$$\min \quad 3x_1 + 2x_2$$

$$\text{s.t.} \quad 8x_1 - x_2 \leq 8$$

$$2x_1 + 3x_2 \leq 5$$

$$x_1 \in \{1, 2, 5\}$$

$$x_2 \in \{2.5, 1.5, 0.25\}$$

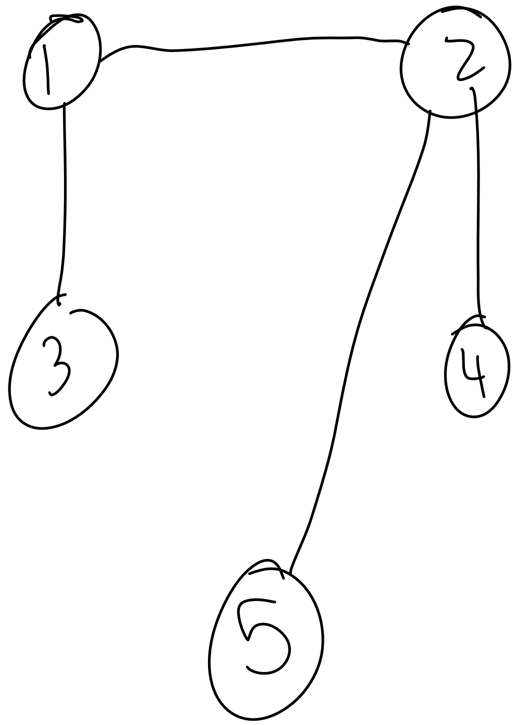
$$x_1 = 1\lambda_1^1 + 2\lambda_1^2 + 5\lambda_1^5$$

$$x_2 = 2.5\lambda_2^{2.5} + 1.5\lambda_2^{1.5} + 0.25\lambda_2^{0.25}$$

$$\lambda_1^1 + \lambda_1^2 + \lambda_1^5 = 1$$

$$\lambda_2^{2.5} + \lambda_2^{1.5} + \lambda_2^{0.25} = 1$$

$$\lambda \in \{0,1\}^6$$



$$\min \sum_{e \in E} c_e x_e$$

s.t. $\sum_{e \in \delta(i)} x_e = 2, \; i \in N$

$x \in X^1$

1-tree : a tree (no-loops) + a degree of 2 at the first-node

$$x_e = \sum_{t : e \in E^t} \lambda_t$$

E^t = set of edges in the t^{th} one-tree

cost of the one-tree t

$$\min \sum_{t=1}^{T_1} (c^t x) \lambda_t$$

s.t. $\sum_{t=1}^{T_1} d_i^t \lambda_t = 2$

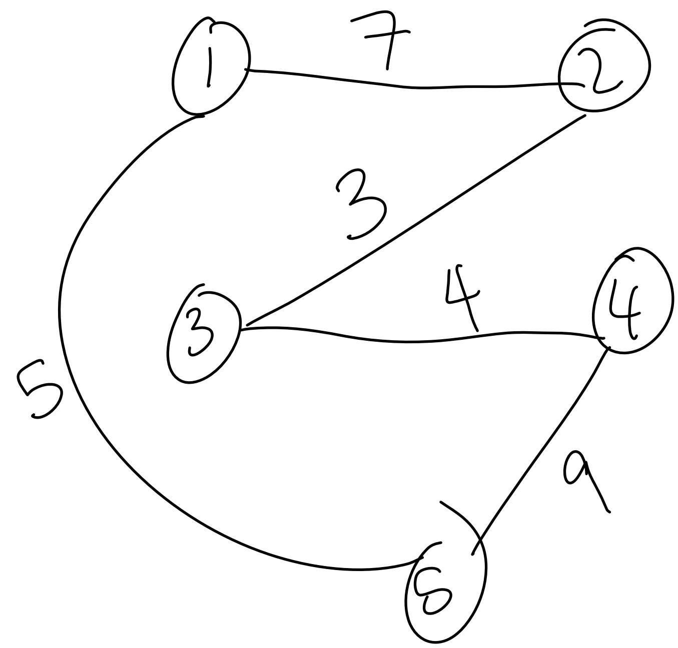
d_i^t = degree of node i in one-tree t

$\sum_{t=1}^{T_1} \lambda_t = 1$

$\lambda \in R_+^{T_1}$

LP master

λ_1 :



Solve Master #1, λ = primal soln

u = dual soln

Use dual soln within Lagrange relaxation as an estimate of Lagrange multipliers.

$$\min \sum_{e \in E} c_e x_e$$

s.t. $\displaystyle\sum_{e \in \delta(i)} x_e = 2 \quad , i \in N$

$x \in X'$

dualize

$(i,j) = e$

$$\min \sum_{e \in E} (c_e - u_i - u_j)\, x_e$$

s.t. $x \in X'$

include $\displaystyle\sum_{k \in N} u_k \Big(2 - \sum_{e \in \delta_k} x_e\Big)$

$$c_e - u_i - u_j = \begin{bmatrix} - & 7-u_1-u_2 & 2-u_1-u_3 & 1-u_1-u_4 & 5-u_1-u_5 & \\ & - & 3-u_2-u_3 & 6-u_2-u_4 & 8-u_2-u_5 & \\ & & - & 4-u_3-u_4 & 2-u_3-u_5 & \\ & & & - & 9-u_4-u_5 & \\ & & & & - & \\ & & & & & - \end{bmatrix}$$

Master 1:

$$c_e - u_i - u_j = \begin{bmatrix} - & -\frac{87}{8} & -\frac{91}{8} & -\frac{133}{8} & -\frac{111}{8} \\ & - & \frac{19}{2} & \frac{33}{4} & 9 \\ & & - & \frac{43}{4} & \frac{15}{2} \\ & & & - & \frac{41}{4} \\ & & & & - \end{bmatrix}$$

total = -6.75

$22.5 - 6.75 = 16.75$ lower bound

22.5 upper bound

$$\min \sum_{e \in E} c_e x_e$$

$$\text{s.t.} \quad \sum_{e \in \delta(i)} x_e = 2, \quad i \in N$$

$$x \in X'$$

Dualize $\sum_{e \in \delta(i)} x_e = 2$. Let u_i be its Lagrange multiplier

$$\min \sum_{e \in E} c_e x_e - \sum_{i \in N} u_i \left(\sum_{e \in \delta(i)} x_e - 2 \right)$$

$$\text{s.t.} \quad x \in X'$$

$$\min \sum_{e \in E} \underbrace{(c_e - u_i - u_j)}_{e=(i,j)} x_e + \sum_{i \in N} 2u_i$$

$$\text{s.t.} \quad x \in X'$$

$$\min \sum_{t=1}^{T} (c^T x^t) \lambda_t$$

$$\text{s.t.} \quad \sum_{t=1}^{T} d_i^t \lambda_t = 2, \quad i \in N$$

$$\sum_{t=1}^{T} \cancel{\lambda_t} = 1$$

$$\lambda \in \mathbb{R}^T_+$$

$$\max \sum_{i \in N} 2u_i \cancel{+ \mu}$$

s.t. Dual constraints

Don't need this constraint

$$\sum_{t=1}^{T_1} \lambda_t = 1$$ * Primal problem

At node 1: $$\sum_{t=1}^{T_1} 2\lambda_t = 2$$ * By construction of 1-tree

$$\sum_{t=1}^{T_1} d_i^t \lambda_t = 2$$