Aircraft Energy Management: Finite-time Optimal Control with Dynamic Constraints

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Traditional aircraft design has been based on peak power and peak thermal loads. With the introduction of more-electric aircraft, increasing requirements for on-demand high-quality power for flight controls and electro-mechanical actuation devices will pose further challenges for energy efficiency and mission capability. This paper examines a new dynamic optimization strategy for energy management of more-electric aircraft based on hybrid systems theory. An expressive framework is developed to address the pertinent issues for energy management in aircraft systems, creation of optimization metrics, implementation of necessary computational tools and initial verification through simulation.

Nomenclature

\begin{align*}
c & = \text{constraints} \\
f, g, h & = \text{generic functions} \\
i & = \text{discrete mode index} \\
j & = \text{electric network bus index} \\
k & = \text{discrete time index} \\
l, m, n & = \text{space dimensions} \\
p, r, s & = \text{optimization weights} \\
q & = \text{discrete states} \\
t & = \text{continuous time index} \\
u & = \text{control inputs} \\
x & = \text{continuous state vector} \\
y & = \text{measurements} \\
B & = \text{bound} \\
F & = \text{fuel rate} \\
I & = \text{current} \\
J & = \text{objective function} \\
M & = \text{Mach number} \\
V & = \text{voltage} \\
\mathbb{R} & = \text{real field} \\
\sigma & = \text{discrete events} \\
\delta & = \text{discrete transition function} \\
\varphi & = \text{cost function}
\end{align*}

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I. Introduction

The energy management problem has paramount importance during the design and operation stages of both military and commercial aircrafts. Traditionally, the aircraft design addresses the peak power and peak thermal demands and the aircraft components are selected to accommodate the peak loads. While this approach offers future growth potential, today’s aircrafts already boast 3 to 5 times more heat load than that of legacy aircraft. As more-electric aircraft vision progresses, increasing complexity of the aircraft design, as well as increasing requirements for on-demand high-quality power for flight controls and electro-mechanical actuation devices will pose further challenges for energy efficiency and mission capability. There is a new and pressing need for dynamic optimization for energy management of more-electric aircraft. Furthermore, this modern energy management and control systems should ensure reduced fuel rate, increased range and performance, and capable of producing and distributing mega-watt level power on-demand, robustly and adaptively. This will pave the path for next generation, energy optimized aircrafts with enhanced capabilities and economical operation.

There are several important aspects to consider for aircraft energy management. For example cost, space and weight limitations make it impractical to provide dedicated energy storage for every load. Instead it’s advantageous to direct the stored and/or generated power to the active loads on as-needed basis. Shared generation/storage presents unique opportunities but it also raises significant control and coordination issues. If properly designed and managed, the energy management system would help reducing fuel consumption and minimizing the reactive power with its attendant losses and voltage transients, while still providing the mission power requirements. Moreover, recent developments in efficient energy generation concepts and new energy storage devices are attracting attention to lower operational costs. Consequently, the management system must be able to optimize a mix of power sources to realize potential cost benefits. On the other hand, subsystems housed in aircrafts have stringent metrics that put a high emphasis on the ability to accommodate changing mission requirements. This can be achieved both through system design and proper energy management including power network reconfiguration. In order to do this reliably, without sacrificing mission capability, the master control system requires monitoring of the overall energy use of the platform and harbor advanced control processes for integrated subsystems in order to supervise and coordinate the system operation in a variety of mission profiles. Therefore, a significant consideration in the development of aircraft energy management problem relates to a mathematical framework with a variable cost function that can be optimized, while addressing individual mathematical relationships to each subsystem. The energy management problem is further complicated by both the fact that the power system may be operated close to stability limits and that many subsystems have local protection mechanisms which interfere with the control.

