Within and Across Country Inequality in a Model of Trade and Endogenous Growth

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Abstract

This paper addresses the dynamics of income inequality, both within and across countries. In an endogenous growth model with North-South trade, the dynamics of income inequality depend on the ability of workers to adapt to new technologies, captured by the quality of education. For developing countries with low quality of education, Southern trade liberalization leads to an inverted U-shape transition of within country income inequality, and transitional divergence in terms of income relative to the North. However, in cases where the South has a high quality of education, workers are better equipped to adapt to new technologies, and trade liberalization induces an U-shape dynamic transition of within country income inequality, and convergence in the short and long run. This paper highlights the critical role the quality of education plays in explaining the variations in the observed dynamics of income inequality in developing countries.

Keywords: Endogenous Growth, North-South Trade, Income Inequality, Quality of Education

JEL Classification: O15, O33

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1 Introduction

The distribution of income across individuals and across countries has long been an important issue in economics. This paper links income inequality both within and across countries, and presents a mechanism that can explain the varied dynamics of inequality among countries and regions. Within most developed and developing countries, the growth of inequality accelerated substantially during the 1980s and 1990s. Focusing on developing and emerging economies, the growth in inequality was more severe in Latin America and Africa compared to many East Asian countries.¹ Inequality across countries have also varied. Per capita GDP in Latin America and Africa, on average, fell relative to the U.S., while East Asia largely converged.² In a general equilibrium model of North-South trade with endogenous innovation (conducted by the North) and imitation (conducted by the South), I account for the varied dynamics of within and across country inequality by focusing on the quality of education and its interaction with technological progress. In the model, a higher quality of education improves the ability of workers to adapt to new technologies. Trade liberalization alters the pace of technological progress which, if the quality of education is low (high), induces a growth (decline) of within country inequality and divergence (convergence) in terms of per capita income during the dynamic transition.

In his seminal contribution, Kuznets (1955) suggests within country inequality will rise in the early stages of development where investment in physical capital is the engine of growth, yet decline in latter stages as human capital becomes the primary growth mechanism. To address the continued growth of inequality within developed countries, the theoretical literature often identifies skill-biased technical change and globalization as sources of changing inequality. Galor and Tsiddon (1997), Greenwood and Yorukoglu (1997), Caselli (1999), Lloyd-Ellis (1999), Aghion et al. (2002) and Aghion (2002) focus on technological revolutions that give rise to an increase in demand for skilled workers which puts upward pressure on their relative wage. Acemoglu (1998) argues a sharp (exogenous) increase in the supply of skilled workers raises the return to innovations targeted at skill-intensive sectors which leads to an increase in their relative wage. Galor and Moav (2000) introduce the idea that the rate of technological progress determines the relative demand, and reward, for skilled labor.

The globalization argument (see Wood, 1994) stems from the Stopler-Samuelson theorem where the reduction in impediments to trade with skill-scarce countries increases the relative demand for skilled workers in the skill-abundant countries, and therefore raises the skill premium. The theory also suggests trade liberalization decreases the relative demand and premium for skills in less developed countries,

¹Uses inequality data from the University of Texas Inequality Project (available at http://utip.gov.utexas.edu/data.html). The empirical evidence will be addressed in more detail in the following section.
²Data on GDP per capita is taken from Penn World Tables (Heston et al., 2006).
which is not consistent with empirical evidence. Dinopoulos and Segerstrom (1999, 2006), Sener (2001), Acemoglu (2003), Grieben (2005) and Zeira (2007) provide more unified models of technology and trade that avoid the pitfalls of the Stolper-Samuelson theorem. Ripoll (2005) develops a general equilibrium model of trade and finds that initial conditions, such as the skilled-unskilled labor ratio, are important to the dynamics of income inequality following trade liberalization. While these papers emphasize the relationship between technology, trade and inequality, this paper focuses on why the dynamics of inequality differ among developing countries and also considers the implications for inequality across countries.

In addition to addressing the dynamics of inequality within countries, this paper also seeks to capture inequality across countries. The economic growth literature primarily focuses on the convergence hypothesis. The hypothesis generally asserts that differences in per capita income between any pair of countries will be transitory as long as they possess identical technologies, preferences and population growth rates. Empirical papers have found evidence that after controlling for savings rates and population growth rates, countries with low initial levels of per capita output tend to grow faster than those with higher initial levels of per capita output. Quah (1996a, b) and Galor (1996), among others, offer a contrasting view in which locally stable convergence clubs emerge, where there is a wide range of long run per capita output levels.

To capture the varied patterns of inequality within and across countries, I focus on the interaction of the quality of education and the pace of technological progress. Workers are differentiated on a continuum of innate ability and make a discrete choice at each point in time as to whether acquire education and become a skilled worker, or remain unskilled, based on their individual expected income. To become skilled, the worker forfeits a fraction of their labor endowment to acquire education. This cost of education is decreasing in their innate ability, or, education is “cheaper” for those with a higher ability.3 The benefits of education are two-fold. First, skilled workers have a productivity advantage in production relative to unskilled workers which is increasing in the rate technological progress. This flows from Ferguson (1993) and Bartel and Sicherman (1999) who find that technological progress increases the return to education.

The second benefit to acquiring costly education is a reduction in the time workers spend adapting, or learning, new technologies. In this model, both skilled and unskilled workers spend a portion of their labor endowment, not in production, but in this learning process. Bartel and Sicherman (1998) show that an increase in the rate of technological progress increases the need to (re)train workers, especially

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3This assumption is line with Griliches and Mason (1972) and Murnane, Willett and Levy (1995) who find individual earnings increase with ability. In this model, because the cost of education is decreasing in ability, higher ability workers devote more of labor endowment to income earning production.
low skill workers. This feature is captured in this model as the portion of time learning is increasing in the pace of technological progress. For skilled workers, this learning cost is reduced. This introduces the role of the quality of education. A higher quality of education reduces the time spent learning new technologies for those who have made the discrete decision to acquire education.

Faster rates of technological progress have competing effects on the overall, effective human capital supplied to production. First, an increase in the rate of technological progress increases the relative productivity of skilled workers inducing more workers to acquire education and become skilled. This effect increases overall human capital. Second, faster rates of technological progress increases the time needed to learn and adapt to the new technologies, reducing the effective labor supply to production which decreases overall human capital. A higher quality of education reduces the strength of this second effect, and increases the marginal impact of an increase of technological progress on overall human capital. The overall impact of faster rates of technological progress is determined by the quality of education which is a key feature of the model.

The model is solved numerically using reasonable parameter values. I focus on the income inequality both within and across countries following Southern trade liberalization. I consider two specific cases referring the quality of education in the South. The first is when the South has a low quality of education and the second is when the South has a higher quality of education.

In the long run, Southern trade liberalization increases the rate of technological progress and economic growth under both cases by a similar magnitude. Long run inequality within countries unambiguously increases in both the North and South due to the increased pace of technological progress, while long run inequality across countries decreases. The per capita income of Southern workers increase relative to Northern workers, however total profits for Northern producers of intermediate goods increase relative to Southern producers of intermediate goods. The intuition is that Northern producers face lower costs of exporting intermediate goods to the South following trade liberalization. Overall, the quality of education in the South does not significantly change the long run, qualitative implications of Southern trade liberalization.

The transitional dynamics of inequality within and across counties, however, do significantly vary based on the quality of education in the South. In the case in which the South has a low quality of education, the rate of technological progress jumps initially, continues to increase in the initial periods of the transition, before declining to an overall higher rate of technological progress in the long run steady state. In contrast, when the quality of education is relatively high in the South, after an initial

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4In this class of models, the South refers a developing or emerging economy, whereas the North represents a developed country. Northern trade liberalization and bilateral trade liberalization experiments are possible in the context of this model. In the present paper, I focus on Southern trade liberalization.
jump, the rate of technological progress declines in the early portion of the transition before increasing to a higher long run equilibrium. The nonmonotonic transitional dynamics are directly due to the quality of education in the South. When the quality of education is low, the initial jump in the rate of technological progress following trade liberalization reduces overall human capital in the South (for the reasons discussed above), which limits their ability to imitate and further increases the incentives for Northern firms to innovate. The initial jump in the rate of technological progress increases overall human capital when the quality of education is high. This increases the ability to imitate in the South, reducing the incentives for Northern firms to innovate, and leads to the decline in the rate of technological progress in the early stages of the dynamic transition.

Inequality within countries follow the nonmonotonic transition of the rate of technological progress. Focusing on the initial periods following trade liberalization, within country inequality declines when the quality of education in the South is high, but increases when the quality of education is low. In addition, the per capita income of Southern workers converge to that in the North in both the transition and in the long run when their quality of education is high. Per capita income for Southern workers, conversely, diverges during the initial periods of the transition when the quality of education is low. The quality of education emerges as an important determinant of the varied dynamics of within and across country inequality observed in the data. Empirical evidence supports this claim. Using data compiled by Hanushek and Kimko (2000) which provides a measure for the quality of education based on math and science scores, Hall (2010) finds a higher quality of education both directly reduces the growth of within country inequality and the marginal impact of technological diffusion on inequality, which is consistent with results of this model. In addition, Hanushek and Kimko use their measure to show the quality of education is positively related to the economic growth in developing countries.

The remainder of the paper is organized as follows: Section 2 provides an overview the empirical motivation; Section 3 introduces the model; Section 4 details the steady-state and transitional dynamics; Section 5 discusses the implications of Southern trade liberalization on within and across country inequality; and Section 6 concludes.

