After finding the optimal soln

① Let i = row of an integer variable whose optimal value is noninteger.

② Add a cut on the i^{th} row:

$$X_{B_i} = \bar{a}_i^0 - \sum_{j \in NB} \bar{a}_{ij} x_j$$

B = set of basic variables/indices

NB = set of nonbasic variables/indices

$$x_{B_i} + \sum \bar{a}_{ij} x_j = \bar{a}_i^0$$

$$-\quad X_{B_i} + \sum_{j \in NB} \lfloor \bar{a}_{ij} \rfloor x_j \leq \lfloor \bar{a}_i^0 \rfloor$$

$$\sum_{j \in NB} \left(\bar{a}_{ij} - \lfloor \bar{a}_{ij} \rfloor \right) x_j \geq \bar{a}_i^0 - \lfloor \bar{a}_i^0 \rfloor$$

$$\sum_{j \in NB} \left(\bar{a}_{ij} - \lfloor \bar{a}_{ij} \rfloor \right) x_j - S = \bar{a}_i^0 - \lfloor \bar{a}_i^0 \rfloor$$

$$S = -\left(\bar{a}_i^0 - \lfloor \bar{a}_i^0 \rfloor \right) + \sum_{j \in NB} \left(\bar{a}_{ij} - \lfloor \bar{a}_{ij} \rfloor \right) x_j$$

3 Solve the new problem

a Use the Dual Simplex method to get feasibility

For each pivot, pick a row where a basic variable is negative.

Using the negative coefficients in that row, calculate the ratios for $\frac{\text{objective fn coeff}}{\text{Constraint coeff}}$

Take the smallest and pivot.

b If necessary, use the Primal Simplex Method to get optimality max profit - fixed costs

s.t. each client assigned to exactly one location

only open locations are assigned clients

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x_{ij} = assignment of client i to location j

y_j = whether or not location j is available

$$\max \sum_{ij} c_{ij} x_{ij} - \sum_j f_j y_j$$

$$\text{s.t.} \quad \sum_j x_{ij} = 1 \quad \forall i$$

$$x_{ij} \le y_j \quad \forall i, j$$

$$x \geq 0, \; y \in \{0,1\}^5$$

$$\max \sum_{ij} c_{ij} x_{ij} - \sum_{j} f_j y_j + \sum_{i} u_i \left(1 - \sum_{j} x_{ij}\right)$$

$$\text{s.t.} \quad x_{ij} \le y_j \quad \forall i, j$$

$$x \ge 0, \quad y \in \{0,1\}^5$$

$$\sum_{ij} c_{ij} x_{ij} + \sum_{i} \left(u_i - \sum_{j} u_i x_{ij}\right)$$

$$= \sum_{ij} c_{ij} x_{ij} + \sum_{i} u_i - \sum_{ij} u_i x_{ij}$$

$$= \sum_{ij} (c_{ij} - u_i) x_{ij} + \sum_{i} u_i$$

$$z(u) = \max \sum_{ij} (c_{ij} - u_i) x_{ij} + \sum_{i} u_i - \sum_{j} f_j y_j$$

constant

$$\text{s.t.} \quad x_{ij} \le y_j \quad \forall i, j$$

$$x \ge 0, \quad y \in \{0,1\}^5$$

$$z(u) - \sum_i u_i = \max \sum_{ij} (c_{ij} - u_i) x_{ij} - \sum_j f_j y_j$$

$$\text{s.t.} \quad x_{ij} \le y_j \quad \forall i,j$$

$$x \geq 0, \; y \in \{0,1\}^5$$

Break up the problem by location

$$z_j(u) = \max \sum_i (c_{ij} - u_i) x_{ij} - f_j y_j$$

$$\text{s.t.} \quad x_{ij} \le y_j \quad \forall i$$

$$x \geq 0, \; y_j \in \{0,1\}$$

e.g. Start with $u = (5, 6, 3, 2, 5, 4)$

$$c_{ij} - u_i = \begin{bmatrix} 6-5 & 2-5 & \\ 4-6 & 10-6 & \\ 3-3 & 2-3 & \cdots \\ 2-2 & 0-2 & \\ 1-5 & 8-5 & \\ 3-4 & 2-4 & \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -3 & -4 & -2 & 0 \\ -2 & 4 & -4 & 0 & -5 \\ 0 & -1 & 1 & -2 & 0 \\ 0 & -2 & 2 & -1 & 2 \\ -4 & 3 & 1 & -3 & 0 \\ -1 & -2 & 0 & 4 & -3 \end{bmatrix}$$

Look at $j=2$

If $y_2=0$ then $x_{i2}=0 \ \forall i$

$z_2(u)=0$

If $y_2=1$ then

$x_{22}=1$ $\quad x_{12}=0$

$x_{52}=1$ $\quad x_{32}=0$

$x_{42}=0$

$x_{62}=0$

$z_2(u)=4+3-4=3$

Decision: $y_2=1$ $\quad z_2(u)=3$

$x_{22}=1$

$x_{52}=1$

Let $j=1$

If $y_1=0$, then $x_{i1}=0 \ \forall i$

$z_1(u)=0$

If $y_1=1$ $\quad x_{11}=1$, $x_{21}=x_{31}=x_{41}=x_{51}=x_{61}=0$

$z_1(u)=1-2=-1$

$j=3$

If $y_3=1$, $x_{33}=1$

$x_{43}=1$

$x_{53}=1$

$$z_3(u)=1\cdot 1+2\cdot 1+1\cdot 1-5=-1$$

$y_3=0$, $x_{i3}=0$

$j=4$

If $y_4=1$, $x_{64}=1$

$$z_4(u)=4\cdot 1-3=1$$

$y_4=1$, $x_{64}=1$, $x_{14}=x_{24}=x_{34}=x_{44}=x_{54}=0$

$j=5$

If $y_5=1$, $x_{45}=1$

$$z_5(u)=2\cdot 1-3=-1$$

$y_5=0$, $x_{i5}=0$ $\forall i$

$y_1 = 0$

$y_2 = 1 \qquad x_{22} = 1, \; x_{52} = 1$

$y_3 = 0$

$y_4 = 1 \qquad x_{64} = 1$

$y_5 = 0$

Total profit $(u) = z(u) = 3 + 1 + 25 = 29$