I build a duopoly model of multi-product firms which interact repeatedly in two countries, separated by trade costs, to study the impact of collusion on trade and welfare. Each firm produces two goods but has a competitive cost advantage in one. In this framework, an international cartel can reduce costs by shutting down production of the cost-inefficient good and importing it instead, or by shutting down trade and producing both goods domestically. At the same time, a cartel can extract surplus by exploiting its market power. The efficiency gains from rationalizing production and trade can dominate the deadweight losses due to monopoly pricing. There always exists a set of marginal cost differences and transportation costs under which maximal collusion welfare-dominates Cournot competition. Furthermore, depending on costs and the degree of product differentiation, collusion can even promote trade relative to competition. Using extensive data on 173 international cartels, my empirical analysis reveals that the average effect of cartels on trade is positive and significant. Moreover, the impact of multi-product cartels on trade is positive, significant and statistically larger than the effect of single-product cartels. Lastly, the positive impact of multi-product cartels on trade is more pronounced when goods are sufficiently distant substitutes, in line with the theoretical model.

**JEL Classification:** D43, F10, F12, F13, F15, F42, L12, L13, L41  
**Keywords:** multi-market, multi-product cartels, self-enforcing collusion, trade costs

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“cartels, and particularly international cartels, are a true scourge of the world economy.”

Joel I. Klein, Assistant Attorney General, Antitrust Division, Department of Justice

1 Introduction

Since 1990 the European Commission has discovered and prosecuted 122 international cartels, imposing fines of about €26 billion and shedding light on the nature of their collusive practices. One common characteristic of such cartels is that they are comprised of multi-product firms that differ in productivities. One notable example is the ‘great global vitamins conspiracy’ (Connor, 2006) when major vitamins producers entered a price-fixing and market-sharing agreement during the 1990s. The participants produced a number of vitamin products and diet supplements, such as Vitamin A, E, C, D3, H, Folic Acid, Thiamine, Riboflavin, Calpan, Beta Carotene. The vitamins industry is highly concentrated and the manufacturing process requires considerable production experience (which acts as a barrier to entry). Moreover, as Kovacic et al. (2007) explain: “Although the major producers have similar production technologies, the chemical synthesis processes involve substantial ‘learning by doing.’ Each producer becomes better, through time, at debottlenecking the chemical synthesis process at any given plant.” Furthermore, the actual list of multi-product cartels is extensive, yet we know little about the effects of such cartels on output, trade or national welfare. Especially when the cartel members exhibit different production efficiencies.

The above quote from Joel I. Klein exemplifies the general perception about cartels. However, the Organization of Economic Co-Operation and Development (OECD) suggests that there might be collusive agreements that generate certain positive economic effects:

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3More information on the characteristics of modern international cartels are available in Agnosteva (2017) Data Appendix.

“Some horizontal agreements between companies can fall short of a hard core cartel, and in certain cases may have beneficial effects. For example, agreements between competitors related to research & development, production and marketing can result in reduced costs...”

In this paper, I seek to show that not all cartels are created equal and that not every cartel should be considered “a true scourge of the world economy.” Specifically, I focus on international cartels comprised of multi-product firms with differing technologies to address the following questions. First, is it possible for multi-product, multi-market cartels to promote international trade relative to Cournot-Nash competition? I examine this issue both theoretically and empirically using an extensive dataset on international cartels. Second, are there circumstances under which multi-product, multi-market cartels can enhance welfare (total surplus) relative to Cournot competition? Third, are such collusive agreements sustainable and can they overcome changes in trade costs or changes in firms’ production costs?

To address these questions, I build a model of two firms, each located in a different country, producing two goods and competing in quantities internationally. Marginal costs of production are constant, but heterogeneous across goods and across firms. Specifically, each firm produces one of the goods at a lower marginal cost than its competitor and, therefore, enjoys a competitive advantage in the production of that good. Moreover, exporting products from one country to the other incurs per-unit trade costs.

Under Cournot competition, the duopolists may produce and export even their respective high-cost goods as they do not internalize the cost inefficiency and thus also inflict a pecuniary externality onto each other. Under collusion, each firm always produces its low-cost good domestically. To supply the high-cost product, the cartel decides whether to import it from the country that has a competitive advantage in it or to produce it locally. If the cost of producing the high-cost good domestically exceeds the cost of importing it, then under the cartel agreement, each country specializes completely in the production of its efficient

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5 More information is available at http://www.oecd.org/competition/cartels/

6 Henceforth, “competition” and “monopoly” are used interchangeably for short with the phrases “Cournot-Nash competition” and “unconstrained maximal collusion,” respectively. Moreover, throughout the analysis, I use monopoly, maximal collusion and unconstrained collusion interchangeably.
good and exports it, while importing the inefficient one. If the trade costs exceed the cost heterogeneity, the cartel shuts down trade and manufactures both goods domestically. Each cartel member internalizes the cost inefficiency and never exports his respective high-cost good as it is subject to both higher production costs and trade costs. By rationalizing production of the inefficient good, collusion raises profits and also generates efficiency gains.

The first novel insight of the model is that multi-product, multi-market cartels can promote trade of the cost-efficient good. Under collusion, each cartel member always supplies its competitive-advantage good in positive quantities domestically. If the cost of producing the inefficient good locally exceeds the cost of importing it, then the cartel will foreclose its production. Moreover, if the products are sufficiently unrelated, shipping more of one good will exert only a negligible negative externality onto the marginal revenue of the other. On the other hand, under competition, the firms do not internalize the pecuniary externality they inflict upon each other and can trade even their respective cost-inefficient goods. Collusion can provide a mechanism for more efficient production and distribution of both goods and can enhance trade relative to competition. This interesting result stands in sharp contrast with previous work on multi-market cartels (Bond and Syropoulos, 2008).

A second finding is that collusion can also enhance national welfare relative to competition. While the cartel generates efficiency gains by shutting down inefficient plants and foreclosing on costly trade, it also reduces consumer surplus by exploiting its market power. I identify conditions on trade costs and the degree of product differentiation that ensure that the welfare gain due to improved production efficiency under collusion can dominate the loss in consumer surplus. I analyze welfare when trade costs take the form of transportation costs in the main text, while the case of import tariffs is discussed in details in Appendix A.

If the cost heterogeneity is sufficiently larger than transportation costs and the goods are sufficiently distant substitutes, collusion can be welfare-superior to competition. The former condition allows for complete specialization in production and exports of the low-cost good.

