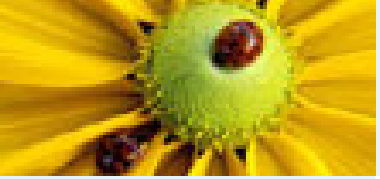
Week 9

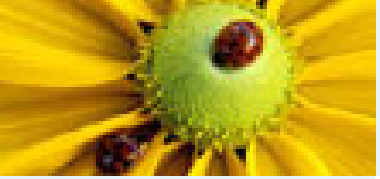
Integer Programming: Algorithms - 3

OPR 992

Applied Mathematical Programming



Column Generation Algorithms



Dantzig-Wolfe Reformulation

Consider an IP of the form

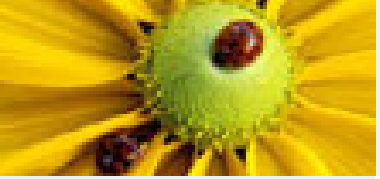
$$\begin{array}{lll} \max & \sum_{k=1}^{K} (c^k)^T x^k & \\ \text{s.t.} & \sum_{k=1}^{K} (A_k)^T x^k & \leq \quad b^k \\ & x^k & \in \quad X^k, \quad k = 1, \ldots, K \end{array}$$

Assuming that

$$X^k = \{x^{k,t}\}_{t=1}^{T_k}, \quad k = 1, \ldots, K,$$

we can introduce $\lambda_{k,t} \in \{0, 1\}$ such that

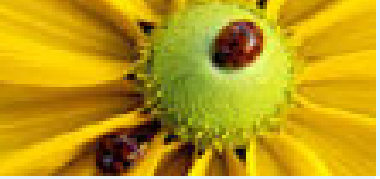
$$X^k = \sum_{t=1}^{T_k} \lambda_{k,t} x^{k,t}, \text{ and } \sum_{t=1}^{T_k} \lambda_{k,t} = 1.$$



Example

Solve an instance of STSP with the following distance matrix:

$$\begin{bmatrix} - & 7 & 2 & 1 & 5 \\ - & - & 3 & 6 & 8 \\ - & - & - & 4 & 2 \\ - & - & - & - & 9 \\ - & - & - & - & - \end{bmatrix}$$

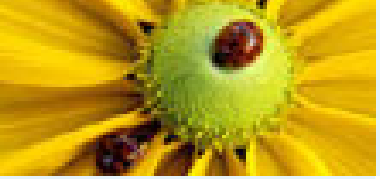


Strength of the Linear Programming Master

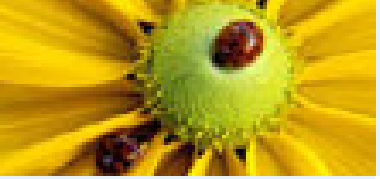
$$z^{LPM} = \max\{\sum_{k=1}^{K} c^k x^k : \sum_{k=1}^{K} A^k x^k = b, x^k \in conv(X^k), k = 1, \ldots, K\}.$$

$$z^{LPM} = w_{LD} = z^{CUT}.$$

Choose the right algorithm based on speed.



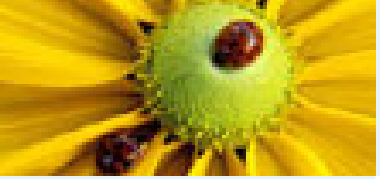
Heuristic Algorithms



Introduction

When should we use a heuristic?

- A solution is required rapidly.
- The instance is too large to formulate as whole problem of reasonable size.
- Once formulated, known algorithms cannot solve it in real time.
- It is much easier to find solutions by inspection than by solving using a general-purpose algorithm.



Tabu Search

Local search for a constrained problem of the form

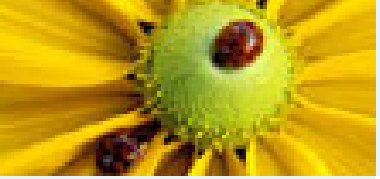
$$\min\{c(x) : g(x) = 0\}$$

can involve a goal function

$$c(x) + \alpha g(x).$$

For different values of α, the local search can cycle among solutions.

Solution: Make certain solution forbidden to avoid cycling.



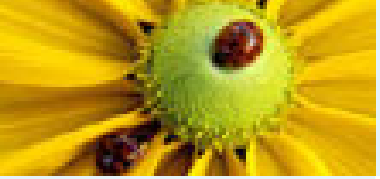
Simulated Annealing

Choose a neighbor randomly.

The probability of choosing a neighbor depends on the effect on the goal function.

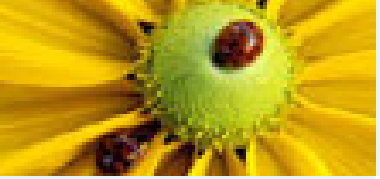
Parameters of a simulated annealing algorithm:

1. The initial temperature T
2. The cooling ratio r
3. The loop length L
4. The definition of “frozen,” or the stopping criterion.



Simulated Annealing - The Algorithm

1. Get an initial solution S.
2. Get an initial temperature T and a cooling ratio r with $0 < r < 1$.
3. While not yet frozen, do the following:
   (a) Perform the following loop L times:
      i. Pick a random neighbor S' of S.
      ii. Let $\Delta = f(S') - f(S)$.
      iii. If $\Delta \leq 0$, set $S = S'$.
      iv. If $\Delta > 0$, set $S = S'$ with probability $e^{-\Delta/T}$.
   (b) Reduce the temperature by setting $T \leftarrow rT$.
4. Return the best solution found.

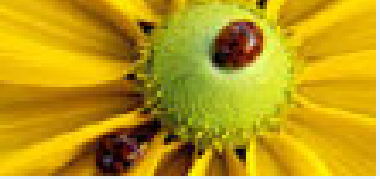


Genetic Algorithms

Start with a finite population of solutions and evolve it from one generation to the next.

Each iteration performs the following steps:

1. Evaluation: The fitness of the individuals is evaluated.
2. Parent Selection: Certain pairs of solutions (parents) are selected based on their fitness.
3. Crossover: Each pair of parents combines to produce one or two new solutions (offspring).
4. Mutation: Some of the offspring are randomly modified.
5. Population Selection: Based on their fitness, a new population is selected replacing some or all of the original population by an identical number of offspring.



Local Search Wrap-Up

In any local search algorithm, it is important to balance:

1. Communication
2. Diversification
3. Intensification