2 Empirical Motivation

2.1 Within Country Income Inequality

The empirical literature on within-country income inequality is extensive. By most measures, inequality within the United States and other industrialized countries rose from the 1970s until 2000 (Wood, 1994;
Machin, 1996; Autor et al., 2005). However, among developing countries there is no such clear pattern. Latin America, for example, experienced a growth in income inequality, while inequality within many East Asian countries declined. Hanson and Harrison (1999) show the skilled/unskilled wage gap grew in Mexico during the 1980s. Robbins (1996) and Wood (1997) find inequality also grew in Hong Kong, Argentina, Chile, Colombia, Costa Rica, Uruguay and Mexico, but fell in Korea, Taiwan, Singapore and Malaysia during the same period. Das (2002), similarly, finds rising income inequality in Mexico and Chile, and falling inequality within Philippines, Singapore and Taiwan. Duryea and Szekely (2000) and Behrman, Birdsall and Szekely (2000), on-the-other-hand, show inequality fell within Brazil, Mexico, Venezuela, Argentina and Bolivia, and was constant in Chile and Costa Rica during the 1980s and 1990s. Michaely et al. (1991) show a rise and then fall of inequality within Singapore and Sri Lanka. Zhu and Trefler (2001) find that out of 29 developing countries, inequality is rising in 16 countries and falling (or remaining constant) in 13 countries.

The empirical literature focusing on the dynamics of within country inequality is certainly mixed. To provide a more uniform consideration of within country inequality, Figure 1, taken from Hall (2010) shows the median of the log change of the Theil index for five country cohorts: advanced countries, Middle Eastern and North African countries, Latin American countries, Sub-Saharan African countries and Asian countries (UTIP-UNIDO, 2002). The data spans from 1960 until 1995 and is broken up into six ten-year periods. This measure, first introduced by Theil (1967), is a consistent measurement of inequality based on industrial wages in the manufacturing sector published annually by the United Nations Development Organization.5

The median growth of inequality was negative or zero for each of the five groups of countries between 1965 and 1975, as well as from 1970 to 1980. However, the growth of inequality accelerated substantially during the 1980s for most groups of countries. The relative magnitude of the growth of inequality in the 1980s is also important. Inequality grew substantially more in Latin America and Africa relative to the other three cohorts, while the growth of inequality was lowest in the Asian countries for the periods 1980-1990 and 1985-1995.

2.2 Across Country Income Inequality

When addressing inequality across countries, one approach is to view countries as a unit of measure. Under this assumption, the empirical convergence literature established divergence in GDP per capita,

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5Another commonly used measure of inequality is the Gini coefficient (Deininger and Squire, 1996; 1998). However, this measures the concentration of income based on household surveys (See Deininger and Squire, 1996). Conceicao and Galbraith (2000) and others also find this measure to be incomplete in terms of country coverage over time and largely inconsistent with other measures of inequality.
Figure 1: Median log change in the Theil index over ten year intervals for five country cohorts: Advanced economics, Middle East/North Africa, Africa, Latin America, and Asia. For each country the data were averaged around years given. For example, the Theil for 1980 is the average of 1978-1982 and the value for 1990 is the average of 1988-1992 for each country.

most notably due to the poor economic performance of many Latin American and African countries. Specifically, the growth rates of poor countries have been lower than the growth rates of rich countries, and the dispersion of income per capita across countries has increased over time.

This finding of divergence, however, is not robust when considering the individual as the unit of measure. The convergence of more populous regions, including China and India, drives a decline in global inequality across all individuals. Overall, recent empirical evidence suggests that after peaking around 1979 global inequality is declining (See Sala-i-Martin, 2006).

Abstracting from the convergence debate, the focus of this paper is why certain countries diverged and others converged in terms of average GDP per capita relative to that in the United States. Figure 2 uses data from the Penn World Tables 6.3 (Heston et al., 2009) to show the GDP per capita relative to the U.S. GDP per capita for a subset of African, Latin American and East Asian countries from 1960 to 2002. Figure 2 shows East Asia, on average, converged substantially in terms of living standards, while both Latin America and Africa experienced divergence. Also, the convergence of East Asia and the divergence of Latin America and Africa accelerated during the 1980s. This decade is known for widespread trade liberalization among developing countries, which suggests that opening to trade and new technologies that flow into the country may be an important source of the dynamics of income

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6 $\beta$-divergence in Barro and Sala-i-Martin (1992) terms.
7 $\sigma$-divergence in Barro and Sala-i-Martin (1992). See the “twin peaks” literature following Quah (1993).
2.3 Quality of Education

The quality of education varies widely across countries. Standard measures for educational quality, including adult literacy rates, teacher-pupil ratios, or expenditures per student, are typically insignificant in cross-country growth studies, and are notoriously poor measures for the quality of education in the labor force. A potential reason is that these measures do not directly capture the cognitive ability of the labor force. Hanushek and Kimko (2000) address this issue and develop measures for the quality of the labor force derived from a number of international mathematics and science tests between the years 1965 through 1991. While test score data is available for only 38 countries, Hanushek and Kimko use consistent estimators to forecast labor force quality for a large number of countries based on country specific characteristics. In all, they produce quality measures for 90 countries across the development spectrum. Table 1 in the appendix provides the quality indices for a sample of countries taken from Hanushek and Kimko (2000).

The quality of the labor force is consistently higher in Asia relative to Latin America, and the measure performs well in cross-country growth regressions. Using the Hanushek and Kimko measure, the poorest quality of the labor force is 18.26 (Iran), while the highest is 72.13 (Singapore). The mean for the entire sample of countries is 51.28. Overall, this data supports the idea that the quality of
education varies significantly and systematically across countries and regions.

3 The Model

The model features North-South trade with Schumpeterian endogenous growth through creative destruction in continuous time. The North represents a developed country, while the South represents a less developed country. Innovations increase the quality, or productivity, of intermediate goods used in final goods production. State-of-the-art quality levels are only discovered through research and development (R&D) efforts in the North, but once an innovation occurs, the South undertakes R&D to imitate the Northern frontier technology.

Human capital is an input for final goods production. As introduced by Galor and Moav (2000), the rate at which new state-of-the-art technologies enter the production process determines, in part, the level of effective human capital in the economy. The effective human capital in the economy is a weighted sum of skilled and unskilled workers whose productivity and effective supply to production are determined by the quality of the education and the rate of technological progress.

3.1 Final Goods Sector

The final goods production function includes a conventional quality ladder model, a la Grossman and Helpman (1991), Aghion and Howitt (1992), and Barro and Sala-i-Martin (1997, 2004). In this setup technology is embedded within the productivity, or quality, of the intermediate goods used in producing a final good. Denote the productivity of a given intermediate good industry $j$ to be $q^{k_j}$, where $q$ is the incremental rise in productivity per innovation, and $k$ is the number of innovations. Assume a continuum of intermediate goods, $j \in [0, 1]$.

Final goods production in each region $m \in [N \ (North), S \ (South)]$ takes the Cobb-Douglas functional form,

$$ Y_m = A_m (L_m H_m)^{1-\alpha} \int_0^1 \left( q^{k_j x_{mk_j}} \right)^\alpha \ dj $$

where $\alpha$ is the share of capital in production, $A_m$ is the total factor productivity parameter in the final goods sector, $x_{mk_j}$ is the physical quantity of intermediate good $j$ with quality level $k$, and $q^{k_j x_{mk_j}}$ is the quality adjusted input for the intermediate good from industry $j$. The inclusion of $N$ in $q^{k_j}$ indicates that in equilibrium only the highest quality of intermediate good will be used in final goods production, which by definition, is discovered only through Northern innovative activity. $H_m$ represents the effective human capital of each country. Embedded within $H$ are the contributions of
both skilled and unskilled workers, whose supply and productivity are both endogenous and determined in equilibrium. $L_m$ is the size of the population, normalized to $L_N = 1$ in the North. For simplicity, there is no population growth and workers are immobile across countries.

The inverse demand for an intermediate good $x$ from industry $j$ is

$$P_j = \alpha \tilde{P}_m (L_m H_m)^{1-\alpha} x_{mkj} q^{kN,\alpha}$$

where $P_j$ is the price of the intermediate good from industry $j$, and $\tilde{P}_m$ is the price of the (nontradeable) final good. The price of the Northern final good, $\tilde{P}_N$, is the numeraire, $\tilde{P}_N \equiv 1$. Therefore, $\tilde{P}_S$ is defined as the relative price of the final good in the South. As later sections discuss, $\tilde{P}_S$ is endogenous, and adjusts in every period to balance trade.

### 3.2 Human Capital

Individuals choose between working as skilled or unskilled based on their expected income, thus the supply of each type of labor is endogenous. Workers are differentiated by their innate, cognitive ability which is reflected in their individual cost of education. A uniformly distributed continuum of individuals $i$ in each region is indexed by ability $a_{mi}$. Each worker is endowed with one unit of labor at every point in time. To become skilled, the individual devotes a fraction of their labor endowment to the acquisition of education. The effective supply of labor to production for individual $i$ as a skilled, $h_{mi}$, or unskilled, $l_{mi}$ worker takes the form of:

$$h_{mi} = a_{mi} - \frac{1}{\delta_m} (1 - a_{mi}) p_I$$

or

$$l_{mi} = 1 - (1 - a_{mi}) p_I$$

efficiency units of labor to final goods production. The first term on the right hand side of equations (3) and (4) captures the labor endowment net costs of education. An unskilled worker supplies their full labor endowment of 1 to production, while a skilled worker supplies $a_{mi}$ to production. The remaining $1 - a_{mi}$ is lost due to the cost of education. The second term on the right hand side is the time cost required for worker $i$ to adapt to new technologies. This time cost is increasing in $p_I$ which is the (endogenous) instantaneous probability of innovation, which captures the pace at which new technologies are entering the production process. Faster rates of innovation reduce the effective supply of labor to production. This learning cost is decreasing in ability (ie. higher ability workers learn and
adapt to new technologies at a faster pace). Finally, $\delta_m > 1$ is the quality of education received by skilled workers who made the costly investment into acquiring education. A higher quality of education reduces the time that skilled workers need to learn new technologies.