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7The case of import tariffs needs to be examined separately as tariffs generate revenues, which further contribute to national welfare.
under collusion, while the latter ensures collusion delivers sufficient welfare gains. If the cost heterogeneity is sufficiently smaller than the transportation costs, then collusion improves welfare relative to competition for any degree of substitutability. In this scenario, the cartel shuts down the costly international exchange and produces both goods domestically in each country. Again, collusion welfare-dominates competition as the welfare gain due to higher profits more than offsets the lower consumer surplus.\footnote{Although different in method, Deltas et al. (2012) also find that cartels can improve welfare relative to competition. They use a duopoly model of horizontal differentiation à la Hotelling, where markets are separated by per-unit trade costs and the firms face identical constant marginal production costs. In this setting, Deltas et al. (2012) study the equilibrium outcomes under price competition and maximal collusion. They show that the cartel can enhance welfare as it reduces the waste of resources in the form of unnecessary transportation costs. In their model, collusion can even increase consumer surplus as long as it decreases the price of the efficiently produced domestic good so as to keep a fraction of the marginal consumers.}

These welfare implications of multi-product collusion remain valid even if trade costs take the form of import tariffs, but with some caveats.\footnote{In standard segmented-markets duopoly models with \textit{perfect substitutes} welfare under monopoly might exceed welfare under competition if trade costs only take the form of transportation costs, but not of tariffs. Here, the possibility of welfare-enhancing collusion exists regardless of the type of trade costs considered.} Collusion can be welfare-superior to competition only when each country specializes completely in the production of its competitive-advantage good. The welfare gain due to enhanced production efficiency and (possibly) larger tariff revenues more than compensates for the loss of consumer surplus due to higher prices. However, if the tariff level exceeds the cost heterogeneity, complete geographic separation is attained with both goods being produced only for domestic consumption and not exported. The higher cartel profits fail to offset the loss of tariff revenues and the lower consumer surplus. Further, there exists a threshold level of the degree of substitutability above which competition is weakly welfare-superior to collusion in the case of tariffs.

Collusion in this model is the result of repeated firm interactions over an infinite time horizon. The literature has already established that repeated interactions over time allow firms to sustain collusion by acting cooperatively until one of them deviates and retaliation ensues afterwards (Friedman, 1986). Multi-market contact can facilitate cartel agreements by allowing members to pool their incentive constraints across markets (Bernheim and Whinston, 1990). Multi-market contact increases the frequency of firm interactions and alleviates
any existing asymmetries across firms. Furthermore, the Folk theorem stipulates that if firms value future profits highly, maximal collusion is sustainable. Nevertheless, to confirm the validity of the results, I verify the stability of the cartel agreement in this framework and study its dependence on trade costs and the cost asymmetry in details in Appendix A.

The prediction of the model about the implications of collusion for trade depends crucially on the parameters. With the help of a novel dataset, which combines information on a large sample of international cartels and bilateral trade at the most disaggregated level available, I provide a reduced-form analysis of the impact of collusion on trade. The data cover 173 private discovered and prosecuted international cartels, which operated between 1958 and 2010 and included participants from 48 countries. However, the sample size for this study is largely determined by the availability of the trade data at the 6-digit Harmonized System (HS) product level and therefore spans between 1988 and 2012 and focuses on OECD countries only. About 37% of the cartels in the final dataset involve more than one 6-digit HS product. The data include information on the countries of nationality of the cartel members, the duration of the infringement, the product code of each of the cartelized goods, various instruments of collusion and cartel characteristics. Appendix B describes the data.

To test whether collusion actually enhances trade, I employ the most successful model in the empirical trade literature, the gravity equation, and estimate it using the Poisson Pseudo Maximum Likelihood (PPML) estimator, standard proxies for trade costs (such as distance, common language, contiguous border, colonial ties, RTAs), as well as a rich set of fixed effects. First, I analyze the average treatment effect of collusion on trade and find it to be positive and significant, suggesting that on average the existence of cartels promotes trade between cartel members. Second, I focus specifically on the impact of multi-product cartels and offer evidence that collusion between multi-product firms exerts a positive and significant effect on trade. This effect is statistically larger (at the 1% confidence level) than the impact of single-product cartels on trade.

Third, I test whether the effects of multi-product cartels and single-product cartels differ
depending on the degree of product substitutability. To proxy for the degree of substitutability, I employ Rauch’s (1999) classification of goods in the main specifications and experiment with Broda and Weinstein’s (2006) elasticity estimates in the sensitivity tests. In line with the theory, I find that the positive and significant effect of multi-product cartels on trade becomes much more pronounced for differentiated goods relative to homogeneous goods. On the other hand, the impact of multi-product collusion on trade when the goods are relatively closer substitutes is still positive, but insignificant. The effect of single-product cartels on trade does not vary with the degree of substitutability. These results are robust to a series of sensitivity experiments, where I use different sets of fixed effects, an alternative proxy for product substitutability, as well as different year intervals. The results are consistent with the model’s predictions and provide empirical evidence that multi-product international cartels can promote trade between members, especially when the goods are sufficiently unrelated.

This paper contributes to several strands of the existing literature. First, I add to the literature examining the impact of economic integration on multi-market collusion by focusing on cartels comprised of multi-product firms, characterized with heterogeneous marginal production costs. Second, I contribute to works studying the effects of cost asymmetries on collusive behavior. However, in the model presented in this paper, differences in marginal costs are coupled with heterogeneity in trade costs, which offers novel insights about the implications of collusion for trade and welfare. Third, I add to the scant empirical literature on collusion by employing an extensive dataset on international cartels to analyze the predictions of the model. The following section provides a more detailed literature review.

2 Related Literature

The literature has studied the link between international cartels and bilateral trade mostly from a theoretical standpoint. Due to the lack of data, the empirical literature on international cartels has been either purely descriptive, providing anecdotal evidence of certain
collusive practices, or has focused on single episodes of collusion and studied them in depth.

A large body of the literature studies the impact of trade policy on multi-market collusion. Pinto (1986) extends Brander and Krugman’s (1983) model by allowing for repeated firm interactions and demonstrates that for certain values of the discount factor there will be no trade if firms choose the monopoly output. Lommerud and Sørgard (2001) construct a homogeneous-good duopoly model where collusion is sustained through trigger strategies and analyze the effects of trade costs on cartel stability. Schröder (2007) focuses on ad valorem tariffs and fixed trade costs and shows that trade liberalization can be detrimental to cartel stability. Bond and Syropoulos (2008) build a homogeneous-good segmented-markets duopoly model, but allow quantity-setting firms to pool their incentive constraints. They show that cartel agreements may support the emergence of intra-industry trade and economic integration may enhance collusive stability and reduce welfare. However, in these models cartels can neither promote trade, nor improve national welfare relative to competition.

