

Applications of NLP

Lecture 5

OPR 992: Applied Mathematical Programming

You, Me, and Dupree

- Dan and I have a nest egg that we're looking to invest.
- At retirement, we want:



- Of course, we first have to pay for little Dupree's Ivy League education....

The Problem Setup

- We have a set of N investment options to choose from.
- We have a set of $T = \{1, 2, \dots, T\}$ time periods until retirement/college.
- Let w_{it} be the random return on investment i at time period t .
- If we exceed our goal G , Dan and I can invest the extra money at an interest rate of q .
- If we fall short of our goal, Dan and I will have to borrow money at an interest rate of r so that Dupree can get an Ivy League education.
- We have $\$b$ to invest.

The Deterministic Formulation

$$\begin{aligned} &\text{maximize} && qy - rw \\ &\text{subject to} && \sum_{i \in N} x_{i1} = b \\ & && \sum_{i \in N} w_{it} x_{i,t-1} = \sum_{i \in N} x_{it}, \forall t \in T \setminus 1 \\ & && \sum_{i \in N} w_{iT} x_{iT} - y + w = G \\ & && x_{it} \geq 0, \forall i \in N, t \in T \\ & && y, w \geq 0. \end{aligned}$$

The Stochastic Formulation

$$\begin{aligned} & \text{maximize} && \sum_{s \in S} p_s (qy_s - rw_s) \\ & \text{subject to} && \sum_{i \in N} x_{i1s} = b, \forall s \in S \\ & && \sum_{i \in N} w_{its} x_{i,t-1,s} = \sum_{i \in N} x_{its}, \forall t \in T \setminus 1, \forall s \in S \\ & && \sum_{i \in N} w_{iT_s} x_{iT_s} - y_s + w_s = G, \forall s \in S \\ & && x_{its} \geq 0, \forall i \in N, t \in T, \forall s \in S \\ & && y_s, w_s \geq 0, \forall s \in S. \\ & && x_{its} = x_{its'}, \forall i \in N, t \in T, \forall s \in S, \forall s' \in S \end{aligned}$$

Nonanticipativity Constraints

- Scenarios with a common history must have the same set of decisions
- Computationally Challenge:
Small dense problem vs. Large sparse problem
- Verbal Challenge:
You try saying *nonanticipativity* five times in a row!