**Assumption 1:** I assume that in equilibrium $1 > p_I \geq 0$. The second part of this inequality ensures that the endogenous instantaneous probability of innovation is non-negative. The first part of the inequality ensures that the worker with the lowest ability will have a non-negative supply of production labor (after subtracting the time spent learning new technologies) as either a skilled or unskilled worker.

In the spirit of Galor and Moav (2000), the aggregate effective human capital $H$ is given by a weighted sum of the endogenous aggregate effective labor supply of skilled and unskilled workers. The effective human capital takes the form of,

$$H_m = (1 + \delta_m p_I) h_m + l_m,$$

where $h_m = \int_{h_{mi}} h_{mi} d a_{mi}$ and $l_m = \int_{l_{mi}} l_{mi} d a_{mi}$ are the total effective supply of labor to production for skilled and unskilled workers. The higher quality of education, $\delta_m$, increases the relative productivity of skilled workers who made the costly investment into acquiring education. Finally, faster rates of technological progress also increase the relative productivity of skilled workers.

Using equations (1) and (5), the competitive wages for skilled ($s$) and unskilled ($u$) workers in country $m$ are

$$w^s_m \equiv \omega_m (1 + \delta_m p_I) \text{ and } w^u_m \equiv \omega_m,$$

where $\omega_m \equiv (1 - \alpha) \frac{P_m Y_m}{H_m}$ is the marginal product of overall human capital. Income is the wage rate times the efficiency units of labor supplied. The individual with ability $a_{mi}$ earns income $I^s_{mi}$ working as a skilled worker, or $I^u_{mi}$ if unskilled:

$$I^s_{mi} = w^s_m h_{mi} = w^s_m (a_{mi} - \frac{1}{\delta_m} (1 - a_{mi}) p_I)$$

$$I^u_{mi} = w^u_m l_{mi} = w^u_m (1 - (1 - a_{mi}) p_I).$$

At each point in time, worker $i$ makes the discrete choice of whether to acquire education and work as skilled or remained unskilled by maximizing potential income as skilled, $I^s_{mi}$, or unskilled, $I^u_{mi}$.

**Proposition 1:** There is a single threshold level of ability, $a^*_m$, such that all workers with ability $1 > a_{mi} > a^*_m$ choose to acquire education and work as skilled, while all workers with ability $0 < a_{mi} < a^*_m$
remain unskilled.

Proof: It is straightforward to show that \( I_m^s (a_i = 0) < 0, I_m^u (a_i = 0) > I_m^s (a_i = 0) \) and \( I_m^u (a_i = 1) > I_m^u (a_i = 1) \). Since income for skilled and unskilled workers are increasing in ability in a linear fashion, there is a single worker whose ability is such that \( I_m^u (a_i = a^*) = I_m^s (a_i = a^*) \).

The threshold ability level of the worker indifferent from becoming skilled or remaining unskilled is given by \( a^*_m \):

\[
a^*_m = \frac{1 - p_f + (1 + \delta_m p_f) \frac{\mu_f}{\delta_m}}{1 - p_f + (1 + \delta_m p_f) \frac{\mu_f}{\delta_m} + \delta_m p_f}.
\]

(8)

Any worker with ability \( 1 > a_m > a^*_m \) will choose to become skilled, while the rest, \( 0 < a_m < a^*_m \), choose to remain unskilled. \( a^* \) is interpreted as the fraction of workers of workers who choose to remain unskilled for a given quality of education and the rate of technological progress.

Figure 3.2 plots the skilled and unskilled incomes across individuals. The dark line shows the maximum potential income for each individual.

The threshold level of ability decreases in the quality of education which implies that an increase in the quality of education will increase the fraction of workers which choose to acquire education:
\[
\frac{\partial a_m^*}{\partial \delta_m} = - \frac{p_I (1 - p_I) + \frac{p_I^2}{\delta_m} (2 + \delta_m p_I)}{(1 - p_I + (1 + \delta_m p_I) \frac{p_I}{\delta_m} + \delta_m p_I)^2} < 0. \tag{9}
\]

An increase in the rate of technological progress also decreases the threshold level of ability based on Assumption 1. Faster rates of technological progress increase the fraction of workers choosing to acquire education:

\[
\frac{\partial a_m^*}{\partial p_I} = - \frac{\delta_m (1 - p_I^2)}{(1 - p_I + (1 + \delta_m p_I) \frac{p_I}{\delta_m} + \delta_m p_I)^2} < 0. \tag{10}
\]

Aggregating across individuals yields

\[
\begin{align*}
 h_m &= \int_{a_m^*}^{1} a_m^* - \frac{1}{\delta_m} (1 - a_m^*) p_I da_m = \frac{1}{2} \left[ 1 + \frac{p_I}{\delta_m} \right] \left( 1 - (a_m^*)^2 \right) - \frac{p_I}{\delta_m} (1 - a_m^*) \\
 l_m &= \int_{0}^{a_m^*} 1 - (1 - a_m^*) p_I da_m = \frac{p_I}{2} (a_m^*)^2 + a_m^* (1 - p_I). \tag{11}
\end{align*}
\]

The total effective supply of skilled and unskilled workers is their total labor endowment less the cost of education for skilled workers (first term in the far right equations), less their total learning cost (final term in above equations). The total effective human capital level is given by substituting equations (11) and (8) into equation (5). Rearranging (5) yields

\[
H_m = \left[ \frac{1}{2} (1 + \delta_m p_I) \left( 1 - (a_m^*)^2 \right) + a_m^* \right] - \left[ \frac{p_I}{2} \left( 1 + \delta_m p_I \right) \frac{(1 - a_m^*)^2}{\delta_m} + a_m^* (2 - a_m^*) \right]. \tag{12}
\]

Equation (12) breaks the overall effective human capital into two primary components. The first set of brackets is the productivity adjusted labor supply of skilled and unskilled workers less the cost of education for skilled workers. The second set of brackets is the productivity adjusted loss of labor supply due to the time spent learning new technologies. The derivative of $H_m$ with respect to $p_I$ is ambiguous and depends on the quality of education. An increase in the rate of technological progress unambiguously increases the first bracket in two ways. First, skilled workers become more productive, and second, more workers make the choice to become skilled. Faster rates of technological progress, however, reduces the second set of brackets because workers spend more time adapting to new technologies rather than in production. This effect is partially offset as more workers become skilled because skilled workers are better able to adapt to new technologies given $\delta_m > 1$.  

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Consider how overall effective human capital depends on $p_I$:

$$\frac{\partial H_m}{\partial p_I} = \left[ \frac{\delta m}{2} (1 - a_m^2) \right] + \left[ (1 - a_m (1 + \delta m p_I)) \frac{\partial a^*_m}{\partial p_I} \right]$$

$$- \left[ \frac{1}{2} (1 + \delta m p_I) \left( \frac{1 - a_m^*}{a_m^*} \right)^2 + a_m^* (2 - a_m^*) \right] - \left[ p_I \left( 1 - \frac{a_m}{a_m} + a_m (1 + p_I) \right) \frac{\partial a^*_m}{\partial p_I} \right] \quad (13)$$

The partial derivative $\partial H_m/\partial p_I$ is positive or negative depending on the rate of technological progress and the quality of education. From equation (13), the first bracket is the gain in productivity for skilled workers, the second bracket is the change in the composition of the workforce, the third bracket is the loss due to the increased time needed to adapt to new technologies, and the fourth bracket is the benefit of more skilled workers lowering the overall amount of time workers need to learn new technologies. In addition, a higher quality of education, $\delta_m$, increases the positive impact of the first bracket, ensures that the second bracket is positive, lowers the magnitude of the third bracket and increases the magnitude of the final bracket. Figure 3.2 shows how the effective human capital depends on the interaction of the quality of education and the rate of technological progress.

For the discussion that follows, for a given change in $p_I$, a country with education parameters such that $\partial H/\partial p_I < 0$ is considered to have a low quality of education, while a country with parameters such that $\partial H/\partial p_I > 0$ is considered to have a high quality of education. Equation (13) implies that an increase in the rate of technological progress will increase the effective human capital for countries with
a sufficiently high quality of education, but could decrease if the quality of education is sufficiently low.

3.3 Income Inequality Within Countries

This section derives the relationship between technical innovation, the quality of education, and income inequality within countries. Inequality across countries will be addressed in a subsequent section.

Within country inequality is defined to be the average income of skilled workers relative to the average income of unskilled workers. Exploiting the linearity of incomes with respect to ability, average incomes for skilled, $\tilde{I}_m^s$, and unskilled, $\tilde{I}_m^u$ workers are

\[
\tilde{I}_m^s = \frac{I_m(a_m^*) + I_m(0)}{2} = \frac{1}{2} w_m^s \left( 1 + a_m^* - \frac{p_l}{\delta_m} \right) (1 - a_m^*)
\]

(14)

\[
\tilde{I}_m^u = \frac{I_m(a_m^*) + I_m(0)}{2} = \frac{1}{2} w_m^u \left( 1 + a_m^* - p_l (2 - a_m^*) \right).
\]

Using the average incomes for the two types of labor, within-country inequality for both the North and South is expressed as

\[
\Omega_{s/u}^s = \frac{\tilde{I}_m^s}{\tilde{I}_m^u} = (1 + \delta_m p_l) \left( 1 + a_m^* - \frac{p_l}{\delta_m} (1 - a_m^*) \right).
\]

(15)

3.4 Intermediate Goods Sector

The intermediate goods sector follows a two-stage process: 1) the process of research and development; and 2) monopolistic competition given the stage one R&D results. The first stage is the allocation of resources into the research and development (R&D) of new technologies. The North, by assumption, alone possesses the capability to invent a new state-of-the-art technology. If there is R&D success in the North in a given industry $j$, then the quality of that intermediate good rises by a constant size, $q$, from $q_{kj}$ to the new quality level $q_{kj}^{k+1}$, where $k$ is the number of innovations. Since Northern firms have the ability to create new technologies, they must innovate in order to expand the world’s technology frontier. R&D in the South is conducted to imitate frontier technologies.

