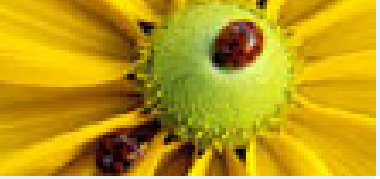
Week 1

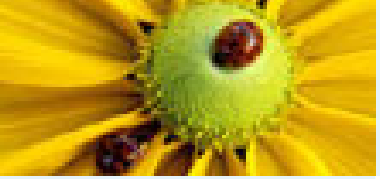
Introduction to Nonlinear Programming

OPR 992

Applied Mathematical Programming



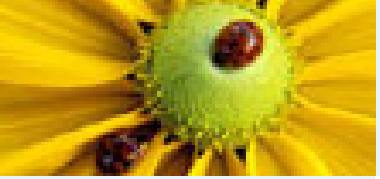
Introduction



What is Nonlinear Programming (NLP)?

- In OPR 620, we studied *linear programming* (LP) problems.
- For an LP, our goal was to maximize or minimize a linear function subject to linear constraints.

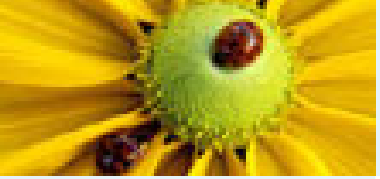
- In many interesting, real-world problems, the objective function may not be a linear function, or some of the constraints may not be linear constraints.
- Optimization problems that involve nonlinearities are called *nonlinear programming* (NLP) problems.



Beaver Creek Pottery

	Resource Requirements		
Product	Labor (hr/unit)	Clay (lb/unit)	Profit ($/unit)
Bowl	1	4	40
Mug	2	3	50
Available	40	120	

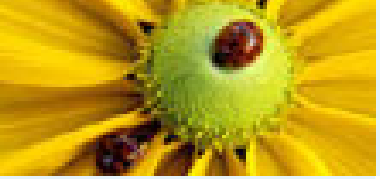
How many bowls and mugs should be produced to maximize profits given labor and materials constraints?



The LP Model for Beaver Creek Pottery

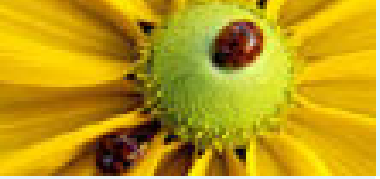
- x_1 = number of bowls to produce per day
- x_2 = number of mugs to produce per day

$$
\begin{aligned}
\text{maximize} \quad & 40x_1 + 50x_2 \\
\text{subject to} \quad & 1x_1 + 2x_2 \leq 40 \\
& 4x_1 + 3x_2 \leq 120 \\
& x_1, x_2 \geq 0.
\end{aligned}
$$



Assumptions of the Beaver Creek Model

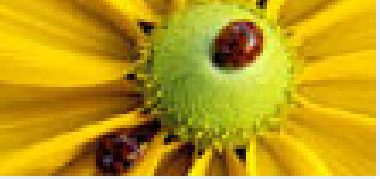
- We can sell every unit produced.
- Each unit requires the same amount of labor.
- Each unit requires the same amount of clay.



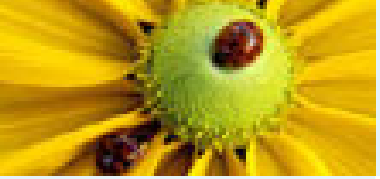
NLP in General Form

$$\begin{aligned} \text{minimize} \quad & f(x) \\ \text{subject to} \quad & g(x) \geq 0. \end{aligned}$$

- The objective function can be maximized or minimized.
- The constraints can be $\leq$, $=$, or $\geq$.
- Many NLPs do not have constraints. They are called *unconstrained* NLPs.



Examples

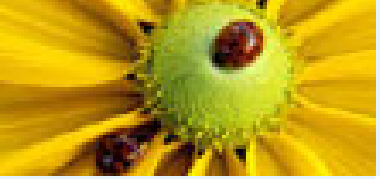


Example 1: Production and Advertising

If Widgetco charges a price p for a product and spends $\$a$ on advertising, it can sell

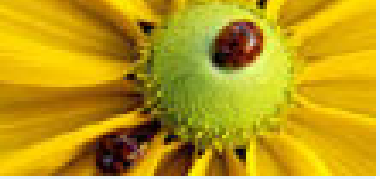
$$10,000 + 5\sqrt{a} - 100p$$

units of the product. If the product costs \$10 per unit to produce, how can the company maximize profits?



Example 2: Capital and Labor

If K units of capital and L units of labor are used, a company can produce KL units of a manufactured good. Capital can be purchased at \$4/unit, and labor can be purchased at \$1/unit. A total of \$8 is available to purchase capital and labor. How can the firm maximize the quantity of good that can be manufactured?

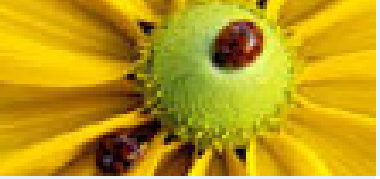


Example 3: Facility Location

Truckco is trying to determine where they should locate a single warehouse. The positions in the x-y plane (in miles) of their four customers and the number of shipments made annually to each customer are as follows:

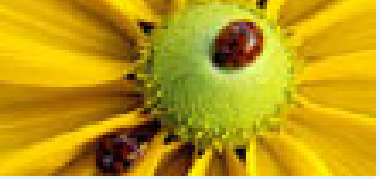
Customer	x-Coordinate	y-Coordinate	Number of Shipments
1	5	10	200
2	10	5	150
3	0	12	200
4	12	0	300

Truckco wants to locate the warehouse to minimize the total distance trucks must travel annually from the warehouse to the four customers.



Example 4: Ads of our Lives

QH Company advertises on soap operas and football games. Each soap opera ad costs \$50,000, and each football ad costs \$100,000. Giving all figures in millions of viewers, if S soap opera ads are bought, they will be seen by $5\sqrt{S}$ men and $20\sqrt{S}$ women. If F football ads are bought, they will be seen by $17\sqrt{F}$ men and $7\sqrt{F}$ women. QH company wants at least 40 million men and at least 60 million women to see its ads. Formulate an NLP that will minimize QH's cost of reaching sufficient viewers.

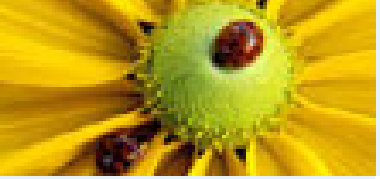


Example 4: Ads of Our Lives

Suppose that the number of women reached by F football ads and S soap opera ads is

$$7\sqrt{F} + 20\sqrt{S} - 0.2\sqrt{FS}.$$

Why might this be a more realistic representation of the number of women viewers seeing QH's ads?



Example 5: No Left Turns in Jersey

Each morning during rush hour, 10,000 people want to travel from New Jersey to New York City. To do so, they can take a train or they can drive. A train trip lasts 40 minutes. If x thousand people drive, it takes each person driving $20 + 5x$ minutes to make the trip. How many people need to travel by road to minimize the average travel time per person?