

## BASIC INTEGRATION TECHNIQUES

IN CHAPTER 8 WE INVESTIGATE MORE SERIOUS "TECHNIQUES OF INTEGRATION" (FINDING ANTIDERIVATIVES) THAN THOSE WE HAVE SEEN SO FAR.

8.1 IS BASICALLY A LIST OF INTEGRATION FORMULAS AND SOME PROBLEMS THAT REVIEW WHAT WE ALREADY KNOW.

### BASIC TABLE OF INTEGRALS

$$1. \int dx = \int 1 dx = x + C$$

$$2. \int x^n dx = \frac{x^{n+1}}{n+1} + C$$

PROVIDED  $n \neq -1$

$$3. \int \frac{1}{x} dx = \ln|x| + C$$

$$4. \int e^x dx = e^x + C$$

$$5. \int b^x dx = \frac{b^x}{\ln b} + C$$

$$6. \int \sin x dx = -\cos x + C$$

$$7. \int \cos x dx = \sin x + C$$

$$8. \int \sec^2 x dx = \tan x + C$$

$$9. \int \csc^2 x dx = -\cot x + C$$

$$10. \int \sec x \tan x dx = \sec x + C$$

$$11. \int \csc x \cot x dx = -\csc x + C$$

$$12. \int \tan x dx = \ln|\sec x| + C$$

$$13. \int \cot x dx = \ln|\sin x| + C$$

$$14. \int \sinh x dx = \cosh x + C$$

15.  $\int \cosh x dx = \sinh x + C$

16.  $\int \operatorname{sech}^2 x dx = \tanh x + C$

17.  $\int \operatorname{csch}^2 x dx = -\operatorname{coth} x + C$

18.  $\int \operatorname{sech} x \tanh x dx = -\operatorname{sech} x + C$

19.  $\int \operatorname{csch} x \operatorname{coth} x dx = -\operatorname{csch} x + C$

20.  $\int \frac{1}{\sqrt{1-x^2}} dx = \operatorname{Arcsin} x + C$

21.  $\int \frac{1}{1+x^2} dx = \operatorname{Arctan} x + C$

22.  $\int \frac{1}{x\sqrt{x^2-1}} dx = \operatorname{Arcsec} x + C$

NOTE: WE ALSO DERIVED SOME ADDITIONAL  
FORMULAS INVOLVING INVERSE HYPERBOLIC FUNCTIONS,

E.G.,

$$\int \frac{1}{\sqrt{x^2+1}} dx = \sinh^{-1} x + C$$

$$= \ln(x + \sqrt{x^2+1}) + C.$$

SINCE WE WILL HAVE GENERAL METHODS FOR  
HANDLING ALL OF THESE SHORTLY, THEY  
NEED NOT BE MEMORIZED.

NOW, JUST FOR PRACTICE, WE'LL DO SOME EXAMPLES.

1.  $\int \sqrt{4+9x} dx$

$$u = 4+9x$$

$$du = 9 dx$$

$$\frac{1}{9} \int (4+9x)^{\frac{1}{2}} (9 dx) = \frac{1}{9} \int u^{\frac{1}{2}} du = \frac{1}{9} \cdot \frac{2}{3} u^{\frac{3}{2}} + C = \frac{2}{27} (4+9x)^{\frac{3}{2}} + C$$

$$2. \int 4x \tan(x^2) dx$$

$$u = x^2$$

$$du = 2x dx$$

$$2 \int \tan(x^2) (2x dx) = 2 \int \tan u du = 2 \ln |\sec u| + C \\ = 2 \ln |\sec(x^2)| + C$$

$$3. \int \frac{1}{4+9x^2} dx$$

$$\int \frac{1}{4(1+(\frac{3x}{2})^2)} dx = \frac{1}{4} \frac{2}{3} \int \frac{1}{1+u^2} du$$