In the second stage, the successful firms set prices and realize the rents from innovation or imitation. Within intermediate industries, competing firms holding different quality grades of a substitutable intermediate good engage in Bertrand price competition. Under the condition that $q < 1/\alpha$, firms follow a limit price strategy.\(^9\) Following Grossman and Helpman (1991), limit pricing drives lower quality grades within a given industry out of the market.

The location of production for intermediate $j$ depends on the stage one R&D results. Following a

\(^9\)If this inequality does not hold, successful innovators charge monopoly prices and successful imitators engage in limit pricing strategies.
successful innovation in industry $j$, the Northern firm serves as the global source for that good. That firm supplies the domestic demand and the foreign demand through exporting. The Northern firm holds the market until either a competing Northern firm makes a further innovation, or the quality grade is successfully imitated by a Southern competitor. Following successful imitation in industry $j$, the Southern firm serves, again, as the global source by exporting to the North. By assumption, Southern firms have a marginal cost advantage and are able to drive Northern competitors out of the market. Assuming a continuum of industries with a mass of one, $n_{NN}$ is defined as the share of industries served by a firm located in the North whose competitor with the next highest quality grade is a fellow Northern firm, $n_{NS}$ is the share of industries served by a firm located in the North whose competitor with the next highest grade is located in the South, and $n_S$ are the share of industries with production in the South.\footnote{Intermediate producers in the South only face competition from the North. Bertrand price competition drives prices down to marginal costs, thus fellow intermediate firms in the South have no incentive to devote resources to imitate a good that has already been imitated.}

**Stage 2: Expected Profits**

Profits are determined by the type of competition faced by each type of firm. Due to the assumption of nondrastic innovations, firms engage in limit pricing strategies. Bertrand price competition results in prices set just low enough to drive the closest competitor (the firm with the next highest quality grade) out of the market. Northern firms facing Northern competition choose the lowest price at which the previous innovator, or closest competitor, could sell before earning negative profits. Since new innovations are $q > 1$ units more productive than the next best good (in this case held by a competing Northern firm), the innovating firm charges a price $q$ times the marginal cost of their rival in order to completely capture the market for that industry. Northern firms facing Northern competition charge a domestic price $P_{NN} = (q - \epsilon) MC_N = q$, where $\epsilon$ is an arbitrarily small positive amount. Notice, the marginal cost of intermediate producers is equal to the price of the final good in that region. In the North, the price of the final good and therefore the marginal cost for intermediate firms in the North is unity, while the price of the Southern final good and marginal costs for Southern intermediate producers adjust to balance trade, $\tilde{P}_S = MC_S < 1$. This price is sufficient to capture the entire market for that industry. In the export market, similar intuition implies $P_{NN}^* = q(1 + \tau_{XS})$, where $\tau_{XS}$ is the tariff imposed by the South.

For a Northern firm facing Southern competition, the innovating firm charges a price $q - \epsilon$ over the marginal cost of the Southern competitor. In the Northern market, the effective marginal cost for a Southern firm is $MC_S (1 + \tau_{XN})$, where $\tau_{XN}$ is the tariff imposed by the North. Thus, Northern firms
facing Southern competition charge the domestic price, $P_{NS} = qMC_S (1 + \tau_{XS}) = q\tilde{P}_S (1 + \tau_{XS})$. In the Southern market, the marginal cost for a Southern firm is simply $MC_S$, so the export price is $P_{NS}^* = qMC_S = q\tilde{P}_S$.

Finally, Southern firms always face Northern competition, and capture the global market by charging a price $1 - \epsilon$ times the marginal cost of the Northern firm. In the Northern market, the marginal cost for a Northern firm is $MC_N = 1$, and the price is $P_j^* = 1$. In the South, Southern firms price at $P_S = 1 + \tau_{XS}$.

Table 2 summarizes the limit pricing schedule and the effective marginal cost of production for each type of firm.

<table>
<thead>
<tr>
<th>Type</th>
<th>Domestic Price</th>
<th>Export Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>$NN$</td>
<td>$P_{NN} = q$</td>
<td>$MC_{NN} = 1$</td>
</tr>
<tr>
<td>$NS$</td>
<td>$P_{NS} = q(1 + \tau_{XS})$</td>
<td>$MC_{NS} = 1 + \tau_{XS}$</td>
</tr>
<tr>
<td>$S$</td>
<td>$P_S = 1 + \tau_{XS}$</td>
<td>$MC_S = \tilde{P}_S$</td>
</tr>
</tbody>
</table>

The pricing strategies for intermediate firms reveals the role of barriers to trade in the model. Trade liberalization alters the limit prices for the three types of firms, and thus, changes demand, expected profits and ultimately the incentives for innovation and imitation. Rearranging equation (2) and using Table 2, the domestic and foreign demand for each type of firm is given by

$$
n_{NN} \Rightarrow X_{NN} = H_N \left( A_N\alpha q^{k_{Nj}/\alpha} \frac{1}{q} \right)^{1/(1 - \alpha)}
X_{NN}^* = L_S H_S \left( A_S\alpha q^{k_{Nj}/\alpha} \frac{\tilde{P}_S}{q(1 + \tau_{XS})} \right)^{1/(1 - \alpha)}
$$

$$
n_{NS} \Rightarrow X_{NS} = H_N \left( A_N\alpha q^{k_{Nj}/\alpha} \tilde{P}_S \right)^{1/(1 - \alpha)}
X_{NS}^* = L_S H_S \left( A_S\alpha q^{k_{Nj}/\alpha} \frac{\tilde{P}_S}{q\tilde{P}_S} \right)^{1/(1 - \alpha)}
$$

$$
n_{S} \Rightarrow X_S = L_S H_S \left( A_S\alpha q^{k_{Nj}/\alpha} \frac{\tilde{P}_S}{1 + \tau_{XS}} \right)^{1/(1 - \alpha)}
X_S^* = H_N \left( A_N\alpha q^{k_{Nj}/\alpha} \frac{1}{1} \right)^{1/(1 - \alpha)}
$$

Successful innovators and imitators earn the sum of domestic and export profits. Total profits are

$$
\pi_j = (P_j - MC_j) X_j + (P_{j}^* - MC_{j}^*) X_{j}^*
$$

where $P_j$, $P_{j}^*$, $MC_j$ and $MC_{j}^*$ are given in Table 2 for $j \in \{NN, NS, S\}$. I assume trade barriers are sufficiently low that export profits are always positive.
The average expected profits in the North and South are summarized as

\[ \pi_N \equiv \bar{\pi}_N Q_N, \quad \text{where} \quad \frac{\partial \bar{\pi}_N}{\partial \tau_{XS}} < 0, \quad \text{and} \quad \frac{\partial \bar{\pi}_N}{\partial \tau_{XN}} < 0 \]

and

\[ \pi_S \equiv \bar{\pi}_S Q_N, \quad \text{where} \quad \frac{\partial \bar{\pi}_S}{\partial \tau_{XS}} > 0, \quad \text{and} \quad \frac{\partial \bar{\pi}_N}{\partial \tau_{XN}} < 0 , \]

where \( \bar{\pi}_N \) and \( \bar{\pi}_S \) are the quality adjusted expected profits for firms in the North and South, respectively, and \( Q_N = \int_0^1 q_{kNj}^{(1-\alpha)/(1-\alpha)} dj \) is the average quality level on the frontier. Equations (17) and (18) are dependent on the barriers to trade, \( \tau_{XS} \) and \( \tau_{XN} \), and the rate of technological progress which is embedded within the effective human capital in the North and South. See the appendix for additional details.

**Stage 1: Research & Development**

Intermediate firms decide the amount of resources to devote to R&D based on the expected present value of profits for successful research. This, in turn, depends on the probabilities of innovation (I) and imitation (C for copying). This section will draw heavily on the work of Connolly and Valderrama (2005, 2007), most notably by incorporating the concept of “learning-to-learn,” which allows for past R&D experience to increase the ability to innovate or imitate.

Let \( p_{Ik} \) and \( p_{Ck} \) denote the instantaneous probability of innovation and imitation, respectively, for industry \( j \) with current quality level \( k \). The probabilities of research success per unit of time follow a Poisson process depending on the resources devoted to R&D and the quality level in that industry. For the aggregate economy, it is sufficient to characterize the rates of technical progress by looking at the average quality. \( Q_N \) is the average frontier technology, and \( Q_S \) is average quality of Southern intermediates.

In the North, the instantaneous probability of innovation is given by

\[ p_I = \phi_I f (Q_N) Z_N \]

where \( \phi_I \) is the productivity parameter for the Northern R&D sector, \( Z_N \) are the resources devoted to innovative R&D in the North and \( f (Q_N) \) is a function that captures the effect of the current technology on the probability of innovation. For simplicity, the function \( f \) is defined as

\[ f (Q_N) = Q_N^{-1} . \]
This specification captures the idea that new innovations become increasingly complex and, thus, innovation becomes more difficult as the average quality level rises. In other words, the easiest innovations are discovered first making it increasingly difficult to innovate over time.

