NEW MATHEMATICS CLASS

FALL, 2006

MATH 680

INTRODUCTION TO ALGEBRAIC TOPOLOGY

$$\Omega^0(M) \rightarrow \Omega^1(M) \rightarrow \Omega^2(M) \rightarrow \Omega^3(M) \rightarrow \dots$$

$$H^k(M) = \textit{ker}\ d_k / \textit{im}\ d_{k-1}$$

Linear Algebra is the study of sets with enough mathematical structure to make sense of the notion of a linear map between them (i.e., vector spaces). Topology is the study of sets with enough mathematical structure to make sense of the notion of a continuous map between them (i.e., topological spaces). Both subjects are vast, intimidating, and with literally hundreds of applications to virtually every area of mathematics, including to each other. This course will explore just one small, but powerful and very beautiful point of contact between the two, arising from the notion of a *differential form*. We will develop the rather considerable algebraic machinery required to define and study differential forms on open sets in $\mathbf{R}^n$. From these we construct the so-called *de Rham cohomology groups*. These are basic topological invariants with which we will be able to prove some of the most renowned theorems of classical topology:

- Every continuous map from the closed ball in $\mathbf{R}^n$ to itself has a fixed point (**Brouwer Fixed Point Theorem).**

- There is a nonvanishing, continuous vector field on the n-dimensional sphere $\mathbf{S}^n$ if and only if n is odd.

- If Σ is a topological copy of $\mathbf{S}^{n-1}$ in $\mathbf{R}^n$, then $\mathbf{R}^n - \Sigma$ has precisely two connected components, one bounded, the other unbounded and with Σ the boundary of both (**Jordan-Brouwer Separation Theorem**).

Instructor: Greg Naber