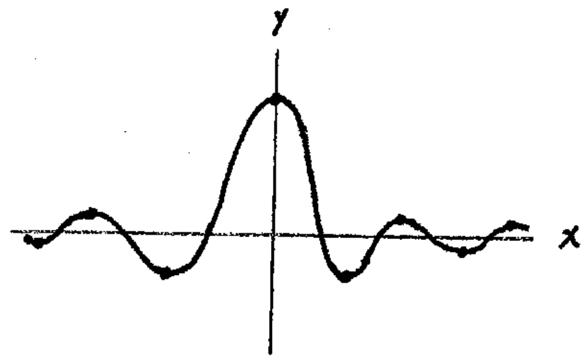
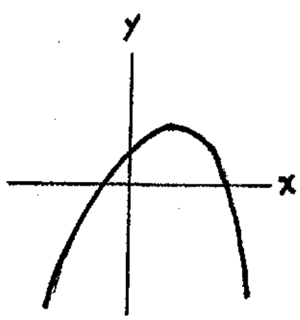
ABSOLUTE MAXIMA AND MINIMA



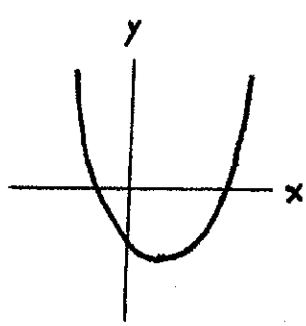
LOTS OF RELATIVE MAXIMA AND MINIMA. ONE "ABSOLUTE MAXIMUM." TWO "ABSOLUTE MINIMA."

<u>DEFINITION</u>: $f(x)$ HAS AN <u>ABSOLUTE</u> <u>MAXIMUM</u> AT $x = x_0$ IF $f(x) \leq f(x_0)$ FOR EVERY x IN THE DOMAIN OF f. $f(x)$ HAS AN <u>ABSOLUTE</u> <u>MINIMUM</u> AT $x = x_0$ IF $f(x) \geq f(x_0)$ FOR EVERY x IN THE DOMAIN OF f.

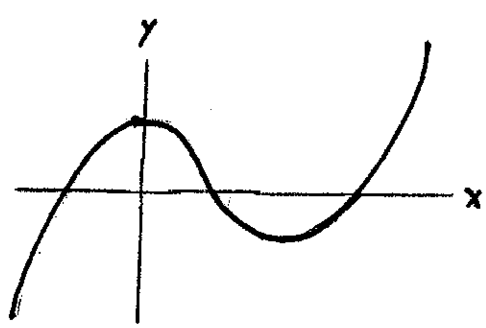
SOME FUNCTIONS HAVE THESE AND SOME DO NOT.



ABSOLUTE MAXIMUM
NO ABSOLUTE MINIMUM

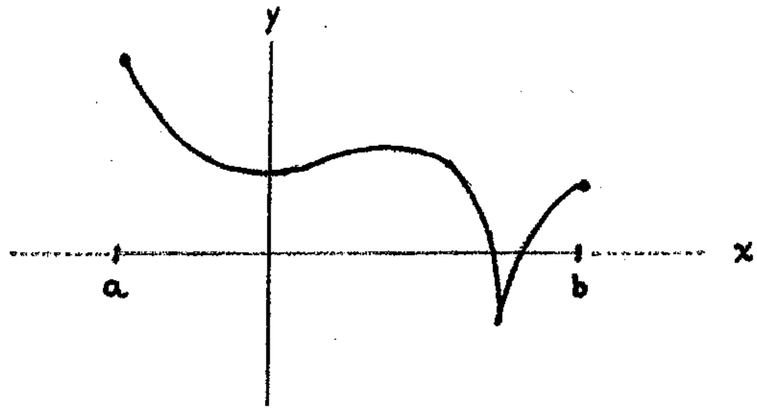


ABSOLUTE MINIMUM
NO ABSOLUTE MAXIMUM



NEITHER

THERE IS ONE SITUATION IN WHICH BOTH AN ABSOLUTE MAXIMUM AND AN ABSOLUTE MINIMUM MUST EXIST.