Bond and Syropoulos (2012) develop a multi-country and multi-firm, homogeneous-goods Cournot model and study the impact of trade liberalization on multi-market collusion. Akinkosoye, Bond, and Syropoulos (2012) focus on a model of price competition with differentiated goods and find that if goods are close substitutes and trade costs are high, trade liberalization facilitates collusion. Ashournia et al. (2013) employ a segmented-markets differentiated-goods Cournot duopoly model and reveal that reductions in trade costs enhance collusion as long as the initial level of trade costs is sufficiently low. Bhattacharjeya and Sinha (2015) explore multi-market collusion in homogeneous-good price-setting framework with territorial allocation of markets. Agnosteva, Syropoulos and Yotov (2016) incorporate a third country to the standard segmented-markets Cournot duopoly model and show that

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when the incentive constraint is binding, all markets become strategically linked. Regional economic integration need not necessarily improve welfare in all markets and the absence of intra-union trade might be welfare-superior to free trade. Deltas et al. (2012) adopt a Hotelling model of horizontal differentiation and find that collusion can raise both national welfare and consumer surplus relative to price competition. In contrast, I focus on multi-market collusion between multi-product firms with cost heterogeneity and show that international cartels can both enhance national welfare and promote trade.\footnote{Abstracting from the repeated firm interactions component of the model, I can view collusion as a cross-border merger between firms. Neary (2002) develops a tractable two-country, multi-sector, homogeneous-good Cournot oligopoly model in general equilibrium to study patterns of trade and specialization. The model is later used in Neary (2003) and Neary (2007). Neary (2003) shows that trade liberalization might lead to profitable cross-border mergers when a low-cost firm acquires a high-cost competitor. Neary (2007) focuses on a single industry, where domestic and foreign firms exhibit exogenous cost differences and shows that takeovers of a high-cost firm by a low-cost foreign competitor are more likely the more concentrated the market in both countries. Further, Neary (2007) proves that the effect of cross-border mergers on aggregate welfare is likely to be positive. My partial equilibrium analysis does not examine the incentives for the formation of international mergers (or international cartels in my case). Nevertheless, I show that a change from duopoly to monopoly (whether due to collusion or a merger) can enhance national welfare (and also promote bilateral trade in my model) when firms exhibit heterogeneity in (production and trade) costs.}

The literature also analyzes heterogeneity in firms’ costs and its potential effects on collusion. The general consensus is that asymmetry in cost functions hinders collusive stability as it makes the more efficient firm difficult to discipline (Bain, 1948; Scherer, 1980; Tirole, 1988). Bae (1987) and Harrington (1991) also find that cost heterogeneity obstructs collusion even if inefficient allocations are allowed to equalize firms’ short-term deviation gains. On the other hand, heterogeneity in firms’ marginal costs has been considered a justification for a merger between competitors in the same market.\footnote{The U.S. Horizontal Merger Guidelines recognize these (potential) positive effects of mergers and stipulate: “a primary benefit of mergers to the economy is their potential to generate significant efficiencies” (https://www.ftc.gov/sites/default/files/attachments/merger-review/100819hmg.pdf)} Such mergers can increase prices by reducing competition, but can also give rise to efficiency gains and cost savings (Williamson, 1968). Nonetheless, the literature has not yet explored the impact of cost heterogeneity on collusion sustained across multiple segmented markets, separated by trade costs.

The empirical literature on international cartels lags behind its theoretical counterpart. Most of the related empirical studies either examine the determinants of collusive stability
using hazard models\textsuperscript{13} or focus on single cartel episodes, where price and quantities data are available.\textsuperscript{14} A few case studies offer a descriptive overview of interesting collusive practices and cartel characteristics.\textsuperscript{15} The papers most closely related to the empirical analysis presented here are Levenstein et al. (2015) and Agnosteva, Syropoulos and Yotov (2016). Levenstein et al. (2015) concentrate on only seven international cartels and first analyze the impact of collusive dissolution on prices and industry concentration, using a difference-in-difference approach, and then study the effects of cartel break-up on trade. Their results suggest that cartel death exerts little or no significant effects on the spatial patterns of trade. Agnosteva, Syropoulos and Yotov (2016) offer a two-step estimation procedure to first examine the impact of a number of organizational features, trade costs and trade liberalization proxies on collusive stability, and then analyze the relation between cartel existence, collusive discipline and bilateral trade. This paper, on the other hand, investigates the effect of multi-product cartels on trade and its dependence on the degree of product substitutability. Before presenting the empirical analysis, the next section describes the theoretical model, its main predictions along with the intuition behind these results.

3 Theoretical Analysis

In this section I first present the model of two multi-product firms, which interact repeatedly in national markets separated by per-unit trade costs and face heterogeneous marginal production costs. I define the global profit of the representative firm in the case of Cournot-Nash competition, maximal collusion, and optimal deviation. Then, I describe the conditions under which maximal collusion can promote trade relative to Cournot competition. I also analyze the circumstances under which collusion can welfare-dominate competition. Lastly, I discuss the validity of these results when the goods are imperfect substitutes.

\textsuperscript{13}See Levenstein and Suslow (2011), Zhou (2012).


3.1 The Model

The model features two multi-product firms, which both produce two goods, indexed \( k = 1, 2 \), for possible sale in two countries, labeled \( \text{home} (h) \) and \( \text{foreign} (f) \). The domestic firm manufactures its products in the domestic market, while the foreign firm produces its products in the foreign market. Henceforth, “∗” will denote all variables pertaining to deliveries to the foreign market. Firms face marginal production costs \( c_h^k \) and \( c_f^k \) for \( k = 1, 2 \).

Without loss of generality, suppose that the firms have “mirror image” costs: that is, good 1 is less costly to produce for the domestic firm \( (c_h^1 < c_h^2) \), while good 2 is less costly to produce for the foreign firm \( (c_f^1 > c_f^2) \). Also, assume that \( c_1^f = c_2^f < c_1^h = c_2^h \). For simplicity, define \( s \equiv c_2^h - c_1^h = c_1^f - c_2^f \) and let \( c_1^f = c_2^f = 0 \). It follows that \( s = c_2^h = c_1^f \). All other aspects of the two countries be identical. The sunk costs that must be incurred in setting up production and exports are exogenous. Let the per-unit trade costs, \( t \), be the costs of delivering goods from one country to the other. Firms interact in quantities and view markets as segmented.

Let \( Q_k \equiv x_k + y_k \) be the total quantity of good \( k \) consumed in the home country, where \( x_k \) \( (x_k^*) \) denotes the quantity supplied locally by the domestic (foreign) firm of good \( k \) and \( y_k \) \( (y_k^*) \) denotes the quantity exported by the foreign (home) firm of good \( k \) to the home (foreign) market. Each consumer in the domestic market has the same quasi-linear utility function over the two goods: \( U = u(Q_1, Q_2) + q_0 \), where \( q_0 \) denotes consumption of the numeraire good and \( u(Q_1, Q_2) \) takes the following form:\(^16\)

\[
u(Q_1, Q_2) = \alpha(Q_1 + Q_2) - \frac{1}{2}(Q_1^2 + Q_2^2) - \gamma Q_1 Q_2
\]

where that \( \alpha > 0 \) and \( 0 \leq \gamma \leq 1 \). In this setting, \( \alpha \) denotes consumer’s maximum willingness to pay, while \( \gamma \) is an index of degree of product substitutability: if \( \gamma = 0 \), the products are independent; if \( \gamma = 1 \), the products are perfect substitutes. Optimization in consumption yields the following inverse demand functions for \( Q_k \geq 0, k = 1, 2 \):

\[
p_1(Q_1, Q_2) = \max(0, \alpha - Q_1 - \gamma Q_2)
\]