$$u = \frac{3x}{2}$$

$$du = \frac{3}{2} dx$$

$$= \frac{1}{6} \arctan u + C$$

$$= \frac{1}{6} \arctan\left(\frac{3x}{2}\right) + C$$

$$4. \int \frac{\sec(\ln x) \tan(\ln x)}{x} dx$$

$$u = \ln x$$

$$du = \frac{1}{x} dx$$

$$\int \sec(\ln x) \tan(\ln x) \left(\frac{1}{x} dx\right) = \int \sec u \tan u du$$

$$= \sec u + C$$

$$= \sec(\ln x) + C$$

$$5. \int \frac{x}{\sqrt{1-x^4}} dx$$

$$u = x^2$$

$$du = 2x dx$$

$$\frac{1}{2} \int \frac{1}{\sqrt{1-(x^2)^2}} (2x dx) = \frac{1}{2} \int \frac{1}{\sqrt{1-u^2}} du = \frac{1}{2} \text{ARCSIN } u + C$$

$$= \frac{1}{2} \text{ARCSIN } (x^2) + C$$

$$6. \int \frac{e^{\arctan x}}{1+x^2} dx$$

$$u = \arctan x$$

$$du = \frac{1}{1+x^2} dx$$

$$\int e^{\arctan x} \frac{1}{1+x^2} dx = \int e^u du = e^u + C$$

$$= e^{\arctan x} + C$$

$$7. \int (3x+1) \cot(3x^2+2x) dx$$

$$u = 3x^2 + 2x$$

$$du = (6x+2) dx$$

$$= 2(3x+1) dx$$

$$\frac{1}{2} \int \cot(3x^2+2x) 2(3x+1) dx = \frac{1}{2} \int \cot u du$$

$$= \frac{1}{2} \ln |\sin u| + C$$

$$= \frac{1}{2} \ln |\sin(3x^2+2x)| + C$$

$$8. \int \frac{dx}{x \ln x}$$

$$u = \ln x$$

$$du = \frac{1}{x} dx$$

$$\int \frac{1}{\ln x} \frac{1}{x} dx = \int \frac{1}{u} du = \ln |u| + C = \ln |\ln x| + C$$

$$9. \int \sec(\sin \theta) \tan(\sin \theta) \cos \theta d\theta$$

$$u = \sin \theta$$

$$du = \cos \theta d\theta$$

$$\int \sec u \tan u du = \sec u + C = \sec(\sin \theta) + C$$

$$10. \int \frac{\cos(\ln x)}{x} dx$$

$$u = \ln x$$

$$du = \frac{1}{x} dx$$

$$\int \cos(\ln x) \frac{1}{x} dx = \int \cos u du = \sin u + C \\ = \sin(\ln x) + C$$

$$11. \int \frac{\sinh(x^{-\frac{1}{2}})}{x^{3/2}} dx$$

$$u = x^{-\frac{1}{2}}$$

$$du = -\frac{1}{2} x^{-3/2} dx$$

$$\begin{aligned} -2 \int \sinh(x^{-\frac{1}{2}}) (-\frac{1}{2} x^{-3/2} dx) &= -2 \int \sinh u du \\ &= -2 \cosh u + C \\ &= -2 \cosh(x^{-\frac{1}{2}}) + C \end{aligned}$$

$$12. \int \frac{e^x}{\sqrt{4-e^{2x}}} dx$$

$$\begin{aligned} \int \frac{1}{\sqrt{4-(\frac{e^x}{2})^2}} e^x dx &= \frac{1}{2} \int \frac{1}{\sqrt{1-(\frac{e^x}{2})^2}} e^x dx \\ u &= \frac{e^x}{2} \\ du &= \frac{1}{2} e^x dx \end{aligned}$$

$$\begin{aligned} &= \frac{1}{2} \cdot 2 \int \frac{1}{\sqrt{1-u^2}} du \\ &= \arcsin u + C \\ &= \arcsin\left(\frac{e^x}{2}\right) + C \end{aligned}$$

$$13. \int 2^{\pi x} dx$$

$$\begin{aligned} u &= \pi x \\ du &= \pi dx \end{aligned}$$

$$\frac{1}{\pi} \int 2^u du = \frac{1}{\pi} \frac{2^u}{\ln 2} + C = \frac{2^{\pi x}}{\pi \ln 2} + C$$