In the South, the instantaneous probability of imitation for the aggregate economy is

\[ p_C = \phi_C g(Q_S, Q_N) Z_S \quad (20) \]

where \( \phi_C \) is the productivity parameter for the Southern R&D sector, \( Z_S \) are the average resources devoted to R&D and \( g(Q_S, Q_N) \) captures three effects of the current technological environment on the probability of imitation. The first effect is the positive effect of \( Q_S \) reflecting learning-to-learn in the South.\(^{11}\) Higher \( Q_S \) implies greater experience with the imitative process which reduces the costs of imitation. The second effect is the increasing difficulty in imitating good of a higher quality. As the frontier expands, innovations are increasingly complex, and costs of imitation increase. Finally, imitation becomes increasingly costly as the aggregate quality in the South catches up to the Northern aggregate quality. The relative quality of the South is defined as \( \hat{Q} \equiv Q_S/Q_N \). Intuitively, as the South approaches the North in terms of quality, the pool of potential imitations shrinks, with only the most complex innovations left available for imitation. Thus, the costs of imitation increase as \( \hat{Q} \) increases. Considering these three effects, the functional form of \( g \) is defined as

\[ g(Q_S, Q_N) = Q_S Q_N^{-2} \hat{Q}^{-\sigma} = \hat{Q}^{1-\sigma} Q_N^{-1} \]

where \( \sigma > 1 \) represents how quickly the imitation rates fall as the South approaches the aggregate quality level in the North. The inclusion of \( \hat{Q} \) guarantees smooth transitional dynamics. Furthermore, if the South completely catches up to the North in terms of average quality, or \( Q_S = Q_N \), the function \( g \) is equivalent to the function \( f \).

### 3.5 Income Inequality Across Countries

Total income in each country is the sum of worker income and firm profits. Total per capita income (controlling for the relative size of the South, \( L_S \)) is given by

\[
\begin{align*}
\text{North} & \Rightarrow \int_0^{a_N} I_N^w(a_{Ni}) \, da_{Ni} + \int_{a_N}^1 I_N^r(a_{Ni}) \, da_{Ni} = (1 - \alpha) \bar{Y}_N Q_N \\
\text{South} & \Rightarrow \frac{1}{L_S} \left[ \int_0^{a_S} I_S^w(a_{Si}) \, da_{Si} + \int_{a_S}^1 I_S^r(a_{Si}) \, da_{Si} \right] = \frac{1}{L_S} \left[ (1 - \alpha) \bar{Y}_S \right] Q_N,
\end{align*}
\]

\(^{11}\)For more details on learning-to-learn effects refer to Connolly (2003) and Connolly and Valderrama (2005, 2007).
and total per capita firm profits are given by

\[
\begin{align*}
\text{North} & \Rightarrow [n_{NN} \bar{\pi}_{NN} + n_{NS} \bar{\pi}_{NS}] Q_N \\
\text{South} & \Rightarrow \frac{1}{L_S} [n_S \bar{\pi}_S] Q_N,
\end{align*}
\]

where \( \bar{Y}_N = \frac{Y_N}{Q_N}, \bar{Y}_S = \frac{Y_S}{Q_N}, \bar{\pi}_{NN} = \frac{n_{NN} \pi_{NN}}{Q_N}, \bar{\pi}_{NS} = \frac{n_{NS} \pi_{NS}}{Q_N}, \text{ and } \bar{\pi}_S = \frac{n_S \pi_S}{Q_N} \) are the aggregate final goods and profits divided by the aggregate quality index.

In the subsequent analysis, I will focus on two measures of inequality across. The first only considers the per capita income of the workers, while the second includes the profits of the firms. Breaking the inequality apart in this way will make the analysis below more clear as to what is driving the dynamics of inequality across country. Consider the following two measures of inequality across countries:

\[
\begin{align*}
\Omega_{1/S}^N &= \frac{\bar{Y}_N}{\bar{Y}_S} \\
\Omega_{1/S}^S &= \frac{(1-\alpha)Y_N + n_{NN} \bar{\pi}_{NN} + n_{NS} \bar{\pi}_{NS}}{(1-\alpha)Y_S + n_S \bar{\pi}_S}
\end{align*}
\]

Notice the level of the aggregate quality of intermediate goods has no effect on inequality across countries. In both the North and the South, aggregate output and profits for each type of firm grow at an equal rate of \( \dot{Q}_N \). Total income and profits in the South are divided by \( L_S \) to transform this measure of inequality into per-capita terms.

### 3.6 Consumers

Consumers live in either the North or the South and are immobile across countries. Consumer \( i \) makes consumption and savings decisions to maximize the present value of lifetime utility. There is no trade in final goods and so the consumers only have access to domestically produced final goods. The general consumer problem is

\[
\max_{C_{mi}, b_{mi}} \int_0^\infty u(C_{mi}) e^{-\rho t} dt
\]

subject to

\[
\dot{b}_{mi} = I_{mi} + r_m b_{mi} - \bar{P}_m C_{mi},
\]

where \( C_{mi} \) is the consumption of individual \( i \) in region \( m \), \( r_m \) is the endogenously determined country specific interest rate, \( I_{mi} \) is the income for individual \( i \) in region \( m \), and \( b_{mi} \) is the net assets for person \( i \) in region \( m \). Using a constant elasticity of substitution utility function:

\[
u(C_{mi}) = \frac{C^{1-\theta}_{mi} - 1}{1 - \theta},
\]
the usual expressions for consumption growth are:

\[
\frac{\dot{C}_N}{C_N} = \frac{1}{\theta} (r_N - \rho) \\
\frac{\dot{C}_S}{C_S} = \frac{1}{\theta} \left( r_S - \frac{\hat{P}_S}{P_S} - \rho \right)
\]

where \(1/\theta\) is the constant elasticity of substitution for all consumers in both regions. The growth rates of consumption are independent of the individual income level and are equal across all individuals within the region.

4 The Steady-State and Transitional Dynamics

4.1 The Steady-State

The model is defined by a system of five dynamic equations, given two aggregate resource constraints and the balanced trade condition. In steady-state, the relative aggregate quality level of the South must be constant, or, \(\dot{Q}/\hat{Q} = 0\); the distribution of intermediate firms must be constant, or, \(\dot{n}_{NN} = \dot{n}_{NS} = \dot{n}_S = 0\); and the growth of consumption must equal the growth rate of technology, or, \(\dot{C}_N/C_N = \dot{C}_S/C_S = \dot{Q}_N/Q_N\).

Aggregate Resource Constraint and Balanced Trade

Substituting for the prices and demand of intermediate goods, aggregate output for the North and South, respectively, is:

\[
Y_N = H_N \Lambda_N \left( \frac{q}{\alpha} \right) \left[ n_{NN} + n_{NS} \left( \frac{1}{P_S (1 + \tau_{XN})} \right)^{1/\alpha} + n_S q^{1/\alpha} \right] Q_N
\]

\[
Y_S = L_S H_S \Lambda_S \left( \frac{q}{\alpha} \right) \left( \frac{\hat{P}_S}{1 + \tau_{XS}} \right)^{1/\alpha} \left[ n_{NN} + n_{NS} \left( \frac{1 + \tau_{XS}}{P_S} \right)^{1/\alpha} + n_S q^{1/\alpha} \right] Q_N
\]

where \(\Lambda_N = A_N^{1/(1-\alpha)} (\frac{a}{q})^{1/\alpha} \) and \(\Lambda_S = A_S^{1/(1-\alpha)} (\frac{a}{q})^{1/\alpha}\). The aggregate output in each region is determined, in part, indirectly by the probabilities of innovation and imitation. As with the profit equations, \(p_I\) and \(p_C\) are embedded within the effective human capital, the balanced trade determination of \(\hat{P}_S\), and the distribution of firms.

The two aggregate resource constraints reflect that the final goods are used for domestic consump-
tion, R&D, or transformed into intermediate goods:

\[ Y_N = C_N + X_{NN} + X_{NN}^* + X_{NS} + X_{NS}^* + Z_N \]
\[ Y_S = C_S + X_S + X_S^* + Z_S \]  

(30)

where \( X_{NN} + X_{NN}^* \) is the total intermediate output for Northern firms facing Northern competition supplied to both the domestic and foreign markets. \( X_{NS} + X_{NS}^* \) is the total intermediate supply from Northern firms facing Southern competition, and \( X_S + X_S^* \) is the total supply of intermediate goods from Southern producers. Tariff revenues are used by the government for no gain in utility or income for the individuals. Using the two resource constraints, I summarize the expressions for \( Z_N \) and \( Z_S \) as (see the appendix for details)

\[ Z_N = Y_N - C_N - (X_{NN} + X_{NN}^* + X_{NS} + X_{NS}^*) \equiv \bar{Z}_N Q_N \]  

(31)

and

\[ Z_S = Y_S - C_S - (X_S + X_S^*) \equiv \bar{Z}_S Q_N \]  

(32)

where \( \bar{Z}_N \) and \( \bar{Z}_S \) are the quality adjusted expenditures on R&D.

The relative price of the Southern final good, \( \tilde{P}_S \), adjusts to balance trade at all times. The trade balance equates the value of Northern intermediate good exports and the value of intermediates produced in the South and exported to the North,

\[ \text{Value of N. Exports} \]
\[ \text{Value of S. Exports} \]

\[ \text{TB} = P_{NS}^* n_{NS} X_{NS}^* + P_{NN}^* n_{NN} X_{NN}^* - P_{SN}^* n_{SN} X_{SN}^* = 0. \]  

(33)

Equation (33) implicitly solves for the relative prices of the Southern final goods, \( \tilde{P}_S \). See the appendix for additional details.