In lieu of the above discussion, the energy management problem for aircrafts can be formulated in hybrid systems (i.e. composed of both continuous dynamics and discrete logic) framework. This hybrid logic – dynamics abstraction allows a complete system description and the solution to be articulated in a systematic setting. The underlying continuous dynamics of the subsystems are usually well-understood; however, this is not sufficient due to the existence of discrete transitions such as discrete action of protection devices, switches and lower level control systems, and external disturbances such as human inputs. The essence of the idea is that the discrete acting subsystem is naturally associated with a set of logical conditions or logical specifications, and the continuous part of the dynamics are usually described by differential-algebraic equations. This abstraction provides a computational framework for optimization problems involving monitoring and control of integrated subsystems – specifically to ensure economic system operation and maximize range based upon specific operational demands required from the aircraft.

In this paper, we construct an expressive framework to pose some of the pertinent issues for energy management in aircraft systems. Based on this hybrid systems framework, an appropriate optimization metric is created, and initial verification of the developed controllers are provided through simulation. This framework addresses all the relevant technical challenges in order to meet the demands of modern aircraft systems. The novelty in this approach is that it not only addresses the necessary conditions for mission capability (the ability of the control system to discern the functioning topology) and capacity (the ability of the generation to meet the load requirements) but also incorporates other critical conditions dictated by the dynamics (the ability to satisfy electrical power quality) and information management (the ability of the system to provide information in a timely manner to both upstream and downstream components). The systematic approach allows that new integrated control architecture is capable of minimizing energy costs, rapid redistribution of power, and as necessary, to coordinate the operation of aircraft subsystems.

This paper is organized in five sections including the present one. Section II elaborates the approach for hybrid systems control theory and optimization. Section III formulates the aircraft energy management problem in terms of
II. Approach for Hybrid Systems and Optimization

A formulation of the aircraft energy management problem is developed in a hybrid systems setting; composed of both the continuous nonlinear dynamics and the discrete dynamics. Hybrid system control design utilizes a model formulation that characterizes the discrete transition behavior by logical specifications, which can then be transformed to a set of mixed integer inequalities, and nonlinear continuous dynamics in the form of differential or algebraic differential equations.

A hybrid system is composed of a continuous dynamical system interfaced with a discrete event system. A hybrid system is depicted that consists of a switched dynamical system, a finite state machine and an interface.

The discrete event system is modeled by a Deterministic Finite Automaton (DFA). The automaton is specified as a 5-tuple \( Q = (Q, \Sigma, \delta, q_0, Q_m) \), where \( Q \) is the finite set of automaton states with \( q_0 \in Q \) is the initial state; \( \Sigma \) is the (finite) set of events (also called symbols). The (possibly partial) function \( \delta: Q \times \Sigma \rightarrow Q \) represents state transitions and \( Q_m \subseteq Q \) is the set of output (also known as accepted) states.

The action of the DFA is characterized by the equation

\[
q(k) = \delta(q(k-1), \sigma(k))
\]  

Here \( k \) is a discrete time index that may be synchronous or asynchronous. In the asynchronous case, the index advances when an event occurs. An important generalization of this model is from a deterministic automaton to a nondeterministic one.

The mode selector is a function \( h_m: Q \rightarrow U \)

\[
u = h_m(q_m(k))
\]  

and the event generator is a function \( h_e: X \rightarrow \Sigma \)

\[
\sigma = h_e(x(t))
\]  

A hybrid system is composed of a continuous dynamical system interfaced with a discrete event system. A hybrid system is depicted that consists of a switched dynamical system, a finite state machine and an interface.
modes) can be employed to switch the various lower level actions to achieve adequate subsystem behavior. These low level modes can be generated directly by the lexical translation of high level command directives into mode changes or series of mode changes. The logical specifications can further be written as a set of inequalities involving integer variables, so called IP-formulas. In general, hybrid controllers are capable of employing discrete actions (or mode changes) and making decisions based on control automaton. In our previous work, we have extended this formulation to solve optimization problems. The process involves separating the inequalities into binary and real sets, and eliminating unfeasible solutions to obtain a much smaller domain for optimal search calculations. The hybrid controller computations can be performed in \( O(ab) \) complexity where \( a \) is the number of discrete states and \( b \) is the number of events.

