OPTIMAL PRICING AND PROCUREMENT STRATEGIES IN A SUPPLY CHAIN WITH MULTIPLE CAPACITATED SUPPLIERS

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ABSTRACT

The scenario considered in this paper is a supply chain with multiple suppliers, all of whom have limited capacity. The demand over a finite planning horizon is known, and an optimal procurement strategy for this multiperiod horizon is to be determined. We supplement this scenario with the entry of a new supplier who is looking to price their products competitively. A single pricing decision is made for the duration of the horizon, and the prices offered by the other suppliers are known. Thus, we present here two models: one for the procurement decision maker and the other for the supplier. In fact, the supplier model is a bilevel model which uses the procurement model. We have solved several randomly generated instances of these problems and report encouraging numerical results. We also describe extensions of the model to include Nash games.

Keywords: supply chain management, pricing decisions, mathematical programming.

1. INTRODUCTION

One of the interesting issues in establishing a supply chain is that of having a single supplier vs. multiple suppliers. While stronger relationships can be established with a single supplier and information sharing is easier, it may be possible to secure lower costs due to competition among multiple suppliers and ensure a steady flow of goods and services in the case of an emergency. In this paper, we will concentrate on the case of having multiple suppliers in the supply chain.

We start with the model of Basnet and Leung [3] for optimal procurement lot-sizing with supplier selection. Their model is an extension of Wagner and Whitin's seminal work [24], which is a lot sizing model for a single product, to the multiproduct case. In addition, it is a multiperiod model, considering the possibility of exploiting economies of scale in the procurement process in exchange for accruing inventory costs from one period to the next. Multiperiod models also offer the opportunity to change suppliers for a product from one period to the next. Many supplier selection models, such as [10] [14] [15] and [19], are single period models. One of the important modifications we consider in this paper is that of introducing capacities for the suppliers. Finite supplier capacities may force the demand for a particular product to be satisfied from a combination of suppliers, as the capacity of a single supplier may not be sufficient. Capacities on resources, in general, were considered in the important work of Manne [18], where he introduced a mixed-integer linear programming model in the presence of labor limitations. Supplier capacities play a similar role in a supply chain, and to the best of our knowledge, there are no other works so far which consider a multiperiod, multiple product capacitated supplier selection model.

We then turn our attention to the case of a new supplier entering the market. The model we consider is that of the new supplier's determination of a pricing strategy to maximize their revenue in the supply chain given perfect knowledge of the pricing strategies of their competitors and the strategy of the procurement decision maker. This model is fairly simplistic in its approach to pricing, but illustrates some important optimization techniques that can be used by a supplier to handle a so-called "static" pricing situation, where the demand is known and the prices and procurement strategies of the other players in the supply chain are known and fixed for several periods. Discussion of more complicated dynamic pricing models can be found in [5] and [7], though without a similar multiperiod, multiproduct procurement model.

The pricing model is formulated as a bilevel model, with one optimization problem inside of another. Bilevel models have always received much attention in the optimization and economics communities, mainly due to the famous examples of a Nash and Stackelberg games, as highlighted in the seminal work of Stackelberg [21]. There has been a lot of work recently to solve such models in the presence of nonlinearities. For a thorough overview of bilevel programming, see [6].

One extension of our work is to consider future planning periods which would give all the suppliers a chance to adjust their prices. Such considerations are the subject of the growing field of supply chain auctions. Especially reverse auctions, where a company solicits bids from a group of potential suppliers, have been researched quite intensely in the last several years, mainly due to the emergence and growth of e-business practices which have greatly enabled and enhanced auction technologies. Two case studies illustrating the success of reverse auctions are that of Mars, Inc [HRDKLA03] and of General Electric [16].

In the next section, we describe the procurement model for optimal lot sizing in the presence of supplier selection issues. This model is adapted from [3] and extended to include supplier capacities. In Section 3, we discuss the new supplier's pricing problem, which is a bilevel model with a quadratic objective function. We follow with a description of our solution method for the bilevel model in Section 4 and present numerical results for both the lot sizing problem and the pricing problem. We conclude with a discussion of possible extensions and by introducing the state-of-the-art on competitive pricing strategies, including those used in the electricity markets.