**Relative Quality Level of the South**

Using the definition of \( \hat{Q} = Q_S / Q_N \), the relative average quality of the South evolves according to

\[ \frac{\dot{\hat{Q}}}{\hat{Q}} = (q - \alpha) (p_C - p_I). \]  

(34)

In steady-state the evolution of the relative Southern quality level is constant. Therefore the steady-state probability of innovation is exactly that of the probability of imitation, or, \( p_I = p_C \).
Distribution of Intermediate Firms

Consider, first, the fraction of industries characterized by Northern firms facing Northern competition, $n_{NN}$. The share of this type of firm will fall if a technology level is imitated, as the production shifts to the South. The share of Northern firms facing Northern competition increases with an innovation over a Northern firm previously facing Southern competition, provided that technology is not imitated. These dynamics are captured in equation (35). The share of Northern firms facing Southern competition increases through innovation in the industries where production is currently in the South ($n_{NS}$), but will fall through imitation or further innovation. This is detailed in equation (36). Finally, the share of firms located in the South increases through imitation in any industry where production is currently in the North ($n_{NN} + n_{NS}$) and falls with successful innovation, as captured in equation (37).

$$\dot{n}_{NN} = p_I (1 - p_C) n_{NS} - p_C n_{NN}$$  \hspace{1cm} (35) \\
$$\dot{n}_{NS} = p_I (p_C n_{NN} + n_{S}) - [(1 - p_I) p_C + p_I (1 - p_C)] n_{NS}$$  \hspace{1cm} (36) \\
$$\dot{n}_{S} = (1 - p_I) p_C (n_{NN} + n_{NS}) - p_I n_{S}$$  \hspace{1cm} (37)

In steady-state, $\dot{n}_{NN} = \dot{n}_{NS} = \dot{n}_{S} = 0$. Solving the system of equations, the steady-state share of each type firm is

$$n_{NN} = \frac{p_I (1 - p_C)}{(p_I + p_C - p_I p_C)^2}$$  \\
$$n_{NS} = \frac{p_I p_C}{(p_I + p_C - p_I p_C)^2}$$  \\
$$n_{S} = \frac{p_C (1 - p_I)}{p_I + p_C - p_I p_C}.$$

Using the steady-state result of the equalization of $p_I$ and $p_C$, the steady-state shares of each type of firm (setting $p = p_C = p_I$) is

$$n_{NN} = \frac{1 - p}{(2 - p)^2}$$  \\
$$n_{NS} = \frac{1}{(2 - p)^2}$$  \hspace{1cm} (38) \\
$$n_{S} = \frac{1 - p}{2 - p}$$

A steady-state increase in the probability of innovation and imitation yields an increase in the share of Northern firms facing Southern competition, or in other words, an increase in global competition. Furthermore, the share of Northern firms facing Northern competition and the share of firms located in the South both decrease.

Consumption and Technological Growth
From equations (26) and (27), the growth rates of consumption depend on the country-specific interest rate and the evolution of the relative price of the Southern final good, $\hat{P}_S$, as determined by the balanced trade condition. To determine $r_N$ and $r_S$, two free-entry conditions imply that firms will devote resources to research until the expected value of R&D success equals the R&D costs for the average industry. The Northern and Southern free entry conditions, respectively are

$$p_I\pi_N \int_t^\infty e^{-\int_t^s [r_N(v) + p_C(v) + p_I(v)]dv} ds = Z_N$$

$$p_C\pi_S \int_t^\infty e^{-\int_t^s [r_S(v) + p_I(v)]dv} ds = Z_S$$

The expected value of innovation is the probability of R&D success times the average profits discounted by the interest rate and the probability of rival innovation and Southern imitation. The Southern profits are discounted only by the interest rate and the probability Northern innovation. Differentiating both sides of the free entry conditions using Leibniz’s rule yields the interest rates in both countries:

$$r_N = \frac{p_I\pi_N}{Z_N} + \frac{\dot{Z}_N}{Z_N} - \frac{\dot{p}_I}{p_I} - \frac{\dot{\pi}_N}{\pi_N} - p_C - p_I$$

$$r_S = \frac{p_C\pi_S}{Z_S} + \frac{\dot{Z}_S}{Z_S} - \frac{\dot{p}_C}{p_C} - \frac{\dot{\pi}_S}{\pi_S} - p_I$$

The interest rates determine, in the long run, the rate of growth for output, consumption, and research expenditures in both countries.

The final dynamic expressions represent the conditions for balance growth. Let $\chi_N \equiv C_N/Q_N$ and $\chi_S \equiv C_S/Q_N$ denote the quality adjusted consumption. In steady state, the rate of consumption growth equals the growth rate of the frontier technology level, or $\dot{\chi}_N/\chi_N = \dot{\chi}_S/\chi_S = 0$. The expressions for the North and South, respectively are

$$\frac{\dot{\chi}_N}{\chi_N} = \frac{1}{\theta} (r_N - \rho) - (q^{\alpha\pi} - 1) p_I$$

$$\frac{\dot{\chi}_S}{\chi_S} = \frac{1}{\theta} \left( r_S - \frac{\dot{P}_S}{P_S} - \rho \right) - (q^{\alpha\pi} - 1) p_I.$$ 

As a result, in steady-state the change in the relative price of the Southern final good, $\frac{\dot{P}_S}{P_S}$, is zero, thus, the diffusion of technology from the North to the South is sufficient to equalize the interest rates in the North and South, or $r_N = r_S$ in steady-state.
4.2 Summary of Steady State

In steady state:

1. \( p_C = p_I \) based on equation (34).

2. The distribution of firm types are pinned down by equation (38).

3. The balanced trade condition pins down the relative price of the Southern final good, \( \tilde{P}_S \) by equation (33).

4. The resource constraints in the North and South pin down R&D outlays, \( Z_N \) and \( Z_S \), by equations (31) and (32).

5. The interest rates in the North and South are determined by the free entry conditions.

6. The balanced growth condition for the south, equation (42), pins down the relative technology level in the South, \( \hat{Q} \).

7. The instantaneous probability of Northern innovation is pinned down by the balanced growth condition in the North.

4.3 Transitional Dynamics

The dynamic system of five equations and five unknowns consist of the evolution of \( \hat{Q} \), defined by equation (34); two firm entry and exit conditions, defined by equations (36) and (37); and the consumption growth conditions in the North and South, defined by equations (41) and (42) respectively. Using initial conditions for \( \hat{Q}, n_{NS}, \) and \( n_S \), the transitional dynamics of wage inequality in the North and South are fully characterized. The transition path is solved by log-linearizing the system of equations around the steady-state and using the reverse shooting methodology.

The model is solved using numerical simulation for reasonable parameter values. Parameter values are based on theoretical and empirical priors, and chosen such that they yield saddle path stability. In addition, I select parameters such that in steady state and at all points during the transition that

\[ 0 < p_I < 1, \quad \frac{1 + X S}{q} < \tilde{P}_S < 1, \] and \( 0 < \hat{Q} < 1 \). Other parameter selections potentially lead to non-existent steady-states, or globally divergent transitional paths. The parameters are restricted as to yield saddle paths with all real eigenvectors and three negative eigenvalues in the transitional matrix. See Eicher and Turnovsky (2001) and Connolly and Valderrama (2007) for details. The benchmark parameter values are listed in Table 3.
Table 3: Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>3.0</td>
<td>Inverse of constant elasticity of sub.</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.02</td>
<td>Subjective discount rate</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.3</td>
<td>Capital share in final goods production</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>8.5</td>
<td>Elasticity of $p_C$ w.r.t. $Q$</td>
</tr>
<tr>
<td>$A_N$</td>
<td>2.25</td>
<td>Northern final goods productivity</td>
</tr>
<tr>
<td>$A_S$</td>
<td>1.75</td>
<td>Southern final goods productivity</td>
</tr>
<tr>
<td>$\phi_I$</td>
<td>0.15</td>
<td>Northern innovation productivity</td>
</tr>
<tr>
<td>$\phi_C$</td>
<td>0.075</td>
<td>Southern imitation productivity</td>
</tr>
<tr>
<td>$q$</td>
<td>1.5</td>
<td>Constant size of quality improvements</td>
</tr>
<tr>
<td>$L_S$</td>
<td>2.0</td>
<td>Relative size of the South</td>
</tr>
</tbody>
</table>

The parameter values for the trade barriers, $\tau_{XS}$ and $\tau_{XN}$, and quality of education, $\delta_m$ are discussed in the next section.

5 Southern Trade Liberalization

Assume the global economy is in a steady-state in which there are positive tariffs on Southern intermediate imports, $\tau_{XS}$. For simplicity, I set $\tau_{XN} = 0$.\(^{12}\) Trade liberalization removes the barriers to trade, specifically $\tau_{XS}$ drops from $\tau_{XS} = 0.35 \Rightarrow 0.0$.\(^{13}\)

I differentiate between two cases based on the quality of education in the South. First case I assume a low value for quality of education parameter $\delta_{S}^{low}$, and in the second case the South has a higher quality of education, $\delta_{S}^{high}$. In both cases I assume the North has a higher quality of education, and it follows that $\delta_N \geq \delta_{S}^{high} > \delta_{S}^{low}$. The quality of education in the South is the only difference between case one and case two. Table 4 summarizes the quality of education parameters.

<table>
<thead>
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<th>Table 4: Educational Quality Values</th>
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<tr>
<td>North</td>
</tr>
<tr>
<td>-------</td>
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<tr>
<td>$\delta$</td>
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</table>

The quality of education in the South is the only difference between the two cases.

5.1 Steady State Implications

The long run implications of Southern trade liberalization do not significantly change based on the quality of education in the South. In steady state, Southern trade liberalization unambiguously increases the incentives to innovate in the North. The average profits of Northern intermediate firms are decreasing in the level of the Southern tariff. Intuitively, the effective marginal cost of a Northern firm

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\(^{12}\)Northern trade liberalization and bilateral trade liberalization experiments are possible, however, I focus here on Southern trade liberalization.