Now, consider a hybrid system with \( a \) number of modes, which are the discrete states of the discrete event subsystem, designated by the Boolean (True, False) variables, \( q_1, \ldots, q_a \). Corresponding to these are binary (0,1) variables, \( \delta_{q_1}, \ldots, \delta_{q_a} \). The number of transitions between modes are \( b \), and they are described by Boolean variables, \( \sigma_1, \ldots, \sigma_b \), or the corresponding binary variables, \( \delta_{\sigma_1}, \ldots, \delta_{\sigma_b} \). The \( n \)-dimensional continuous state is denoted by \( x \).

The dynamics of the hybrid system evolve according to the equations

\[
\dot{x} = f_q(x, \delta_q, u) \\
q' = \delta(q', \delta_q)
\]  

(5)

Note that, the discrete state vector \( \delta_q \) specifies the mode and determines the dynamical equations. For now, suppose the transition vector \( \delta_q \) consists of all controllable events such that all of its values are specified by the controller.

Eq. (5) also describes the discrete state evolution. In our framework, the mode transition dynamics are modeled by \( (x, \delta_q) \), the continuous (real-valued) and discrete (binary-valued) states prior to a mode transition, and \( \delta_q \) the vector of transition values. All of these are known at time \( t \). The function \( h_q(x) \) defines real quantities that play a role in determining mode transitions. For example, a real quantity like voltage or current could activate a protection system. It is assumed that such variables can be expressed as functions of the real state. \( q' \) is the mode following a transition. If \( \delta_q \) is uniquely defined, the discrete event subsystem is said to be deterministic. Even if this is not the case, for each non-deterministic automaton, there exists an equivalent deterministic one.

In this work, we introduce a continuous vector \( c(t) \) that represents dynamic constraints on the continuous subsystem. For synthesis purpose, we model \( c(t) \) as piecewise continuous and bounded functions that satisfy

\[ \|c(t)\| \leq B \]

where \( B \) is a prescribed bound. What is novel in this case is that we allow \( B \) to depend on the mode; so we write \( B(\delta_q) \). The elements of \( c(t) \) may be continuous control commands so we can bound them in certain modes. For example, we could limit the propulsion power command to fractions of rated power in specified modes. The controller could steer the discrete event subsystem to the appropriate reduced power mode as needed. Otherwise the propulsion control is unrestricted. In addition, strict bounds on real variables that are to be enforced may be incorporated using this device. Note that, when synthesizing the controller, we try to ensure system integrity for all possible constraints.

We want to synthesize controllers for the hybrid system defined by Eq. (5). To do this, we seek a control policy \( \delta_q(t) \) on some finite time interval, \( t \in [0, T] \), that results in minimum cost for all possible \( c(t) \), where the cost function is of the form

\[ J(x(0), \delta_q(0)) = \Psi_T(x(T), \delta_q(T)) + \int_0^T \Psi(x(t), \delta_q(t), \delta_q(t), c(t)) dt \]  

(6)

In Eq. (6), \( \Psi_T \) is called the terminal cost and \( \Psi(t) \) the running cost. Based on Eq. (6), the optimal cost for the hybrid system is defined as

\[ J^*(x(0), \delta_q(0)) = \min_{\delta_q} \max_B J(x(0), \delta_q(0)) : \|c(t)\| \leq B(\delta_q) \]  

(7)

Similar constructs can be found in the literature. For example, a related problem in which there is no running cost considers the goal as ‘safety’, i.e., certain modes are to be avoided. In our own prior work, we considered a minimization problem without constraints \( c(t) \). In that case, we used a discrete time version of Bellman’s principle of optimality to compute the optimal control and the minimizing cost. With some modification that procedure can be used to solve the max-min problem given by Eq. (7). We use this method to compute the minimizing control, \( \delta_q \), and the optimal cost \( J^* \). The controller can be designed in several ways. In our approach,
we consider the controller that optimizes $J$ for all control actions in $\delta$. As can be seen, this controller can be ‘suboptimal’ compared to ideal conditions (i.e., without considering $c(t)$) since constraints required from the aircraft (e.g., altitude, Mach number, power demand, etc.) needs to be satisfied first.