2. PROCUREMENT MODEL WITH CAPACITATED SUPPLIER SELECTION

For the procurement model, we follow essentially the same development as [3]. The main difference is that the suppliers considered have capacities. This is a realistic assumption because the suppliers may have production or storage limitations or the mode of transportation from the supplier may determine an upper bound on the number of units shipped. As in [3], we describe the model components:

Indices:
$i = 1, \ldots, I$ index of products (inventory items).
$j = 1, \ldots, J$ index of suppliers.
$t = 1, \ldots, T$ index of time periods.

Parameters:
D_{it} = demand of product *i* in period *t*.
P_{ij} = purchase price of product *i* from supplier *j*.
H_i = holding cost of product *i* per period.
O_j = transaction cost for supplier *j*.
C_{ij} = maximum number of units of product *i* that can be obtained from supplier *j*.

Variables:
X_{ijt} = number of units of product *i* ordered from supplier *j* in period *t*.
Y_{jt} = 1 if an order is placed from supplier *j* in time period *t*, 0 otherwise.
R_{it} = inventory of product *i*, carried over from period *t* to period *t* + 1.

The procurement decision maker's problem can be formulated as

$$\text{minimize} \quad \sum_{\text{ijt}} \mathrm{P_{ij}} X_{ijt} + \sum_{jt} O_j Y_{jt} + \sum_{it} H_i R_{it}$$

$$\text{subject to} \quad R_{it} = \sum_{k=1}^{t}\left(\sum_{j} X_{ijk} - D_{ik}\right), i = 1,\ldots,I, t = 1,\ldots,T \qquad (1)$$

$$\left(\sum_{k=t}^{T} D_{ik}\right) Y_{jt} - X_{ijt} \geq 0, i = 1,\ldots,I,\ j = 1,\ldots,J, t = 1,\ldots,T$$

$$Y_{jt} \in \{0,1\},\ j = 1,\ldots,J, t = 1,\ldots,T$$

$$R_{it} \geq 0, i = 1,\ldots,I, t = 1,\ldots,T$$

$$0 \leq X_{ijt} \leq C_{ij}, i = 1,\ldots,I,\ j = 1,\ldots,J, t = 1,\ldots,T$$

The goal is to minimize total cost, and the objective function consists of the three customary terms for computing cost: purchasing, ordering, and holding costs. The first constraint simply computes the amount of inventory left at the end of each period. The second and third constraints ensure that an ordering cost is charged by the appropriate supplier whenever an order is placed. The fourth constraint signifies that stockouts are not allowed. The last constraint states that the procurement lot size from a supplier cannot exceed the capacity of the supplier. It is this constraint that differentiates our model from that of [3], and, while the modification to the model looks trivial, it has serious consequences in terms of the solution.

Note that the optimization problem stated above is a mixed-integer linear programming problem, and we will solve it using the classic branch-and-bound approach. As there are 2^{JT} possible combinations of *Y* values for this problem, branch-and-bound is used as a tool to efficiently investigate or disregard possible solutions to the problem. During the algorithm, various combinations of 0-1 values are assigned to subsets of the *Y* variables and the rest of the variables are treated as continuous variables. Without a capacity constraint on the suppliers, all infeasibility-inducing combinations of *Y* values have that

$$Y_{j1} = 0,\ j = 0,\ldots,J.$$

Thus, all branches that have such Y values can be fathomed due to infeasibility. As a matter of fact, these are the only branches that can be fathomed in such a way. This is due to the fact that when there is no upper bound on the number of units that can be ordered from a supplier, we can satisfy all demand as long as we have at least one producer to purchase from in the first time period.

In the presence of capacity constraints, however, there are many more combinations of Y values that lead to infeasible solutions. Given the total capacity available in any given period, there is a limit on the amount of inventory that can be carried from one period to the next to meet future demand. Also, given the individual capacities in each period, there is a possibility that the demand for a particular product may not be met by a strict subset of the set of suppliers. Due to these reasons, it is very hard to readily identify infeasible solutions, but their possible abundance allows us to prune the branch-and-bound tree more than we could before. Thus, the solution algorithm needs to pay particular attention to the identification of infeasible subproblems.

While the number of branches pruned due to infeasibility may increase, we would also expect to see a much deeper tree than the uncapacitated model. The reason for this is that due to capacities on the suppliers, the demand for a particular product may not be met from a single supplier. Thus, the second constraint in model (1), combined with the minimization, might produce fractional values for Y. More solutions with fractional values mean that the branch-and-bound tree will be deep, and the number of subproblems will be considerable.