\(^{13}\)Trade liberalization of a lesser magnitude induces the same qualitative results.
exporting to the Southern market falls when the tariff rate is reduced, which increases the returns to successful innovation.

The faster long run rate of technological progress increases within country inequality in both the North and the South, increases the total per capita income for Southern workers relative to Northern workers, but also increases global inequality when also considering the relative profitability of Northern and Southern intermediate firm profits. As mentioned above, profits for Northern firms rise which also increases the total fraction of firms located in the North. In addition, profits for Southern firms fall as does the fraction of firms located in the South. Intuitively, lower trade barriers imply Southern firms must charge a low price to ensure all Northern firms are priced out of the market.

Table 5 details the steady state implication of Southern trade liberalization using the parameters above.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Case 1: Low Quality</th>
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<th>Case 2: High Quality</th>
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<tr>
<td>$p_I = p_C$</td>
<td>Original SS</td>
<td>Final SS</td>
<td>Percentage Change</td>
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<tr>
<td>0.00641</td>
<td>0.01013</td>
<td>58.03</td>
<td>0.006374</td>
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<tr>
<td>$\dot{Q}$</td>
<td>0.935</td>
<td>0.856</td>
<td>-8.45</td>
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<tr>
<td>$H_N$</td>
<td>1.00378</td>
<td>1.01135</td>
<td>0.75</td>
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<tr>
<td>$H_S/L_S$</td>
<td>0.99730</td>
<td>0.99617</td>
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<tr>
<td>$a_N^*$</td>
<td>0.88825</td>
<td>0.83379</td>
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<td>$a_S^*$</td>
<td>0.96878</td>
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<td>1.67205</td>
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Southern trade liberalization significantly increases the rate of technological progress in both cases. When the South has a high quality of education, within country inequality increases by a slightly larger percent in the long run, and using both measurements of across country inequality, a higher quality of education leads to greater convergence. There is a greater amount of additional workers who choose to become skilled when the quality of education is high. Finally, when the quality of education is low, trade liberalization decreases the overall effective human capital in the South, but when the quality of education is high, trade liberalization increases overall effective human capital.
5.2 Dynamic Transition

The quality of education in the South does not greatly change the long implications for inequality following trade liberalization. However, the transitional dynamics depend on the quality of education to a much larger extent. I begin, first, by discussing the dynamics of the instantaneous probabilities of innovation and imitation, and then shift my focus to the dynamics of within and across country inequality. The dynamics of all variables are found in the appendix.

Probabilities of Innovation and Imitation

Figure 5.2a documents the evolution of the rates of innovation and imitation in the transition from the initial steady state with barriers to trade to the open steady-state as the percentage deviation from the initial steady state.

Beginning with the immediate effects, trade liberalization reduces the price of intermediate goods for the Southern final goods market in two ways, \( P_{NN}^* = q (1 + \tau_{XS}) \) and \( P_S = 1 + \tau_{XS} \), thus increasing the demand from the South. Overall, average profits of intermediate good firms in the North increase, which leads to an increase the resources allocated for innovation. The fall in \( P_{NN}^* \) increases the demand for exports from Northern firms facing Northern competition, \( X_{NN}^* \), which decreases the relative price of the Southern final good, \( \tilde{P}_S \). The fall in \( \tilde{P}_S \) increases \( Y_N \) by equation (28), decreases \( \bar{\pi}_N \) and decreases \( X_{NN}^* \). The net total initial effect is an immediate jump in \( Z_N \), and thereby, an increase in \( p_I \). In the South, trade liberalization reduces average profits for successful imitators, and through the balance trade condition, reduces aggregate output, \( Y_S \) by equation (29). The net initial effect is a small drop in
the probability of imitation $p_C$. These initial jumps hold in both the case when the quality of education is high and when the quality of education is low.

The initial increase in the rate of innovation ($p_I$) and decrease in the rate of imitation ($p_C$) also have implications for the effective human capital in both regions. The effective human capital increases in the North, and decreases in the South when the quality of education in the South is low, which, by equation (31), increases Northern aggregate output, and decreases in Southern output which further increases the resources allocated to innovation in the North. This leads to a larger initial gap between $p_I$ and $p_C$ when the quality of education in the South is low.

The initial jump in $p_I$ introduces a dynamic feedback loop. When the quality of education in the South is low, $H_S$ falls, which lessens their ability to successful imitate. Lower rates of imitation extend the duration of Northern firms earning profits, which continues to increase their incentives to innovate. The effective human capital in the North continues to rise, further increasing effective human capital in the North, while the effective human capital in the South continues to fall, further decreasing their ability to imitate. The complete transitional path, however, is non-monotonic. Since $p_I$ is greater than $p_C$ in the transition, $\hat{Q}$ decreases. As $\hat{Q}$ falls, successful imitation becomes easier by equation 20, and the rate of imitation begins to converge to the rate of innovation. As a result $p_I$ begins to fall as the transitional path approaches the new long run steady state.

A very different feedback loop emerges when the quality of education in the South is high. The initial jump in $p_I$ increases the effective human capital in the South when their quality of education is high. This increases their ability to imitate, which decreases the incentives for Northern firms to innovate following the initial jump following trade liberalization. The fall in $p_I$ during the first part of the transition along with the growth in $p_C$ makes imitation increasingly difficult. Thus, $p_C$ increases by a smaller and smaller amount, which then reverses the downward movement of $p_I$. During the last part of the transition, both $p_I$ and $p_C$ converge to a higher steady state.

**Southern Within Country Inequality**

Figure 5.2b documents Southern income inequality within countries for case 1 (low quality of education) and case 2 (high quality of education). The two diagrams are in scale as percentage deviation from the initial steady state.

Figure 5.2b: The dynamics of Southern within country inequality.
The dynamics of within country inequality follow the same nonmonotonic transition path as the instantaneous probability of innovation. As mentioned above, the steady state growth of inequality is higher when the quality of education is high. This conceals, however, the important differences during the transition. During the transition, the growth of inequality is higher when the quality of education is low. In addition, after an initial jump, inequality is declining during the first part of the transition when the quality of education is high, whereas, inequality continues to increase when the quality of education is low.

Inequality Across Countries

In the model I characterize income inequality in two ways, the first considers only the relative per capita income of workers, while the second takes into account the total profits of firms. Using either measurement Southern trade liberalization decreases long run inequality across country. Figure 5.2 shows the percentage change of the per capita income of Northern workers relative to Southern workers, and Figure 5.2 includes firm profits. A decline in either measurement indicates convergence by the South (relative income of Northern workers decline).

Figure 5.2c: Relative Per Capita Worker Income.
Focusing first on Figure 5.2c, income inequality across countries falls immediately following Southern trade liberalization. In the first part of the transition inequality rises, before falling to a lower long run steady state. This holds true for when the quality of education in the South is low or high. In the long run, inequality falls by a slightly greater amount when the quality of education is low. However, the differences in the transition are equally important. The divergence during the first part of the transition is stronger when the quality of education is low compared to the case when the quality of education is high.

This become more clear when looking at Figure 5.2d where firm profits are included. Overall, inequality across countries decreases by a smaller amount. This is driven by an increase in the number of firms in the North and their increased profitability due to trade liberalization. Using this measurement,
inequality across countries decreases by a larger amount when the quality of education in the South is high. Moreover, the decline in inequality is monotonic. When the quality of education in the South is low, inequality increases in the initial periods of the transition following the immediate impact of trade liberalization.

6 Conclusion

This paper emphasizes how the dynamics of income inequality, both within and across countries, are dependent on the interaction between the rate of new technologies entering the production process and the quality of education embedded within the effective human capital. This paper links the two types of inequality within a general equilibrium model of North-South trade and endogenous innovation and imitation, in which agents make the endogenous discrete choice of educational attainment. The quality of education increases the ability of workers (who acquired education) to adapt and learn new technologies. Following Southern trade liberalization, the ability to adapt to faster rates of technological progress has implications for the long run, but more importantly, the short run, dynamics of within and across country income inequality.

In the long run, Southern trade liberalization increases the rate of technological progress, which increases inequality within both the North and the South. In the transition, however, income inequality in the South will increase by a greater percentage when the quality of education is low. In terms of inequality across countries, the total per capita income of Southern workers and firm profits increase relative to the North. This suggests Southern trade liberalization decreases long run inequality across countries. This convergence is stronger when the quality of education in the South is high. When the quality of education is low, there are periods of divergence during the dynamic transition that does not appear when the quality of education is high. The key contribution of this paper is introducing a source of heterogeneity among developing countries that accounts for a wide array of income inequality dynamics.

Broadly speaking, the results provide some intuition as to why, following trade liberalization, developing countries in Latin America and Africa, with a relatively low quality of education have experienced a higher growth of within country income inequality and notably less convergence compared to East Asian economies. In a preliminary paper, Hall (2010) provides empirical support for the relationship between within country inequality, technological change and the quality of education. Specifically, he finds a higher quality of education directly reduces the growth of inequality and indirectly by reducing the marginal impact of faster rates of technological progress. This empirical evidence lends support
that emphasize the critical role of education and technological change in determining the dynamics of income inequality.

7 References


### Table 1: Labor-Force Quality Data

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| Median     | 56.61         | 40.22        | 49.57         | 39.20        | 32.70        |
| Std. Dev.  | 7.15          | 9.60         | 16.74         | 9.43          | 11.43        |
| Obs.       | 27            | 21           | 13            | 15            | 11           |

Table 1: Data is the $QL2$ measure from Hanushek and Kimko (2000).