\(^{16}\)This utility function abstracts from industry income effects and allows for partial equilibrium analysis.
\[ p_2(Q_1, Q_2) = \max(0, \alpha - Q_2 - \gamma Q_1) \]

The inverse demand functions for the foreign market have the same functional form, \( p_k(\cdot) = p_k^*(\cdot) \) for \( k = 1, 2 \). Let \( \Pi \) denote the global profit function of the domestic firm:\textsuperscript{17}

\[ \Pi = [p_1(Q_1, Q_2)]x_1 + [p_1^*(Q_1^*, Q_2^*) - t]y_1^* + [p_2(Q_1, Q_2) - s]x_2 + [p_2^*(Q_1^*, Q_2^*) - s - t]y_2^* \tag{2} \]

To simplify the presentation, in the analysis to follow I assume the goods are completely unrelated (i.e., \( \gamma = 0 \)).\textsuperscript{18} The representative firm’s best-response functions are given by:\textsuperscript{19}

\[ \tilde{x}_1 \equiv \tilde{x}_1(y_1, s) = \left( \frac{\alpha - y_1}{2} \right) \tag{3a} \]

\[ \tilde{x}_2 \equiv \tilde{x}_2(y_2, s) = \max \left( \frac{\alpha - y_2 - s}{2}, 0 \right) \tag{3b} \]

\[ \tilde{y}_1^* \equiv \tilde{y}_1^*(x_1^*, s, t) = \max \left( \frac{\alpha - x_1^* - t}{2}, 0 \right) \tag{3c} \]

\[ \tilde{y}_2^* \equiv \tilde{y}_2^*(x_2^*, s, t) = \max \left( \frac{\alpha - x_2^* - s - t}{2}, 0 \right) \tag{3d} \]

The Cournot-Nash equilibrium quantities in each market can be derived by solving both firms’ best-response functions simultaneously. Given the symmetry of the model and the fact that firms regard national markets as segmented, it suffices to focus on output decisions only in one market. Henceforth, I study home production, \( x_k \), and home imports, \( y_k \), of good \( k = 1, 2 \). Let subscript ‘\( N \)’ identify ‘\( Nash \)’ quantities for the domestic market and suppose, for a moment, that \( t \) and \( s \) are such that all output levels are positive. Then,

\[ x_1^N = \frac{\alpha + s + t}{3} \tag{4a} \]

\[ y_1^N = \frac{\alpha - 2s - 2t}{3} \tag{4b} \]

\[ x_2^N = \frac{\alpha - 2s + t}{3} \tag{4c} \]

\textsuperscript{17}Given the symmetry of the model, the firm subscripts can be omitted to avoid cluttering the notation.

\textsuperscript{18}The proofs in Appendix A allow for the possibility that goods may be imperfect substitutes.

\textsuperscript{19}Firm’s best response functions for good \( k \) do not depend on the quantity of the other good \( l \neq k \), as shown in (3a)-(3d), due to the assumption of a linear-quadratic utility in the non-numeraire good as in [1].
Several observations regarding the Cournot-Nash deliveries stand out. First, trade liberalization ($t \downarrow$) leads to an expansion of imports ($y_k \uparrow$) and a reduction in domestic production ($x_k \uparrow$). Second, reciprocal reductions in trade costs ($t \downarrow$) bring about an increase in domestic availability of every good as $\partial Q_k^N / \partial t < 0$. Third, increases in the cost heterogeneity ($s \uparrow$) induce the firm to expand its total output of the good in which it has a competitive advantage (i.e., $\partial (x_1^N + y_1^N) / \partial s > 0$ and $\partial (x_2^N + y_2^N) / \partial s > 0$) and to reduce its output of the other good (i.e., $\partial (x_2^N + y_2^N) / \partial s < 0$ and $\partial (x_1^N + y_1^N) / \partial s < 0$).

A closer inspection of the Cournot-Nash equilibrium allocations in (4a)-(4d) reveals that the firm might cease trade if trade costs are prohibitively high. Fig. 1 helps visualize the idea. For sufficiently low values of trade costs as in regions $A$ and $A'$ all output levels are positive. However, suppose trade costs are intermediate as in regions $B$ and $B'$. In that case, the representative firm no longer finds it profitable to ship its respective high-cost good abroad. Using (4b), it follows that $y_1^N = 0$ for $t \geq \bar{t}_y \equiv \frac{\alpha - 2s}{2}$.

But if trade costs are even higher, as in region $C'$ of Fig. 1, then neither firm imports any of the goods (not even their respective cost-efficient product) and each country is in a state of autarky, producing both goods for the domestic market only. That is, when $t \geq \bar{t}_x(s) = \frac{\alpha + s}{2}$ with $\bar{t}_y < \bar{t}_x$ then $x_1^N = \frac{\alpha}{2}$, $x_2^N = \frac{-s}{2}$, and $y_1^N = y_2^N = 0$.

Similarly, if the cost asymmetry is initially negligible, then in both countries both goods are produced for domestic consumption and for foreign exports. However, if the cost differential takes some intermediate levels as in regions $B$ and $B'$ of Fig. 1 then the firm stops trading its respective high-cost goods. Specifically, if $s \geq \bar{s}_y(t) = \frac{\alpha - 2t}{2}$, then $y_1^N = 0$, $x_1^N = \frac{\alpha}{2}$,

$$y_2^N = \frac{\alpha + s - 2t}{3}. \tag{4d}$$
Lastly, if the cost heterogeneity is excessively high, $s \geq \bar{s}_x(t) = \frac{\alpha + t}{2}$, neither firm will produce its cost-inefficient good even for the domestic market. In that case, each country completely specializes in the production of its low-cost good and exports it, while importing the other product from abroad: $x_1^N = \frac{\alpha}{2}, y_2^N = \frac{\alpha - t}{2}, x_2^N = y_1^N = 0$. This possibility is illustrated in region $C$ of Fig. 1. Lastly, the symmetry between the relevant regions in Fig. 1 with respect to the 45\degree-degree line is only present because the goods are completely unrelated, i.e. $\gamma = 0$. In this case there is no essential distinction between the two sources of heterogeneity within a firm.

Utilizing the equilibrium output allocations (4a)-(4d), the Nash profit of the representative firm in the home market takes the following form:

$$\Pi^N(t,s) = \begin{cases} 
(\frac{\alpha + s + t}{3})^2 + (\frac{\alpha + s - 2t}{3})^2 + (\frac{\alpha - 2s - 2t}{3})^2 & \text{if } t < \bar{t}_y \\
(\frac{\alpha}{2})^2 + (\frac{\alpha + s - 2t}{3})^2 + (\frac{\alpha - 2s + 2t}{3})^2 & \text{if } \bar{t}_y \leq t < \bar{t}_x \\
(\frac{\alpha}{2})^2 + (\frac{\alpha - s}{2})^2 & \text{if } t \geq \bar{t}_x \\
(\frac{\alpha + s + t}{3})^2 + (\frac{\alpha + s - 2t}{3})^2 + (\frac{\alpha - 2s + 2t}{3})^2 & \text{if } s < \bar{s}_y \\
(\frac{\alpha}{2})^2 + (\frac{\alpha + s - 2t}{3})^2 + (\frac{\alpha - 2s + 2t}{3})^2 & \text{if } \bar{s}_y \leq s < \bar{s}_x \\
(\frac{\alpha}{2})^2 + (\frac{\alpha - t}{2})^2 & \text{if } s \geq \bar{s}_x 
\end{cases}$$

$\Pi^N(t,s)$ is non-monotonic in $t$ ($s$) for $t < \bar{t}_y$ and $\bar{t}_y \leq t < \bar{t}_x$ ($s < \bar{s}_y$ and $\bar{s}_y \leq s < \bar{s}_x$).