3. THE SUPPLIER'S PRICING PROBLEM

Let us now consider the situation where a new supplier is entering the supply chain and looking to develop a pricing strategy in order to maximize their profits. They know the prices offered by their competitors, and that the procurement decision maker uses a model of the form (1). Assume that the competitors' prices are fixed for the remainder of the planning horizon, and that once the new supplier decides on a price, it will also be fixed for the same timeframe. As is the case in a good supply chain, there is information sharing between tiers, and, therefore, the supplier has access to demand information and holding cost. Let us assume that the ordering cost for the supplier is known and is due to production setup, order entry, and fixed transportation costs. The capacity at the supplier is determined by a combination of their production and transportation capabilities.

Let the new supplier be Supplier 0, that is, all parameters and variables with index $j = 0$ refer to the new supplier. The supplier's objective function is

$$\text{maximize} \sum_{it} \mathrm{P}_{i0} X_{i0t} + \sum_{t} O_0 Y_{0t}$$

As stated, the ordering cost O_0 is known. Note that unlike problem (1), the costs P_{i0} are (nonnegative) variables, and the values of X_{i0t} and Y_{0t} come from the solution of the procurement decision maker's optimization problem, which is a modification of (1) to include the new supplier (where $j = 0, \ldots, J$). Thus, the supplier's problem is a bilevel problem, which requires the solution of an optimization problem within another optimization problem. The traditional approach to solving such a problem is to replace the inner optimization problem (in this case, the procurement decision maker's problem) with its optimality conditions. The resulting problem is of the form:

$$
\begin{aligned}
\text{maximize}\quad & \sum_{it} \mathrm{P_{i0}} X_{i0t} + \sum_{t} O_0 Y_{0t} \\
\text{subject to}\quad & R_{it} = \sum_{k=1}^{t}\left(\sum_{j} X_{ijk} - D_{ik}\right), i = 1,\ldots,I, t = 1,\ldots,T \\
& \left(\sum_{k=t}^{T} D_{ik}\right) Y_{jt} - X_{ijt} \geq 0, i = 1,\ldots,I, j = 0,\ldots,J, t = 1,\ldots,T \\
& P_{ij} - \sum_{k=t}^{T} \lambda_{ik} + \rho_{ijt} - \kappa_{ijt} + \pi_{ijt} = 0, i = 1,\ldots,I, j = 0,\ldots,J, t = 1,\ldots,T \\
& O_j - \sum_{i} \rho_{ijt}\left(\sum_{k=t}^{T} D_{ik}\right) - \eta_{jt} + \theta_{jt} = 0, j = 1,\ldots,J, t = 1,\ldots,T \\
& H_i + \lambda_{it} - \delta_{it} = 0, i = 1,\ldots,I, t = 1,\ldots,T \\
& \left(\sum_{\mathrm{ijt}} \mathrm{P_{ij}} X_{ijt} + \sum_{jt} O_j Y_{jt} + \sum_{it} H_i R_{it}\right) - \\
& \qquad \left(-\sum_{i} \lambda_{it}\left(\sum_{k=1}^{t} D_{ik}\right) + \sum_{ijt} \pi_{ijt} C_{ij} + \sum_{jt} \theta_{jt} u_{jt} - \sum_{jt} \eta_{jt} l_{jt}\right) = 0 \\
& \sum_{\mathrm{ijt}} \mathrm{P_{ij}} X_{ijt} + \sum_{jt} O_j Y_{jt} + \sum_{it} H_i R_{it} \leq \text{minCost} \\
& 0 \leq X_{ijt} \leq C_{ij}, i = 1,\ldots,I, j = 0,\ldots,J, t = 1,\ldots,T \qquad (2) \\
& l_{jt} \leq Y_{jt} \leq u_{jt}, j = 0,\ldots,J, t = 1,\ldots,T \\
& R_{it} \geq 0, i = 1,\ldots,I, t = 1,\ldots,T \\
& P_{i0} \geq 0, i = 1,\ldots,I \\
& \rho_{ijt} \geq 0, i = 1,\ldots,I, j = 0,\ldots,J, t = 1,\ldots,T \\
& \eta_{jt} \geq 0, \theta_{jt} \geq 0, j = 0,\ldots,J, t = 1,\ldots,T \\
& \kappa_{ijt} \geq 0, \pi_{ijt} \geq 0, i = 1,\ldots,I, j = 0,\ldots,J, t = 1,\ldots,T \\
& \delta_{it} \geq 0, i = 1,\ldots I, t = 1,\ldots T
\end{aligned}
$$