<u>THEOREM</u> : IF $f(x)$ IS CONTINUOUS ON A <u>CLOSED</u> INTERVAL $[a, b]$, THEN $f(x)$ HAS AN ABSOLUTE MAXIMUM AND AN ABSOLUTE MINIMUM ON $[a, b]$. THESE CAN ONLY OCCUR IN ONE OF TWO POSSIBLE LOCATIONS :

1. A CRITICAL POINT OF f IN THE <u>OPEN</u> INTERVAL (a, b).
2. ONE OF THE ENDPOINTS $x = a$ OR $x = b$ (WHICH NEED <u>NOT</u> BE CRITICAL POINTS).

<u>EXAMPLES</u> : FOR EACH OF THE FOLLOWING FIND THE ABSOLUTE MAXIMA AND ABSOLUTE MINIMA, IF THEY EXIST.

1. $f(x) = x^3 - 3x - 1$ ON $[0, 2]$

THE FUNCTION IS A POLYNOMIAL AND THEREFORE CONTINUOUS ON $[0,2]$.

MUST HAVE BOTH ABSOLUTE MAXIMUM AND ABSOLUTE MINIMUM.

POSSIBLE LOCATIONS:

1. CRITICAL POINTS IN $(0,2)$

$$f'(x) = 3x^2 - 3 = 3(x^2-1)$$
$$= 3(x-1)(x+1)$$

CRITICAL POINTS : $x=-1$, $x=1$

ONLY $x=1$ IS IN $(0,2)$

$$f(1) = 1^3 - 3(1) - 1$$
$$\boxed{f(1) = -3}$$

2. ENDPOINTS :

$x=0$: $\boxed{f(0) = -1}$

$x=2$: $\boxed{f(2) = 1}$

ABSOLUTE MAXIMUM VALUE OF 1 AT $x=2$.

ABSOLUTE MINIMUM VALUE OF -3 AT $x=1$

2. $f(x) = x^3 - 3x - 2$ ON $(-\infty, \infty)$

NOTE THAT

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} (x^3 - 3x - 2)$$

$$= \lim_{x \to \infty} x^3$$

$$= \infty$$

SO THERE CAN BE NO ABSOLUTE MAXIMUM.

SIMILARLY,

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} (x^3 - 3x - 2)$$

$$= \lim_{x \to -\infty} x^3$$

$$= -\infty$$

SO THERE CAN BE NO ABSOLUTE MINIMUM.

3. $f(x) = 3x^4 + 4x^3$ ON $(-\infty, \infty)$

THIS TIME

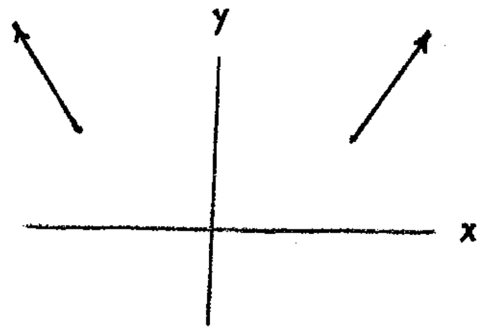
$$\lim_{x \to \infty} f(x) = \infty$$

AND

$$\lim_{x \to \infty} f(x) = \infty$$

SO THERE IS NO ABSOLUTE MAXIMUM.

SINCE THE GRAPH LOOKS LIKE



ON THE ENDS, IT ACHIEVES AN ABSOLUTE MINIMUM SOMEWHERE INBETWEEN (WHERE IT IS CONTINUOUS). THIS MUST OCCUR AT A CRITICAL POINT (WHERE IT TURNS AROUND).

$$f'(x) = 12x^3 + 12x^2$$
$$= 12x^2(x+1)$$

$$x = 0, -1$$

$$\boxed{f(0) = 0}$$

$$\boxed{f(-1) = -1}$$

ABSOLUTE MINIMUM VALUE OF -1 AT $x = -1$

4. $f(x) = \cos(\sin x)$ ON $[0, \pi]$

CRITICAL POINTS IN $(0, \pi)$: $f'(x) = -\sin(\sin x)\cos x$

$$f'(x) = 0$$
$$-\sin(\sin x)\cos x = 0$$
$$-\sin(\sin x) = 0 \quad \text{OR} \quad \cos x = 0$$