Within each interval of $t$ values, as trade costs rise, domestic profits for the representative firm increase, while export profits decrease. The former effect dominates at high initial values of $t$, while the latter effect prevails at low initial values of $t$. Within each interval of $s$ values, as the cost heterogeneity increases, profits from each firm’s respective cost-efficient good rise, while profits from each firm’s respective cost-inefficient good fall. The former effect is stronger at high levels of $s$, while the latter effect is dominant at low levels of $s$. As Lemma 1 explains, depending on the parameters, $\Pi^N(t,s)$ can have multiple local minima.

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23 Using (4b), define the threshold value of the cost heterogeneity such that $y_1^N = 0$: $\bar{s}_y(t) = \frac{\alpha - 2t}{2}$.

24 Clearly, the level of $s$, which makes exports of the costlier good unprofitable will be lower than the level of $s$, which makes domestic production of the same good unprofitable, $\bar{s}_y < \bar{s}_x$.

25 The derivation of the Nash equilibrium for each possible range of trade cost and cost differential values is described in more details in the beginning of Appendix A.
Lemma 1. (Global Profit under Competition) The representative firm’s global profit under Cournot-Nash competition, $\Pi^N(t,s)$, has the following properties:

a) For any given level of cost differential, $s \in [0, \alpha]$,
   
i) there exists a $t^1_{min} \equiv \arg \min_t \Pi^N(t,s) = \frac{2a-s}{10} \in \left[0, \bar{t}_y\right)$ if $s < \frac{\alpha}{3}$ and a $t^2_{min} \equiv \arg \min_t \Pi^N(t,s) = \frac{\alpha+4s}{5} \in \left[\bar{t}_y, \bar{t}_x\right)$ if $s \geq \frac{\alpha}{3}$;
   
ii) $\frac{\partial \Pi^N}{\partial t} \leq 0$ if $t \leq t^1_{min} \in \left[0, \bar{t}_y\right)$ or if $t \leq t^2_{min} \in \left[\bar{t}_y, \bar{t}_x\right)$. Moreover, $\frac{\partial^2 \Pi^N}{\partial t^2} > 0$ for $t \in \left[0, \bar{t}_x\right)$.

b) For any given level of trade costs, $t \in [0, \alpha]$,
   
i) there exists a $s^1_{min} \equiv \arg \min_s \Pi^N(t,s) = \frac{2a-t}{10} \in \left[0, \bar{s}_y\right)$ if $t \in \left[0, \frac{2\alpha}{5}\right)$ and a $s^2_{min} \equiv \arg \min_s \Pi^N(t,s) = \frac{a+4t}{5} \in \left[\bar{s}_y, \bar{s}_x\right)$ if $t \in \left[\frac{\alpha}{5}, \frac{\alpha}{2}\right)$;
   
ii) $\frac{\partial \Pi^N}{\partial s} \leq 0$ if $s \leq s^1_{min} \in \left(0, \bar{s}_x\right)$ or if $s \leq s^2_{min} \in \left[\bar{s}_y, \bar{s}_x\right)$. Moreover, $\frac{\partial^2 \Pi^N}{\partial s^2} > 0$ for $s \in \left(0, \bar{s}_x\right)$.

Next, I examine the profit under collusion, $\Pi^C$, of the representative cartel member. Due to the symmetry of the model, the cartel can be viewed as making decisions to maximize profits in a given market. Thus, it suffices to focus on the home market only:

$$\Pi^C = [\alpha - (x_1 + y_1)](x_1 + y_1) + [\alpha - (x_2 + y_2)](x_2 + y_2) - t \times (y_1 + y_2) - s \times (y_1 + x_2)$$  \hspace{1cm} (6)$$

Cartel members may be able to sustain the most collusive outcome and act as a monopolist in both markets if they value future profits sufficiently. Moreover, as the Folk Theorem stipulates, there always exist discount factors high enough to sustain maximal collusion.

The monopolist decides how much of each good to produce in each market for domestic consumption and for exports, taking into consideration cost asymmetries and trade costs. Under the assumed structure of costs, the cartel will never find it profitable to export the goods that either firm produces inefficiently, $y_1^M = 0$ and $y_2^M = 0$. In the case of free trade, $t = 0$, the cartel produces good 1 (2) only in home (foreign) for domestic and export purposes and the output levels of both goods are equal: $x_1^M = y_2^M = \frac{\alpha}{2}$. The cartel assigns production according to each country’s competitive advantage and complete specialization is attained.

However, when trade costs are non-zero, the cartel again rationalizes production and trade and chooses to either produce both goods in both markets (if trade costs exceed the
cost heterogeneity as in regions $A'$, $B'$ and $C'$ of Fig. 1 or produce good 1 in the domestic market and good 2 in the foreign market and export them to the other country (if the cost differential exceeds trade costs as in regions $A$, $B$ and $C$ of Fig. 1). If $t < s$, the cartel imports the high-cost good from the country that enjoys a competitive advantage in it rather than producing it domestically. In that case, the cartel manufactures good 1 only at home and good 2 only in foreign and trades them internationally. The optimal output allocations are $x^M_1 = \frac{\alpha}{2}$, $y^M_2 = \frac{\alpha - t}{2}$ and $x^M_2 = y^M_1 = 0$. If $s < t$, then the cartel forecloses on the costly international exchange and complete geographic separation of markets is attained. The cartel produces both goods domestically in both markets and the optimal output levels are: $x^M_1 = \frac{\alpha}{2}$, $x^M_2 = \frac{\alpha - s}{2}$ and $y^M_2 = y^M_1 = 0$. When $t = s$, the cartel is indifferent between producing the cost-inefficient good at home or importing it from foreign and the optimal solution is a correspondence: $x^M_1 = \frac{\alpha}{2}$, $x^M_2 + y^M_2 = \frac{\alpha - s}{2} = \frac{\alpha - t}{2}$ and $y^M_1 = 0$. Regions $A$, $B$, and $C$ of Fig. 1 depict the case of $t < s$, while regions $A'$, $B'$, and $C'$ illustrate the case of $s < t$. The optimal global profit for the monopolist is:

