NLP Solvers

CONOPT, MINOS - Reduced Gradient Method

Excel/Solver

1st order

very reliable
very popular
oldest

good for small/medium convex problems

Pennon - interior-point method
- very similar to Lancelot (default)
- engineering - truss optimization
- nonlinear semidefinit programming

PathNLP - Path built for equilibrium problems
- generalization to NLP

Nondifferentiable (Derivative free) Optimization

condor

download & combine with simulation
- UOBYQA - available for download
- direct
- IBM/COIN-OR has dfo solvers

Semidefinite Programming

$$\max \sum_{ij} c_{ij} x_{ij}$$

$$\text{s.t.} \quad \sum_{ij} a_{ij}^k x_{ij} = b^k, \quad k = 1, \ldots, K$$

$$\underbrace{X \succeq 0}$$

X is symmetric positive semidefinite

Def: $y^T X y \geq 0$ for all y

$$\max \sum_{ij} C_{ij} X_{ij} - \mu \log(\det(X))$$

$$\text{s.t.} \quad \sum_{ij} a^k_{ij} X_{ij} = b^k \quad \forall k$$

Say $K=1$

$$A \cdot X = b$$

$$C - A^T y - Z = 0$$

$$X \cdot Z = \mu$$

- $n \times n$ variables $(X \in \mathbb{R}^{n \times n})$
- dense problem
- how do you keep X and Z symmetric?

$$\max \quad c^T x$$
$$\text{s.t.} \quad Ax = b$$

$$X = \begin{bmatrix} x^0 \\ x^1 \end{bmatrix}$$

scalar

vector

$$\|x^1\| \leq x^0$$

$$\sqrt{(x_1^1)^2 + (x_2^1)^2 + \dots + (x_n^1)^2}$$

$$\min \quad b^T y$$
$$\text{s.t.} \quad \|A^T y + c\| \leq d^T x + e$$

SeDuMi - very good at solving second-order cone programs

MOSEK - linear, quadratic, second-order cone, convex nonlinear optimization

commercial

interior-point / simplex code

$$z = \max \ c(x)$$
$$\text{s.t.} \quad x \in X \subseteq \mathbb{Z}^n$$

$\mathbb{Z}$ = set of integers

for example: $X = \{1, 2, 3, 4\}$

$$X = \{x \in \mathbb{Z}^3 : x_j \geq 5, j = 1, 2, 3\}$$

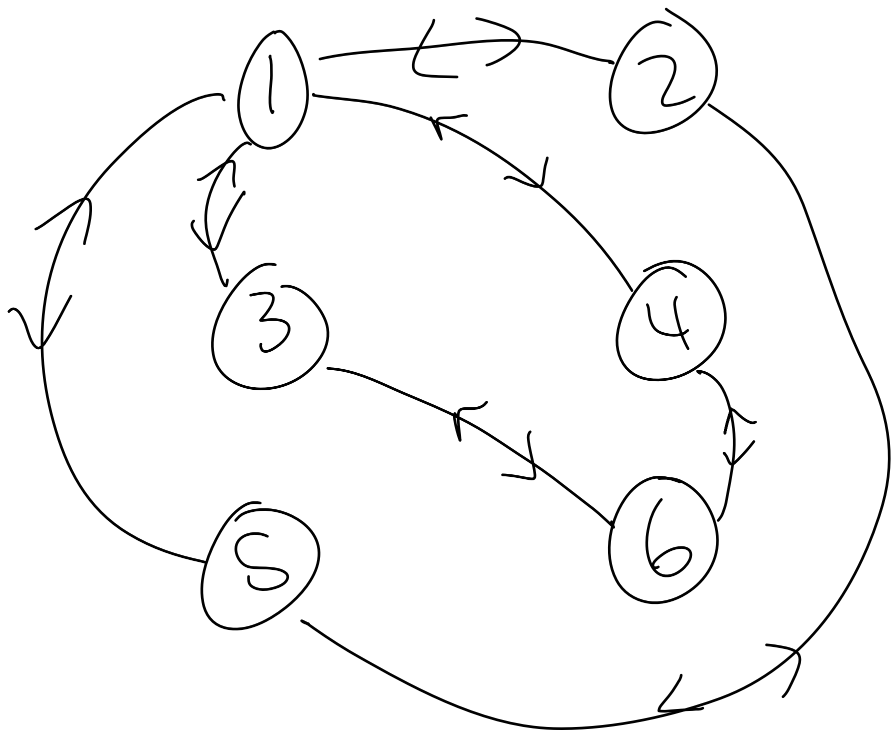
$$X = \{x \in \mathbb{Z}^3 : x_j \geq 5, j = 1, 2, 3, \text{ and } Ax \leq b\}$$

$$\max \quad 12x_1 + 8x_2 + 17x_3 + 11x_4$$
$$\text{s.t.} \quad 4x_1 + 3x_2 + 7x_3 + 5x_4 \leq 9$$
$$x_1, x_2, x_3, x_4 \in \{0, 1\}$$

$$\frac{\text{value}}{\text{unit weight}} = \left(\frac{12}{4}, \frac{8}{3}, \frac{17}{7}, \frac{11}{5}\right) = \left(3, 2\tfrac{2}{3}, 2\tfrac{3}{7}, 2\tfrac{1}{5}\right)$$

$$z \geq 20$$

$$\begin{bmatrix} - & 9 & 2 & 8 & 12 & 11 \\ 9 & - & 7 & 19 & 10 & 32 \\ 2 & 7 & - & 29 & 18 & 6 \\ 8 & 19 & 29 & - & 24 & 3 \\ 12 & 10 & 18 & 24 & - & 19 \\ 11 & 32 & 6 & 3 & 19 & - \end{bmatrix}$$



Uncapacitated Facility Location:

Initial Solution = $\{1,2\}$

Client	Depot	Variable Cost
1	2	2
2	1	1
3	2	2
4	1	9
5	1	7
6	2	3
	+	24

$21 + 16 + 24 = 61$

To consider: $\{1\}$, $\{2\}$, $\{1,2,3\}$, $\{1,2,4\}$

p.32

$\{1\}$: 63

$\{2\}$: 66

$\{1,2,3\}$: 60

$\{1,2,4\}$: 84

New incumbent = $\{1,2,3\}$

Check $\{1,3\}$, $\{2,3\}$, $\{1,2,3,4\}$

$\uparrow$

42

New incumbent = $\{2,3\}$

Check $\{2\}$, $\{3\}$, $\{2,3,4\}$

$\uparrow$

31

$$\begin{aligned} \max \quad & 4x_1 - x_2 \\ \text{s.t.} \quad & 7x_1 - 2x_2 \leq 14 \\ & x_2 \leq 3 \\ & 2x_1 - 2x_2 \leq 3 \\ & x_1, x_2 \geq 0 \end{aligned}$$

$$x \approx (2.86, 3)$$

$$z \leq 8.43$$

$$z \leq 8$$