Problem (2) is a mixed integer quadratic programming problem. This is due to the fact that in addition to the variables of problem (1), we also have the costs P_{i0} as variables. Therefore, the objective function is quadratic. The first five sets of constraints are linear as they arise from the optimality conditions of the inner linear programming problem. The customary nonlinear complementarity conditions have been replaced by a single equivalent condition (quadratic in the variables of the unified problem) that the duality gap should be 0. Note that, in the event of a nonlinear cost function in the procerement decision maker or other nonlinearities appearing in the model, we could resort to using complementarity terms in the optimality conditions of the inner problem. There has been recent interest in solving nonlinear problems with complementarity constraints, as discussed in [4] and [9].

We have also added a constraint that the cost to the procurement decision maker must be no more than the minimum cost they would spend without the new supplier. This is to ensure that the procurement decision maker has an incentive to do business with the supplier 0. The minimum cost is the optimal value of problem (1). The variables $\lambda, \rho, \eta, \theta, \delta, \kappa, \pi$ are the dual variables for the inner problem.

Note that the problem has been expressed in a way that befits the use of a branch-and-bound method. The binary conditions on the variables *Y* have been replaced by bounds *l* and *u*, and we assume that these bounds are changed for each node of the branch-and-bound tree in a consistent manner. The bounds, also, appear in the dual objective function for the lower level mixed integer linear programming problem, and are thus included in the duality gap constraint.

4. SOLUTION OF THE SUPPLIER'S PROBLEM

We have chosen to implement models (1) and (2) in a C routine (available by request), which calls the subroutine library of LOQO[23] to solve the problem. Rather than a traditional modeling environment such as MS Excel/Solver or Lindo for the mixed integer linear programming problem (1) or such as GAMS [2] or AMPL [8] for the mixed integer quadratically constrained quadratic programming (QCQP) problem (2), we opted for a mode tedious, yet flexible approach. The reason for this was the bilevel nature of the problem—unlike a traditional branch-and-bound approach where only bounds on the variable change from one subproblem to the next, we have employed a method that changes the optimality conditions of the inner problem along with the bounds. Thus, for each subproblem, the model changes, and the most straightforward way for us to implement such a change was to have more control over the function evaluations by writing our own C-code. The looping-mechanism such as the one provided in AMPL may allow us to implement our model in a different environment in the future, but for the purposes of this study, we did not pursue this course.

Optimization techniques for linear and nonlinear (including QCQP) programming have become very sophisticated and produced very efficient software such as LOQO [23], KNITRO[1], and SNOPT[11] in the last two decades. There are also several efficient codes for mixed integer linear and mixed integer nonlinear programming, such as CPLEX [13], Baron [20], and Minlp [17]. We have chosen to use the nonlinear interior-point method LOQO to solve our problems. As LOQO was originally developed as a solution algorithm for linear and quadratic programming problems, it is particularly suited for this task. The availability of the source code to the authors of this paper also facilitated the ease of implementation. We implemented a branch-and-bound mechanism in this code to accommodate the integer variables.

We have generated random data from a using the same distributions as [3]. All parameters are uniformly distributed integers in the following intervals:

$$D_{it} \in [1,200]$$
$$P_{ij} \in [20,50]$$
$$H_i \in [1,5]$$
$$O_j \in [1000,2000]$$
$$C_{ij} \in [75,125]$$

The model and data files are available from the author upon request. Again, as in [3], we have generated 15 data sets each for various problem sizes (smaller sized problems due to the more complicated nature of the problem, as discussed in Section 2), and we report average results for each problem size in Table 1. The numerical tests were performed on a PC with a 2.4GHz Intel Pentium 4 processor and 512MB of RAM, running Linux RedHat 9.0. The branch-and-bound approach was developed for LOQO version 6.05. Note that LOQO is an *infeasible* interior-point method, that is, a feasible initial solution is not required and feasibility only needs to be achieved at the optimal solution.