$$
\Pi^M = \left\{ \begin{array}{ll}
\left( \frac{\alpha}{2} \right)^2 + \left( \frac{\alpha - t}{2} \right)^2 & \text{if } t < s \\
\left( \frac{\alpha}{2} \right)^2 + \left( \frac{\alpha - s}{2} \right)^2 & \text{if } s < t \\
\left( \frac{\alpha}{2} \right)^2 + \left( \frac{\alpha - t}{2} \right)^2 & \text{if } t = s
\end{array} \right. \quad (7)
$$

as long as $t < \bar{t}_M$ and $s < \bar{s}_M$. First, if $t < s$, then $\Pi^M(t)$ is decreasing and strictly convex in $t$ and independent of $s$. Second, if $s < t$, then $\Pi^M(s)$ is decreasing and strictly convex in $s$ and independent of $t$. Third, the minimum of $\Pi^M(t)$ (resp., $\Pi^M(s)$) approaches $\min(s, \alpha)$ (resp., $\min(t, \alpha)$). Profits under maximal collusion are higher under free trade than under complete geographic separation, i.e. $\Pi^M(t = 0, s) > \Pi^M(t = s, s)$ for any values of $s$. $\Pi^M$ is at its highest level when $t = s = 0$. Lemma 2 provides a more detailed characterization:

**Lemma 2.** (Global Profit under Monopoly) The global profit of the representative cartel member under maximal collusion, $\Pi^M$, has the following properties:

a) If $t < s$, then $\frac{\partial \Pi^M(t)}{\partial t} < 0$ and $\frac{\partial^2 \Pi^M(t)}{\partial t^2} > 0$, with $\arg\min_t \Pi^M(t) \to s$ and $\arg\max_t \Pi^M(t) = 0$.

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26 From the optimal output allocations, it follows that the prohibitive levels of $t$ and $s$ in the case of maximal collusion are given by $\bar{t}_M = \alpha (y^M_2 = 0)$ and $\bar{s}_M = \alpha (x^M_2 = 0)$. 

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Moreover, $\Pi^M$ is independent of $s$, $\frac{\partial \Pi^M(t)}{\partial s} = 0$.

b) If $s < t$, then $\frac{\partial \Pi^M(s)}{\partial s} < 0$ and $\frac{\partial^2 \Pi^M(s)}{\partial s^2} > 0$, with $\arg\min_s \Pi^M(s) \to t$ and $\arg\max_s \Pi^M(s) = 0$. Moreover, $\Pi^M$ is independent of $t$, $\frac{\partial \Pi^M(t)}{\partial t} = 0$.

Suppose that home’s cartel partner supplies $(x^*_k, y_k)$ for any good $k = 1, 2$. In that case, the best-response component functions for the domestic firm are given in equations (3a)-(3d). The profit under an optimal deviation for the home firm consists of the profit gain from deviating in home plus the profit gain from deviating in foreign. To define home’s best-response global profit, $\bar{\Pi} = \Pi^D$ (superscript ‘$D$’ for ‘deviation’), $I$ (once again due to symmetry) focus solely on the domestic market:

$$\Pi^D = \bar{x}_1^2 + \bar{x}_2^2 + \bar{y}_1^2 + \bar{y}_2^2 \tag{8}$$

First, the deviation profit is decreasing and strictly convex in the outputs $(x_k, y_k)$ for the firm which abides by the cartel agreement. Second, $\Pi^D$ is decreasing and strictly convex in $(t, s)$ in the individual regions identified in Fig. 1 but not necessarily convex for all $(t, s)$ in $(0, \alpha) \times (0, \alpha)$. Third, decreases in trade costs ($t \downarrow$) tend to increase the best-response export volume of the deviating firm, while increases in the cost differential ($s \uparrow$) tend to increase (decrease) the best-response output of the cost-efficient (cost-inefficient) good. Lastly, the best-response output levels as well as the net prices are well-defined for all values of $s$ and $t$ described in Fig. 1 when the goods are completely unrelated.

Before presenting the main predictions of the model, it is useful to discuss whether maximal collusion is even sustainable in this setting. Generally, cartel agreements are unstable due to the tempting profits that any member could earn if he deviates from the agreement. However, Friedman (1986) demonstrates that in the context of infinite firm interactions, collusion is sustainable through ‘grim’ trigger strategies, stipulating continued adherence to the prescribed output and reversion to the Cournot competitive equilibrium upon violation of the agreement. As firms interact repeatedly over time, they pool their incentive constraints
to sustain collusion (Bernheim and Whinston, 1990). Multi-market contact strengthens the cartel agreement as it increases the frequency of firm interactions and alleviates any existing asymmetries between cartel members. The Folk theorem suggests that as long as firms value future profits considerably, full collusion is sustainable and, therefore, the results that follow hold true. Nevertheless, to verify the stability of the collusive outcome and the validity of these findings, I analyze the cartel’s constrained optimization problem. I derive the minimum discount factor, capable of sustaining collusion, and then examine its dependence on trade costs and the cost differential. The analysis is provided in Appendix A.

Using the main features of the model just described, I proceed to examine the impact of unconstrained collusion on trade relative to Cournot-Nash competition. Then, I study the levels of welfare under the two types of market structure and derive conditions under which multi-product, multi-market cartels can improve national welfare.

3.2 Collusion and Trade

A key characteristic of the model is that the cartel, by rationalizing production and trade, may choose to shut down production of the inefficient good in the home country and import it, while exporting the efficient good to the foreign market. Thus, maximal collusion enhances production efficiency, but could it also promote trade relative to the least collusive outcome? To address this question, I compare the volume of trade under collusion with the volume of trade under competition under the assumption that the goods are completely unrelated. Depending on the initial level of trade costs and the cost differential, collusion can lead to such an arrangement that exports of the cost-efficient good from the country, which has a competitive advantage in it, be higher than if the firms act non-cooperatively. Proposition 1 summarizes the conditions under which this result is valid.

**Proposition 1.** (*Volume of Trade*) If the trade cost level $t$ is less than the cost differential $s$ (i.e., $t < s$), then the volume of trade of the cost-efficient good under maximal collusion will exceed the volume of trade of the same product under Cournot-Nash competition (i.e.,
\( y_2^M > y_2^N \) and \( y_1^M > y_1^N \).

(Recall that for the time being, I establish this result under the assumption that the goods are completely unrelated.) This finding stands in sharp contrast to previous models of multi-market collusion and offers a novel insight on the impact of international cartels on trade. Essentially, the cartel weighs the cost of producing the inefficient good domestically (i.e., \( s \)) and the cost of importing that good (i.e., \( t \)). If the cost heterogeneity exceeds trade costs \( (t < s) \), under collusion each country specializes according to its competitive advantage, producing the cost-efficient good only and exporting it to the other market. Moreover, when the goods are completely unrelated, importing more of the cost-inefficient good exerts no negative effect on the marginal revenue of the domestically produced good. On the other hand, under Cournot competition, the duopolists produce and export both goods (unless trade costs or the cost asymmetry exceed specific threshold levels) as they do not internalize the cost inefficiency and thus also generate a pecuniary externality for each other.