### III. Problem Formulation

#### A. Notional Aircraft Model with Electrical and Thermal Subsystems

A notional model was developed for aircraft thermal and electric power systems to allow design, analysis and validation of hybrid controllers developed for this system. The objectives are defined for an aircraft power management system in the context of hybrid control. Notional two-engine aircraft model integrates power generation plant (engines), integrated power unit for electrical power conversion, cooling and thermal management system, electric and thermal loads as well as associated component models for actuation and sensing. The mission profile for this notional aircraft is long-range subsonic operation under nominal conditions. The notional plant model is given in Fig. 2.

The engine model consists of lookup tables for thrust, thrust specific fuel consumption, and engine time constant. The outputs are corrected for altitude using an atmospheric model to calculate relative pressure ratio and relative temperature ratio. The main inputs to the engine model are the throttle position, Mach number ($M$), and the altitude, and the main outputs are the thrust and the fuel flow rate.

There are two generators in the aircraft that serve as primary power sources. Each of them provides power to the AC Bus through a generator control breaker, which is controlled by a local generator control unit. Each generator is rated at 160 kW and 115 $V_{ac}$. Also available is a 120 kW, 115 $V_{ac}$ auxiliary power unit. The electric loads consist of static vital and non-vital loads as well as the dynamic loads such as electric motors used for actuators. Avionic system as well as the electro-optical and radar systems were modeled as resistive loads that generate heat. Therefore, the loads are categorized in two different ways. There are static loads based on the functionality of the load and there are vital and non-vital loads based on mission/safety requirements as provided in Table 1.

The thermal system includes the thermal loads due to the engine operation and the electric loads. It is driven by an oil pump connected to the coolant oil and a heat sink and a fuel pump connected to the fuel tank. Fuel is used in integrated aircraft thermal management systems to cool aircraft subsystems and the engine lubricating oil. Therefore, the aircraft circulates fuel on the airframe to match heat loads with available heat sink. These thermal loads push the fuel temperature as high as 163°C at the inlet to the mainburner and to wetted wall temperatures of 205°C inside the fuel nozzle passages.

#### B. Concept of Operations

The operational mode of the aircraft is determined from the mission by considering a combination of fuel economy, reliability, and survivability. This also involves enforcing the operational requirements in terms of altitude and speed. The following propulsion, electric and thermal configurations were considered:

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Table 1. Electric loads

<table>
<thead>
<tr>
<th>Equipment</th>
<th>Type</th>
<th>Vital Load (kW)</th>
<th>Additional Non-vital Load (kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radar</td>
<td>Static</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>Electro-optical</td>
<td>Static</td>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td>Avionics</td>
<td>Static</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>Fuel Pump</td>
<td>Dynamic</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>Oil Pump</td>
<td>Dynamic</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Actuators</td>
<td>Dynamic</td>
<td>74</td>
<td>0</td>
</tr>
</tbody>
</table>
1. Ground: One engine is online; electrical loads are supplied from ground.
2. Departure: One or more engines are online supplying propulsion and electric power; speed is less than 0.9 $M$; altitude is less than 40 k-ft; sensors are at standby.
3. Cruise: One or more engines are online supplying propulsion and electric power; speed is 0.9 $M$; altitude is 40 k-ft; sensors are online.
4. Landing: One or more engines are online supplying propulsion and electric power; speed is less than 0.9 $M$; altitude is less than 40 k-ft; sensors are at standby.

