Multi-Model Adaptive Regulation with Disparate Zero Structures

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Introduction

- Safety critical systems must operate through degraded conditions & faults.
- Multiple Models address multiple plant configurations.
- The Design Problem: Given a range of plant parameters \( \theta \),
  - Partition the set of possible plants \( \mathcal{P} \).
  - Put controllers \( C \) s.t. each \( \mathcal{P}(\theta) \in \mathcal{P} \) is stabilized by at least one \( C \in \mathcal{C} \).
- Switch Design: A performance signal and logic to switch on a stabilizing controller \( C \).

Preliminaries

- This regulator design procedure applies to LTI systems:
  - plant states \( x = A_x x + B_u u \rightarrow y = C_x x \) outputs
equation system \( e = y - r \) error
- For the Laplace variable, \( s = \sigma + j \omega \), write the Transfer Function

\[
\frac{C(s)}{A(s)} = C(s)A(s)^{-1}B_u + E(s)\tilde{B}_u + G(s)\Gamma(s)
\]

- The zero structure changes for a (Transmission) Zero at \( \lambda \).
- A closed-loop regulator block diagram: \( \mathcal{C} \)

Plants & Partitions

- Nonlinear, parameter dependent system has linearized dynamics

\[
\begin{align*}
x &= f(x, u, \theta) \\
e &= h(x, \theta)
\end{align*}
\]

- Taylor Linearize \( \dot{x} = A_x x + B_u u + E_u\epsilon \)

At the equilibrium point \( p \in \mathbb{R}^n \), such that

\[
\begin{align*}
\end{align*}
\]

- Open-loop regulation equation solvability depends on \( \Gamma \)

\[
\Gamma(\theta) = \begin{bmatrix} X & U \end{bmatrix} - \begin{bmatrix} A_1 & B_1 \end{bmatrix} \begin{bmatrix} X \end{bmatrix} - \begin{bmatrix} E \end{bmatrix}
\]

- Locally \( E, F, M \) must lie in the range of \( X, U \) under \( \Gamma(\theta) \)

- A closed-loop regulator block diagram:

\[
\begin{align*}
\mathcal{C} &= \text{Internal Model} \\
\mathcal{S} &= \text{State Feedback} \\
\mathcal{A} &= \text{Aircraft}
\end{align*}
\]

- Closed Loop dynamics can be factored as

\[
\begin{align*}
\dot{x} &= \begin{bmatrix} A_1 + B_1 K_u B_{1u} & B_1 K_u & B_{1u} & K_2 \\
0 & 0 & 0 & 0 \end{bmatrix} \\
\theta &= \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}
\end{align*}
\]

- Stability is lost when an eigenvalue, \( \lambda(\Gamma(\theta)) \), changes sign \([1]\)

- A Hybrid (Discrete state) form of adaptive regulation is needed.

Control Covering: Multiple Model Adaptive Control

- Continuous Adaptive Control
- Multiple Model Adaptive Control

Control Covering: Comparison with Prior Work

- Matrix Norm Robustness:
  - Frequency Domain [2]
  - Lyapunov Functions [3]

- Our Approach:
  - Identify structural impediments to continuous adaptation
  - "cover" disjoint sets with multiple controllers

Control Covering: Set of Controllers \( \mathcal{C} \)

- Two subproblems remain:
  1. Find a \( K \) for each \( C \).
  2. Find a performance metric to identify a stabilizing \( C \).

- One Linear Matrix Inequality obtains both \( K \) and \( P \) in polynomial time:

\[
\begin{align*}
\text{Consider the system} & \quad x = A x + B u \\
\text{Minimize the output energy} & \quad \int_0^t z^T z\, dt
\end{align*}
\]

- By solving the riccati equation

\[
A^T P + PA + C^T C - PB_1 (D_1^T D_2)^{-1} B_1^T P = 0
\]

- The two subproblem solutions are thus

1. The state feedback gain is \( K = - (D_1^T D_2)^{-1} B_1^T P \).
2. Performance metric is a Common Quadratic Lyapunov function \( V_i = x^T P_i x \)

Switch Logic: Generalized Energy Performance Signal

- \( \frac{d}{dt} V < 0 \) implies stability
- \( V \) is \( \geq 0 \) with Ellipsoidal Level Sets
- \( V = x^T P x \approx E \) Energy, and is scalar

\[
\begin{align*}
\text{Above: Level Sets } V_i = \alpha_i (\text{green}) & \quad \text{state trajectory (red)} \\
\text{Right: Lyapunov Level Sets } V_i(x)
\end{align*}
\]

Switch Logic & Control Covering: Linear Matrix Inequality (LMI)

- An abstraction of the longitudinal aircraft dynamics \([4]\),

\[
A = \begin{bmatrix} p_1 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}, \quad D = 0
\]

- Solve two LMIs for \( CGLP \) and control gain \( K \): \( \det \Gamma(\theta) = 0 \)

- Our Approach:
  - Cover disjoint sets with multiple controllers

Example: Aircraft Flight Path Regulation

- Significant change:
  - \( \theta \) the angle of attack and pitch damping, and pitch stiffness resp.
  - \( \theta_1, \theta_2, \theta_3 \) are unknown constants, \( -5 < \theta_1 < 5, -5 < \theta_2 < 5 \)
  - At \( \theta_3 = 0 \), zero at \( s = 0 \), a pole at the origin, \( B \) is uncontrollable.

- Structural change: \( \theta \in [1, 4] \Rightarrow \Gamma(0) \) or \( \theta \in [-1, -4] \Rightarrow \det \Gamma(0) > 0 \)

- Solve two LMIs for \( CGLP \) and control gain \( K \):

Conclusions

- Zero structure changes partition regulator design for plant families.
- Multiple Model Adaptation accommodates changes in system structure.
- An LMI obtains optimal gain and a useful performance metric.
- Future research will focus on switch logic improvements.

References