How does the insight described in Proposition change if the goods are imperfect substitutes (i.e., \( \gamma > 0 \))? This novel result holds true even if the goods are imperfect substitutes as long as the degree of product substitutability is not too high. More specifically, in Appendix A, I show that there exists a threshold level of the degree of substitutability, \( \hat{\gamma} \), such that for any \( \gamma \in [0, \hat{\gamma}) \) exports of the cost-efficient good are greater under collusion than under competition. As long as the goods are sufficiently unrelated (i.e., \( \gamma < \hat{\gamma} \)), importing more of the high-cost good will exert a negligible negative effect on the marginal revenue of the domestically produced good. On the other hand, under Cournot competition, the duopolists do not internalize the negative price externality that they inflict upon each other as they engage in intra-industry trade. Thus, multi-product, multi-market cartels can enhance trade of the competitive-advantage product relative to Cournot competition as long as the goods are sufficiently distant substitutes.

Graphically, collusion improves trade relative to competition for values of \( t \) and \( s \) in regions \( A \) and \( B \) of Fig. For any intermediate values of \( \gamma \), this result continues to be
valid as long as the two goods are sufficiently unrelated, $\gamma < \hat{\gamma}$. Otherwise, the opposite result holds true. Further, under the same conditions, the value of trade under maximal collusion can be greater than the value of trade under Cournot competition, as shown in Appendix A. Thus, by rationalizing production and trade, the cartel can enhance trade of the cost-efficient good relative to the least collusive outcome.

To the best of my knowledge, this is the first paper to show that international cartels can promote trade. However, this result depends crucially on the interaction between trade costs, the cost heterogeneity and the degree of product substitutability. Therefore, in Section 4, I examine this possibility empirically, using a novel dataset on cartels and trade. Defying the conventional wisdom, I find that multi-product cartels exert a positive and significant impact on trade and that this effect is even more pronounced for relatively unrelated goods. Before discussing the empirical analysis, I study the welfare implications of multi-market, multi-product cartels and demonstrate that collusion can also improve national welfare.

3.3 Collusion and Welfare

The model thus far shows that multi-product international cartels can enhance production efficiency and promote trade. In this section, I examine the welfare effects of such collusive agreements. First, I define national welfare and characterize it in the case of Cournot-Nash competition and maximal collusion. Second, I compare welfare under collusion with welfare under competition when the goods are completely unrelated. Third, I provide a discussion of the validity of the results in the presence of imperfect substitutes.

3.3.1 The Case of Unrelated Goods

To simplify the presentation of the results, henceforth I assume that trade costs take the form of transportation costs.\[^{27}\] In this framework, welfare in the domestic country is given by $W = CS + \Pi$, where $CS$ and $\Pi$ capture consumer surplus and global profit, respectively.

\[^{27}\]Appendix A characterizes welfare under Cournot-Nash competition in the case of import tariffs.
It is easy to verify that $CS = \frac{1}{2}(Q_1^2 + Q_2^2)$. Utilizing these observations in the definition of welfare and simplifying expressions generates the welfare function for home:

$$W = \alpha (Q_1 + Q_2) - \frac{1}{2} (Q_1^2 + Q_2^2) - s \times (Q_2) - t \times (y_1 + y_2) \quad (9)$$

I characterize the dependence of welfare under Cournot-Nash competition, denoted by $W^N$, on trade costs and the cost differential.\footnote{By symmetry, the above expressions also describe welfare in the foreign country.} Welfare under competition, $W^N$, is non-monotonic in $t$ for $0 \leq t < \tilde{t}_y$ and $\tilde{t}_y \leq t < \tilde{t}_x$, as trade costs are resource-using (Brander and Krugman, 1983).\footnote{For low levels of $t$, $\partial W^N / \partial t < 0$, while for high levels of $t$, $\partial W^N / \partial t > 0$. This is driven by the non-monotonicity of $\Pi^N$ in $t$.} When transportation costs are in the neighborhood of the prohibitive level, the waste of resources due to cross-hauling of goods dominates the reduction in prices due to more competition. If transportation costs are in the neighborhood of free trade, the pro-competitive effect outweighs the resource costs. Depending on the values of the parameters, $W^N$ can exhibit multiple local minima in trade costs for $t < \tilde{t}_x$.\footnote{In contrast, in the standard model of reciprocal dumping, the welfare function under Cournot-Nash competition does not exhibit multiple local minima. This novel finding is driven by the interaction between the two sources of heterogeneity in the model.}

Welfare is non-monotonic in the production cost differential for $0 \leq s < \bar{s}_y$ and $\bar{s}_y \leq s < \bar{s}_x$. Increases in the cost heterogeneity lead to higher production of the efficient good and lower production of the inefficient good, with the effect on total output being negative. As the cost asymmetry expands, consumer surplus falls, while profits fall (rise) if the initial level of $s$ is low (high), hence the non-monotonicity of $W^N$ in $s$.\footnote{Similarly as for $t$, for low levels of $s$, $\partial W^N / \partial s < 0$, while for high levels of $s$, $\partial W^N / \partial s > 0$. Again, this result is due to the non-monotonicity of $\Pi^N$ in $s$.}

Lemma 3. \textit{(Welfare in Nash Equilibrium)} Under Cournot-Nash competition, national welfare depends on transportation costs and the cost differential as follows:
a) \( \arg \min_t W^N(t, s) = \frac{2(2\alpha-s)}{11} \in [0, \bar{t}_y) \) if \( s < \frac{\alpha}{6} \); \( \arg \min_t W^N(t, s) = \frac{4\alpha+7s}{11} \in [\bar{t}_y, \bar{t}_x) \) if \( s \geq \frac{\alpha}{12} \;
abla
\)

b) \( \arg \min_s W^N(t, s) = \frac{2(2\alpha-t)}{11} \in (0, \bar{s}_y) \) if \( t < \frac{\alpha}{6} \); \( \arg \min_s W^N(t, s) = \frac{4\alpha+7t}{11} \in [\bar{s}_y, \bar{s}_x) \) if \( t \geq \frac{\alpha}{12} \;
abla
\)

c) \( W^N(0, s, t) > W^N(\bar{t}_x, s, t) \).

The relations documented in Lemma 3 are driven by the main model assumptions: segmented national markets, constant marginal costs, and firms interacting over a single period. What is of greater interest, however, is how welfare under competition compares to welfare under collusion. Before addressing this question, I first characterize welfare under unconstrained collusion in the case of transportation costs.

**Lemma 4.** (Welfare under Maximal Collusion) Under maximal collusion, national welfare in the case of transportation costs, \( W^M \), has the following properties:

a) If \( t < s \), then \( \frac{\partial W^M(t)}{\partial t} < 0 \) and \( \frac{\partial^2 W^M(t)}{\partial t^2} > 0 \), with \( \arg \min_t W^M(t) \rightarrow s \) and \( \arg \max_t W^M(t) = 0 \). Moreover, \( W^M(t) \) is independent of \( s \), \( \frac{\partial W^M(t)}{\partial s} = 0 \).

b) If \( s < t \), then \( \frac{\partial W^M(s)}{\partial s} < 0 \) and \( \frac{\partial^2 W^M(s)}{\partial s^2} > 0 \), with \( \arg \min_s W^M(s) \rightarrow t \) and \( \arg \max_s W^M(s) = 0 \). Moreover, \( W^M(s) \) is independent of \( t \), \( \frac{\partial W^M(s)}{\partial t} = 0 \).