There is another important concept of quality of service (QOS). QOS aims at insuring a reliable supply of power to loads during normal operations. It provides a measure of how well the generation capacity matches with the loads and the severity of the transients associated with mode changes such as shedding non-vital loads. As mentioned before, the loads are divided into two categories: vital loads which cannot be taken offline during a specific mission profile, and the non-vital loads that can be taken offline.

Based on the aforementioned loads and generation alternatives, there is finite number of possible actions at each operational mode:
1. Temporarily shed non-vital loads (only possible if operational mode allows it)
2. Supply power, temporarily, from the auxiliary power unit
3. Temporarily reduce electric generation capacity (to single generator: 160 kW)
4. Use single engine for thrust and electric generation

For the specific optimization problem of a subsonic aircraft, a subset of available states and actions is considered. For example, using the auxiliary power unit and single engine operation are eliminated from the available actions since these actions are not part of nominal aircraft operation. The 6 discrete configurations (states) that are available in optimization are as follows:
- $q_1$ – Generators 1 and 2 are online with full electric load
- $q_2$ – Only Generator 1 is online with full electric load
- $q_3$ – Generators 1 and 2 are online with only vital load
- $q_4$ – Only Generator 1 is online with only vital load
- $q_5$ – Only Generator 1 is online with no load
- $q_6$ – Generators 1 and 2 are online with no load

IV. Optimal Control Synthesis and Validation

A. Optimization for Control Synthesis

The hybrid system formalism was used to design optimal strategies. These strategies are intended to ensure that the aircraft energy management system continues to provide essential functions following discrete mode changes while minimizing the thermal stress, electrical transients and fuel consumption. The controllers themselves are state feedback controllers that trigger controllable discrete events, such as load shedding, to maintain optimal performance in accordance with a specific performance index that could include power production costs. The ability to deal effectively with this changing performance goals using discrete mechanisms is the key to global power management.

The system’s operational context within which the controller operates could change in a manner that cannot be anticipated beforehand. Consequently, the controller synthesis problem is posed as a finite-horizon optimization with a variable objective function. Moreover, the controller should protect system integrity for all allowable control actions while allowing as much flexibility as possible to other decision makers.

In the context of optimal control, an objective function is used to evaluate the feasibility of control actions at each mode of operation and tries to find the optimal path from the initial state to a final configuration that will minimize the optimization criteria. Note that, the control actions correspond to transitions from one mode of operation to another and they are realized as different system configurations. When the system is reconfigured, it undergoes transients and settles to a new steady state condition. Therefore, it is possible to quantify the cost of transients, the previous and the new steady state conditions in terms of critical measurements. If any mode transition violates the QOS metric, the corresponding control action is eliminated from the solution space. This not only helps reducing the computational burden, it also satisfies the dynamic constraints enforced. Furthermore, the fuel consumption is included in the objectives as a function of time. In general, fuel rate is based on produced thrust, which is a function of the discrete mode, and total electric load. Therefore, the overall optimization problem attempts to minimize fuel consumption for a mode of operation or sequence of modes, while insuring QOS constraints are satisfied.

The cost function for this specific example is chosen as follows:
\[ J_i = \int_0^T \left( p_i F_i(t) + \sum_j r_{ij} |V_j(t) - \overline{V}_j| + \sum_j s_{ij} |I_j(t) - \overline{I}_j| \right) dt + sw_i \]  

where \( i \) is the discrete mode index from 1 to 6 and \( j \) denotes network bus numbers. \( F_i(t) \) is the fuel consumption rate and the weights \( p_i, r_{ij}, \) and \( s_{ij} \) are chosen according to the specification of system requirements. \( V_j \) and \( I_j \) correspond to individual voltage and current measurements, respectively, for each bus, where \( (\overline{\cdot}) \) is the known nominal values. The last term \( sw_i \) signifies the cost associated with mode switching.