### 8 Appendix

#### 8.1 Quality of Education

Table 1 summarizes the quality of education measure from Hanushek and Kimko (2000). The measures for the quality of the labor force are derived from a number of international mathematics and science tests between the years 1965 through 1991. While test score data is available for only 39 countries, Hanushek and Kimko use consistent estimators to forecast labor force quality for a large number of countries based on country specific characteristics. The International math and science tests were conducted by the International Association for the Evaluation of Educational Achievement and the International Assessment of Educational Progress. Hanushek and Kimko produce two quality measures for countries across the development spectrum. The first measure converts the test scores to a mean of 50. This, however, assumes that the world performance in math and science in constant over time. The second measure (detailed in Table 1) relaxes this assumption, and allows for the global mean to drift based on the performance of U.S. students over time.
8.2 Detailed Equations

Northern and Southern Average Profits

This appendix provides more details for equations (17) and (18). Firms located in the North whose closest competitor is a Northern firm obtain a flow sum of domestic and export profits,

\[
\pi_{NN} = (q - 1) H_N \Lambda_N Q_N + (q - 1) (1 + \tau_{XS}) \left( \frac{\hat{P}}{1 + \tau_{XS}} \right)^{\frac{1}{1-\alpha}} H_S \Lambda_S Q_N. \tag{43}
\]

where \( \Lambda_N = A_N^{1/(1-\alpha)} \left( \frac{q}{\hat{q}} \right)^{1/(1-\alpha)} \), and \( Q_N = \int_0^1 q_k \beta_{nj}^{\alpha/(1-\alpha)} \text{dj} \) is the average quality level on the frontier. Northern firms facing Southern competition, likewise, earn a sum of import and export profits

\[
\pi_{NS} = \left( \frac{q \hat{P}}{1 + \tau_{XN}} - 1 \right) \left( \frac{1}{\hat{P}} \left( 1 + \tau_{XN} \right) \right)^{\frac{1}{1-\alpha}} H_N \Lambda_N Q_N + \left( \frac{q \hat{P} - (1 + \tau_{XS})}{1 + \tau_{XS}} \right) H_S \Lambda_S Q_N \tag{44}
\]

where the size of the incremental quality increase, \( q \), must be sufficiently large such the limit price exceeds the marginal cost of production. Average profits for intermediate firms in the North is

\[
\pi_N = \frac{n_{NN} \pi_{NN} + n_{NS} \pi_{NS}}{n_{NN} + n_{NS}} \equiv \bar{\pi}_N Q_N \tag{45}
\]

where \( \bar{\pi}_N \) is the profits adjusted for the average quality level on the frontier. Trade barriers and the probability of innovation are embedded within \( \bar{\pi}_N \). A change in the rate of innovation and imitation, alter the effective levels of human capital (\( H_N \) and \( H_S \)), the equilibrium relative price of the Southern final good (\( \hat{P}_S = MC_S \)), and the distribution of firms (\( n_{NN}, n_{NS}, \) and \( n_S \)).

The partial effects of trade liberalization on the Northern average profits is given by,

\[
\frac{\partial \pi_N}{\partial \tau_{XS}} = -\frac{H_S \Lambda_S}{n_{NN} + n_{NS}} \left( n_{NN} (q - 1) \left( \frac{\alpha}{1-\alpha} \right) \left( \frac{\hat{P}_S}{1 + \tau_{XS}} \right)^{1/(1-\alpha)} + n_{NS} \right) < 0
\]

\[
\frac{\partial \pi_N}{\partial \tau_{XN}} = -H_N \Lambda_N \left( \hat{P}_S (1 + \tau_{XN}) \right)^{1/(\alpha-1)} \frac{1 - \alpha q \hat{P}_S (1 + \tau_{XN})}{(1-\alpha)(1 + \tau_{XN})} < 0
\]
Southern flow profits for intermediate producers are

\[
\pi_S = (1 + \tau_{XS} - \hat{P})(\frac{q\hat{P}}{1 + \tau_{XS}})^{\frac{1}{1-\alpha}} H_S \Lambda_S Q_N + \left(1 - \hat{P}(1 + \tau_{XN})\right) q^{\frac{1}{1-\alpha}} H_N \Lambda_N Q_N
\]  

(46)

where \( \Lambda_S = A_S^{1/(1-\alpha)} \left( \frac{\alpha}{\tau} \right)^{1/(1-\alpha)} \), and, as in the North, \( \bar{\pi}_S \) is the quality adjusted profits for Southern imitators. I assume the Northern tariff is sufficiently low such that the export profits for Southern firms, given the limit price, is positive. Since only the state-of-the-art technology is used, any good produced in the South will still have the same quality level as the lead Northern quality frontier.

The partial effects of trade liberalization on the Northern average profits is given by,

\[
\frac{\partial \bar{\pi}_S}{\partial \tau_{XS}} = \left( \frac{q\hat{P}}{1 + \tau_{XS}} \right)^{\frac{1}{1-\alpha}} H_S \Lambda_S Q_N \left[ \hat{P}_S - \alpha (1 + \tau_{XS}) \right]/(1 - \alpha) (1 + \tau_{XS}) > 0
\]

\[
\frac{\partial \bar{\pi}_S}{\partial \tau_{XN}} = -q^{1/(1-\alpha)} H_N \Lambda_N Q_N \hat{P}_S < 0
\]

**Resources Allocated to R&D**

Using the two world resource constraints, the expressions for \( Z_N \) and \( Z_S \) from equations (31) and (32) are, in the North,

\[
Z_N = \left[ H_N \Lambda_N \left( \frac{\alpha}{\tau} \right) \left( n_{NN} + n_{NS} \right) \left( \frac{1}{p_S(1 + \tau_{XN})} \right)^{\frac{1}{1-\alpha}} + n_S q^{\frac{1}{1-\alpha}} \right] Q_N
\]

\[
- n_{NN} \left( H_N \Lambda_N + H_S \Lambda_S \left( \frac{\alpha}{\tau} \right)^{\frac{1}{1-\alpha}} \right)
- n_{NS} \left( H_S \Lambda_S + H_N \Lambda_N \left( \frac{1}{p_S(1 + \tau_{XN})} \right)^{\frac{1}{1-\alpha}} \right) - \chi_N \right] Q_N
\]

\[
\equiv \bar{Z}_N (\tau_{XS}, \tau_{XN}, p_I) Q_N
\]

where

\[
\frac{\bar{Z}_N}{\partial \tau_{XS}} = -\frac{n_{NN} H_S \Lambda_S}{(1 - \alpha) (1 + \tau_{XS})} \left( \frac{\hat{P}}{1 + \tau_{XS}} \right)^{1/(1-\alpha)} < 0
\]

\[
\frac{\bar{Z}_N}{\partial \tau_{XN}} = \frac{n_{NS} H_N \Lambda_N}{(1 - \alpha) (1 + \tau_{XN})} \left( \frac{1}{p_S(1 + \tau_{XN})} \right)^{1/(1-\alpha)} \left[ 1 - \hat{P}_S (1 + \tau_{XN}) \right] > 0
\]
and in the South,

\[ Z_S = \left[ H_S \Lambda_S \left( \frac{q}{2} \right) \left[ n_{NN} \left( \frac{\bar{P}_S}{1 + \tau_{XS}} \right)^{1/\alpha} + n_{NS} + n_S \left( \frac{q\bar{P}_S}{1 + \tau_{XS}} \right)^{1/\alpha} \right] \right. \]

\[ -n_S q^{1/\alpha} \left( H_S \Lambda_S \left( \frac{\bar{P}_S}{1 + \tau_{XS}} \right)^{1/\alpha} + H_N \Lambda_N \right) - \chi_S] Q_N \]

\[ \equiv \bar{Z}_S (\tau_{XS}, \tau_{XN}, p_I) Q_N \]

where

\[ \frac{\partial \bar{Z}_S}{\partial \tau_{XS}} = -\frac{q H_S \Lambda_S}{\bar{P}_S (1 - \alpha)} \left( \frac{\bar{P}_S}{1 + \tau_{XS}} \right)^{1/(1-\alpha)} \left[ n_{NN} + n_S q^{\alpha/(1-\alpha)} \left( 1 - \frac{\bar{P}_S}{1 + \tau_{XS}} \right) \right] < 0 \]

where \( \chi_N = C_N/Q_N \) and \( \chi_S = C_S/Q_N \). \( \bar{Z}_N \) and \( \bar{Z}_S \) are the quality adjusted expenditures on R&D.

**Balanced Trade Condition**

The relative price of the Southern final good adjusts at each point of time to balance trade between the North and South. Expanding equation 33, \( \bar{P}_S \) implicitly solves,

\[ q H_S \Lambda_S \left[ n_{NS} \bar{P}_S + n_{NN} \left( 1 + \tau_{XS} \right) \left( \frac{\bar{P}_S}{1 + \tau_{XS}} \right)^{1/(1-\alpha)} \right] - n_S H_N \Lambda_N q^{1/(1-\alpha)} = 0 \]

where the partial effects of trade liberalization is given by,

\[ \frac{\partial \bar{P}_S}{\partial \tau_{XS}} = \frac{n_{NN} \left( \frac{\alpha}{1-\alpha} \right) \left( \frac{\bar{P}_S}{1 + \tau_{XS}} \right)^{1/\alpha}}{n_{NS} + \frac{1}{1-\alpha} n_{NN} \left( \frac{\bar{P}_S}{1 + \tau_{XS}} \right)^{1/\alpha}} > 0 \]
8.3 Southern Trade Liberalization - All Variables

Panels 1 thru 20 show the evolution of the other key variables. The variables $Y_m$, $X_m$, $Z_m$ and $C_m$ are presented adjusting for aggregate quality. Each, however, grows at a rate equal to the growth rate of technologies on the frontier, $\dot{Q_N}/Q_N$. Additionally, each variable is presented as the percentage change from the initial steady state.

Figure 8.3a: Relative Per Capita Worker Income.
Figure 8.3b: Relative Per Capita Worker Income plus Firm Profits.