental quality increase, \( q \), must be sufficiently large such the limit price exceeds the marginal cost of production. Average profits for intermediate firms in the North is

\[
\bar{\pi}_N = \frac{n_{NN} \pi_{NN} + n_{NS} \pi_{NS}}{n_{NN} + n_{NS}} 
\]

where \( \bar{\pi}_N \) is the profits adjusted for the average quality level on the frontier. Trade barriers and the probability of innovation are embedded within \( \bar{\pi}_N \). A change in the rate of innovation and imitation, alter the effective levels of human capital (\( H_N \) and \( H_S \)), the equilibrium relative price of the Southern final good (\( \tilde{P}_S = MC_S \)), and the distribution of firms (\( n_{NN}, n_{NS}, \) and \( n_S \)).

The partial effects of trade liberalization on the Northern average profits is given by,

\[
\frac{\partial \bar{\pi}_N}{\partial \tau_{XS}} = -\frac{H_S \Lambda_S}{n_{NN} + n_{NS}} \left( n_{NN} (q - 1) \left( \frac{\alpha}{1 - \alpha} \right) \left( \frac{\tilde{P}_S}{1 + \tau_{XS}} \right)^{1/(1-\alpha)} + n_{NS} \right) < 0
\]
\[
\frac{\partial \bar{\pi}_N}{\partial \tau_{XN}} = -H_N \Lambda_N \left( \bar{P}_S (1 + \tau_{XN}) \right)^{1/(\alpha - 1)} \left( 1 - \alpha \bar{q} \bar{P}_S (1 + \tau_{XN}) \right) \left( \frac{1}{(1 - \alpha)} \right) < 0
\]

Southern flow profits for intermediate producers are

\[
\pi_S = \left( 1 + \tau_{XS} - \bar{P} \right) \left( \frac{q \bar{P}}{1 + \tau_{XS}} \right)^{\frac{1}{1-\alpha}} H_S \Lambda_S Q_N + \left( 1 + \bar{P} (1 + \tau_{XN}) \right) q^{\frac{1}{1-\alpha}} H_N \Lambda_N Q_N
\]

where \( \Lambda_S = A_S^{1/(1-\alpha)} \left( \frac{\alpha}{q} \right)^{1/(1-\alpha)} \), and, as in the North, \( \bar{\pi}_S \) is the quality adjusted profits for Southern imitators. I assume the Northern tariff is sufficiently low such that the export profits for Southern firms, given the limit price, is positive. Since only the state-of-the-art technology is used, any good produced in the South will still have the same quality level as the lead Northern quality frontier.

The partial effects of trade liberalization on the Northern average profits is given by,

\[
\frac{\partial \bar{\pi}_S}{\partial \tau_{XS}} = \left( q \bar{P}_S \right)^{\frac{1}{1-\alpha}} H_S \Lambda_S Q_N \left[ \bar{P}_S - \alpha \left( 1 + \tau_{XN} \right) \right] > 0
\]

\[
\frac{\partial \bar{\pi}_S}{\partial \tau_{XN}} = -q^{1/(1-\alpha)} H_N \Lambda_N Q_N \bar{P}_S < 0
\]

**Resources Allocated to R&D**

Using the two world resource constraints, the expressions for \( Z_N \) and \( Z_S \) from equations (26) and (27) are, in the North,

\[
Z_N = \left[ H_N \Lambda_N \left( \frac{q}{\alpha} \right) \left( n_{NN} + n_{NS} \left( \frac{1}{\bar{P}_S (1 + \tau_{XN})} \right)^{\frac{\alpha}{1-\alpha}} + n_S q^{\frac{\alpha}{1-\alpha}} \right) - n_{NN} \left( H_N \Lambda_N + H_S \Lambda_S \left( \frac{\bar{P}_S}{1 + \tau_{XN}} \right)^{\frac{1}{1-\alpha}} \right) - n_{NS} \left( H_S \Lambda_S + H_N \Lambda_N \left( \frac{1}{\bar{P}_S (1 + \tau_{XN})} \right)^{\frac{1}{1-\alpha}} \right) - \chi_N \right] Q_N
\]

where

\[
\frac{\partial \bar{Z}_N}{\partial \tau_{XS}} = -n_{NN} H_S \Lambda_S \left( \frac{\bar{P}_S}{1 + \tau_{XN}} \right)^{1/(1-\alpha)} \left( 1 - \bar{P}_S (1 + \tau_{XN}) \right) < 0
\]

\[
\frac{\partial \bar{Z}_N}{\partial \tau_{XN}} = \frac{n_{NS} H_N \Lambda_N}{(1 - \alpha) \left( 1 + \tau_{XN} \right)} \left( \frac{1}{\bar{P}_S (1 + \tau_{XN})} \right)^{1/(1-\alpha)} \left[ 1 - \bar{P}_S (1 + \tau_{XN}) \right] > 0
\]
and in the South,