Note that while total fuel consumption is not typically impacted by the relatively short period of time needed to run through any sequence of state transitions, the transition response insuring satisfaction of QOS constraints could be very important. Consequently, it is required to synthesize controllers for the hybrid system that minimize the fuel with QOS constraints. To do this, an optimal control policy is sought on some finite time interval, \( t \in [0, T] \), that results in minimum cost, where the cost function is based on the fuel rate \( F \) for each operational mode, which also satisfies the QOS metric. The optimal cost \( J^* \) is defined as

\[ J^* = \min_{i, \epsilon, \theta} \max J_i : |V_j(t) - \overline{V}_j| \leq \epsilon_j \quad \& \quad |I_j(t) - \overline{I}_j| \leq \theta_j \]

where the QOS constraints are enforced based on the threshold values \( \epsilon_j \) and \( \theta_j \). A generic 80-seconds mission profile was chosen for optimization, which includes “ground” for 10 seconds, “departure” for 20 seconds, “cruise” for 20 seconds, “landing” for 20 seconds and another “ground” mode of operation for 10 seconds. Mission constraints were included in the control problem as follows. During “cruise” mode of operation, all loads and all generators have to be operational due to mission requirements and this is built into the controller as a forced transition. The optimization took place on a finite horizon of \( T = 10 \) seconds i.e., the controller decides a new operating state at each 10-second interval for a total of 80 seconds in this generic mission. The optimal controller is pictured in Fig. 3. Notice that the optimal controller turns on only one generator (state \( q_5 \)) as mission starts at “ground”. For the operation mode “departure”, the controller selects state \( q_4 \) which signifies again a single generator in operation and only the vital loads are on. As soon as the operational mode becomes “cruise”, the controller leaps to state \( q_1 \) to satisfy the mission requirements with two generators and all vital and non-vital electric loads. Once the mission progresses to the “landing” stage, the controller chooses to stay with two generators but sheds the non-vital loads (state \( q_3 \)). This is to minimize the transients that would be triggered by turning off a generator. Once the landing is completed, the controller state goes back to \( q_5 \).

![State Transition Diagram](image)

Figure 3. Optimal controller behavior is given as state transition diagram.

B. Controller Validation and Verification

Simulation studies were conducted based on the models and optimal controller developed for control verification. The simulation results for an 80-second mission profile without optimization are shown in Fig. 4. Total fuel consumption at the end of the mission was 58.84 kg and the fuel temperature reached up to 343 degrees Kelvins maximum during the mission.
Figure 4. Simulation outputs of the open-loop plant.

The simulation results for the 80-second mission profile with optimal controller are shown in Fig. 5. Total fuel consumption at the end of the mission was 56.24 kg and the fuel temperature reached up to 328 degrees Kelvins maximum during the mission.

The results indicate that the optimal controller accomplishes 2.6 kg fuel savings and better thermal management in terms of 15 degrees Kelvin less fuel temperature increase during 80 seconds. With some modest fuel savings, the controller enables much better thermal management without compromising QOS for electrical equipment since the transients due to mode changes are all within acceptable tolerances throughout the mission.
This paper presents the optimal hybrid control synthesis and validation for energy management of a notional aircraft’s electrical and thermal systems. The presented approach addresses the necessary elements for aircraft power management problem in a coupled, distributed environment. A control optimization algorithm was developed and implemented in hybrid systems framework. The developed controller was evaluated on the simulation environment. The results showed that the optimal controller accomplished 2.6 kg fuel savings and 15 degrees Kelvin lower fuel temperature during an 80-second test scenario. It was also shown that the controller performance didn’t compromise QOS for electrical equipment since the transients due to mode changes were within acceptable tolerances throughout the mission. This paper shows that hybrid control optimization can be a powerful tool for aircraft thermal and electric power management problem. It is also possible to extend the developed tools for other complex systems that have continuous dynamics and discrete logic elements intertwined.

Figure 5. Simulation outputs using the optimal controller.

V. Conclusion
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