	Procurement Problem		Supplier's Pricing Problem			Procurement Problem		Supplier's Pricing Problem	
Size	Time	Cost	Time	Revenue	Size	Time	Cost	Time	Revenue
1,2,1	0.00	2578	0.01	2173	2,3,1	0.00	9084	0.25	6006
1,2,2	0.01	7189	0.10	6550	2,3,2	0.03	17912	0.68	13068
1,2,3	0.03	10230	0.55	8222	2,3,3	0.07	23596	3.15	16570
1,2,4	0.09	13235	1.25	10900	2,3,4	0.35	32905	13.86	22333
1,2,5	0.26	25861	3.94	14754	2,3,5	1.47	40671	36.54	25904
1,2,6	0.97	23853	4.49	16603	2,3,6	11.66	49339	64.68	32486
1,3,1	0.00	4247	0.09	3396	3,2,1	0.00	6704	0.01	6173
1,3,2	0.01	8293	0.16	6444	3,2,2	0.01	13828	0.26	13203
1,3,3	0.03	14235	0.96	9046	3,2,3	0.03	37356	2.63	26768
1,3,4	0.19	19840	2.46	13758	3,2,4	0.07	46876	13.21	32902
1,3,5	0.47	21866	6.43	12510	3,2,5	0.17	57928	23.14	41072
1,3,6	1.79	29692	8.51	18740	3,2,6	1.38	73718	29.27	50714
2,2,1	0.00	4396	0.00	4278	3,3,1	0.00	11276	0.46	7854
2,2,2	0.01	10044	0.28	9561	3,3,2	0.03	20581	0.68	15852
2,2,3	0.02	18581	1.44	17035	3,3,3	0.17	34806	9.44	24716
2,2,4	0.06	26340	6.14	21568	3,3,4	0.79	48889	24.33	33874
2,2,5	0.18	32501	2.20	27859	3,3,5	3.71	55901	79.21	36747
2,2,6	0.33	53165	11.48	34939	3,3,6	18.43	68271	141.89	44772

TABLE 1. Average results on the procurement decision maker's problem and the supplier's pricing problem. The averages are taken over 15 problems of each size.

Here is a sample solution to the problem with 3 products, 3 suppliers, and 5 time periods. Note that due to the capacities of the suppliers, the optimal procurement strategy must involve multiple suppliers for some of the products. This provides an opportunity for the new supplier to increase their revenue by gaining a market share without having to charge lower costs than the other suppliers.

$$D = \begin{bmatrix} 184 & 87 & 178 & 116 & 194 \\ 136 & 187 & 93 & 50 & 22 \\ 163 & 28 & 91 & 60 & 164 \end{bmatrix} \quad C = \begin{bmatrix} 97 & 120 & 105 & 115 \\ 106 & 110 & 83 & 111 \\ 114 & 89 & 80 & 83 \end{bmatrix} \quad P = \begin{bmatrix} 39 & 34 & 50 \\ 32 & 42 & 25 \\ 33 & 47 & 42 \end{bmatrix}$$

$$X_{\cdot 0 \cdot} = \begin{bmatrix} 97 & 87 & 92 & 97 & 89 \\ 106 & 106 & 93 & 50 & 22 \\ 114 & 28 & 91 & 110 & 114 \end{bmatrix} \quad Y = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad P_{\cdot 0} = \begin{bmatrix} 36.5 \\ 29.1 \\ 36.3 \end{bmatrix}$$

5. CONCLUSION

In this paper, we have formulated a procurement model with supplier selection in the presence of multiple suppliers with limited capacity. The model is an extension of the work of [3]. We then examine the pricing problem of a new supplier entering the supply chain. The bilevel problem is reformulated as a mixed integer quadratic programming model and solved using a branch-and-bound approach with an interior-point method. We report encouraging numerical results on a set of randomly generated problems.

We have assumed in our model that all the information about the procuring company and the other suppliers were available to the new supplier. We have also assumed that the existing suppliers were not allowed to react to the entry of the new supplier by changing their pricing strategy in the rest of the planning horizon. An interesting extension of this problem is to observe the behavior of all suppliers at a time period when they are all allowed to readjust their prices. Such problems are quite common with applications in competitive markets such as for electricity. They are formulated as multiplayer Nash games, which are bilevel problems, and can be solved using the approach outlined in this paper. For an interesting example from the electricity market, see [22].

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