\[ Z_S = \left[ H_S \Lambda_S \left( \frac{q}{\alpha} \right) \left[ n_{NN} \left( \frac{\tilde{P}_S}{1 + \tau_{XS}} \right) \frac{\alpha}{1-\alpha} + n_{NS} + n_S \left( \frac{q_{\tilde{P}_S}}{1 + \tau_{XS}} \right)^{\frac{\alpha}{1-\alpha}} \right] - n_S q^{\frac{1}{1-\alpha}} \left( H_S \Lambda_S \left( \frac{\tilde{P}_S}{1 + \tau_{XS}} \right) \frac{\alpha}{1-\alpha} + H_N \Lambda_N \right) - \chi_S \right] Q_N \]

\[ \equiv \tilde{Z}_S (\tau_{XS}, \tau_{XN}, p_I) Q_N \]

where

\[ \frac{\partial \tilde{Z}_S}{\partial \tau_{XS}} = -\frac{q H_S \Lambda_S}{\tilde{P}_S (1 - \alpha)} \left( \frac{\tilde{P}_S}{1 + \tau_{XS}} \right)^{\frac{1}{1-\alpha}} \left[ n_{NN} + n_S q^{\alpha/(1-\alpha)} \left( 1 - \frac{\tilde{P}_S}{1 + \tau_{XS}} \right) \right] < 0 \]

where \( \chi_N = C_N / Q_N \) and \( \chi_S = C_S / Q_N \). \( \tilde{Z}_N \) and \( \tilde{Z}_S \) are the quality adjusted expenditures on R&D.

**Balanced Trade Condition**

The relative price of the Southern final good adjusts at each point of time to balance trade between the North and South. Expanding equation 30, \( \tilde{P}_S \) implicitly solves,

\[ q H_S \Lambda_S \left[ n_{NS} \tilde{P}_S + n_{NN} (1 + \tau_{XS}) \left( \frac{\tilde{P}_S}{1 + \tau_{XS}} \right)^{\frac{1}{1-\alpha}} \right] - n_S H_N \Lambda_N q^{1/(1-\alpha)} = 0 \]

where the partial effects of trade liberalization is given by,

\[ \frac{\partial \tilde{P}_S}{\partial \tau_{XS}} = \frac{n_{NN} \left( \frac{\alpha}{1-\alpha} \right) \left( \frac{\tilde{P}_S}{1 + \tau_{XS}} \right)^{\frac{1}{1-\alpha}}}{n_{NS} + \frac{1}{1-\alpha} n_{NN} \left( \frac{\tilde{P}_S}{1 + \tau_{XS}} \right)^{\frac{\alpha}{1-\alpha}}} > 0 \]

**8.2 Northern Trade Liberalization**

Northern trade liberalization yields significantly different results due to their different effects on the expected gains from successful R&D. In the long run, Northern trade liberalization decreases the rate of innovation, and thereby decreases steady-state inequality within countries. Figure 8.2a documents the transition for \( p_I \) and \( p_C \) for case 1 and case 2 as defined above.
In case 1, the rate of innovation first decreases, then increases to a lower steady-state level, while in case 2, $p_I$ first increases, then decreases. A reduction in the Northern tariff rate, $\tau_{XN}$, decreases the domestic price for Northern firms facing Northern competition, which lowers the average profits for Northern intermediate firms, and increases the average profits for a Southern intermediate firm. As such the rates of innovation $p_I$ initially drops and the rate of imitation $p_C$ initially increases. This sets in motion a dynamic feedback loop opposite as the case during Southern trade liberalization.

In the case of Northern trade liberalization, when the South has low quality of education, income inequality follows a U-shape transition, but an inverted U-shape transition with the quality of education is high. Ultimately, income inequality in the both the North and South fall, reflecting the long run decline of the rate of innovation.

**The Dynamics of Global Inequality**

In steady-state, Northern trade liberalization decreases the relative wages of Northern workers, thereby decreasing the degree of global inequality. This is, again, in contrast to the results following Southern trade liberalization. The general transitional path is the inversion of the transitional dynamics following Southern trade liberalization. The transitional path is first decreasing, then increasing to a lower steady-state. Figure 8.2b documents the dynamics of global inequality under the cases in which the South is of high or low educational quality.
Figure 8.2b: The dynamics of global inequality.

Below is the evolution of all the variables following Northern trade liberalization. Again, case 1 assumes the South has a low (below threshold) quality of education (Figure 8.2c), while case 2 assumes the South has a high quality of education (Figure 8.2d). All variables are expressed as percentage changes from the initial steady state.

Figure 8.2c: Northern Trade Liberalization, Case 1

* Denote adjusted for Qaulity
† Denote percentage changes from initial steady state
Figure 8.2d: Northern Trade Liberalization, Case 2

* Denotes adjusted for Quality
† Denotes percentage change from initial steady-state