Lemma 4 first reveals that if \( t < s \), then \( W^M(t) \) is decreasing and strictly convex in \( t \) and independent of \( s \). Second, if \( s < t \), then \( W^M(s) \) is decreasing and strictly convex in \( s \) and independent of \( t \). Third, welfare under unconstrained collusion is maximized under free trade or when there are no cost asymmetries.

Next, I compare the levels of welfare under maximal collusion and Cournot competition. The cartel internalizes the cost inefficiency and always produces the low-cost good domestically, while shutting down trade of each country’s respective high-cost good. Depending on the cost differential and the trade costs, under collusion, the cost-inefficient product is either imported from the country that has a competitive advantage in it or produced domestically. Thus, by rationalizing production of the costlier good, the cartel increases profits and at the same time generates efficiency gains. This raises the question of how collusion affects welfare. Monopoly can welfare-dominate competition if transportation costs are sufficiently

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32 Appendix A characterizes welfare under maximal collusion in the case of import tariffs.
larger than the cost differential. Collusion can also welfare-dominate Cournot duopoly if the
cost heterogeneity is sufficiently greater than the trade costs, as described in Proposition 2

**Proposition 2.** (Welfare Comparison – Transportation Costs) Suppose the two goods are com-
pletely unrelated and trade costs take the form of transportation costs. Then, for any \( t \in [0, \alpha) \)
(resp., \( s \in [0, \alpha) \)), there exists a range of values of the cost differential \( s \) (resp., trade cost \( t \)), such
that \( 0 \leq t < s < \bar{s}_x \) (resp., \( 0 \leq s < t < \bar{t}_x \)), which imply collusion welfare-dominates competition
(i.e., \( W^M(t, s) > W^N(t, s) \)). In the former case, under collusion, firms specialize completely in the
good they produce most efficiently and trade it internationally. In contrast, under competition firms
may produce and trade even their inefficient good. In the latter case, under collusion, firms produce
both goods domestically and do not engage in international trade. In contrast, under competition
firms may again produce and trade their inefficient good as well.

Fig. 2 illustrates Proposition 2. The regions in pink depict the combinations of \((t, s)\) under which collusion welfare-dominates competition, while the green sets describe the com-
binations for which the opposite result holds true. The regions in blue identify the \((t, s)\) pairs
for which the welfare levels coincide. When goods are unrelated and trade costs take the
form of transportation costs, the behavior of the welfare functions is symmetric relative to
the 45°-line (where \( t = s \)) because there is no essential distinction between the two sources
of heterogeneity within a firm. The intuition behind Proposition 2 can be summarized as
follows. First, in region \( A \) of Fig. 2 the differential in marginal costs \( s \) is so high as compared
to transportation cost \( t \) that colluding firms find it appealing to replace domestic production
of the good they produce inefficiently with trade. The efficiency gains from shutting down
production more than offset the loss in consumer surplus due to higher prices. This result is
valid both when under competition the firms trade both goods \((t < \bar{t}_y)\) and when they trade
only the cost-efficient good \((\bar{t}_y \leq t < \bar{t}_x)\), as long as the cost asymmetry is substantially
large. In this case, for any transportation cost, \( t \in [0, \alpha) \), there exists a sufficiently high
value of the cost differential such that collusion welfare-dominates competition.\(^{33}\) Second,

\(^{33}\)The exact threshold values of \( t \) and \( s \), which allow maximal collusion to welfare-dominate Cournot
competition are derived in Appendix A in the proof of Proposition 2.
for intermediate values of transportation costs and the cost differential, as in region \( B \) of Fig. 2, the cartel’s efficiency advantage is not strong enough to compensate for the greater consumer burden and competition provides a greater level of welfare.

Third, collusion can be welfare-superior to competition for any value of the cost differential, \( s \in (0, \alpha) \), as long as transportation costs are sufficiently high, as shown in region \( A' \) of Fig. 2. In this case, the cost heterogeneity is negligible relative to transportation costs and competition, albeit providing welfare gains in the form of lower prices, is less cost-efficient than unconstrained collusion. The cartel eliminates the waste of resources in the form of additional transportation costs and produces both goods domestically in each market. As trade costs rise, exporting both products becomes even costlier under competition and profits and consumer surplus decline. Under collusion, however, profits and consumer surplus are independent of transportation costs. For such high levels of trade costs relative to the cost heterogeneity, the positive change in profits dominates the negative change in consumer surplus as the market structure switches from competition to collusion, and the cartel improves national welfare.\(^{34}\) When either transportation costs or the cost asymmetry are prohibitively high (i.e., \( t \in [\bar{t}_x, \alpha) \) or \( s \in [\bar{s}_x, \alpha) \)), each firm is a monopolist in its own market and the levels of welfare are identical under both collusion and competition.

Import Tariffs. The welfare analysis rests on the assumption that trade costs take the form of transportation costs. In the case of import tariffs, some of the welfare results are still valid with crucial caveats. When trade costs take the form of tariffs, collusion welfare-dominates competition if the cost heterogeneity is sufficiently greater than the heterogeneity in trade policy. In this case, the cartel specializes completely in the production of the low-cost good in each market and imports the high-cost one, generating efficiency gains and enjoying larger profits. Moreover, if the initial tariff level is not infinitesimal, collusion creates larger tariff revenues than competition. Thus, under maximal collusion, the reduction in consumer surplus due to cartel prices is more than offset by the rise in profits and tariff revenues.\(^{34}\)

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\(^{34}\)This result holds true when both goods are traded under competition (\( t < \bar{t}_y \)) and when the duopolists only export the cost-efficient good (\( \bar{t}_y \leq t < \bar{t}_x \)), as long as transportation costs are sufficiently high.
Under competition, on the other hand, the firms do not internalize the pecuniary externality that they inflict upon each other and engage in intra-industry trade of either both goods or at least of the cost-efficient one. Therefore, in this scenario, maximal collusion welfare-dominates Cournot competition. In contrast, if tariffs exceed the cost asymmetry, the cartel shuts down trade and produces both goods domestically and no longer accumulates tariff revenues. Under competition, the firms continue cross-hauling at least their respective low-cost good and earn tariff revenues. Thus, the lack of tariff revenues and the loss in consumer surplus under the collusive regime outweigh the gain in profits and collusion fails to enhance welfare relative to competition. Appendix A provides a more detailed characterization of the welfare implications of collusion in the presence of import tariffs.

3.3.2 The Case of Imperfect Substitutes

The analysis so far abstracts from the possibility that the goods can be imperfect substitutes. However, imperfect substitutability has important implications for collusive behavior and welfare. It is well-known from previous literature (Bond and Syropoulos, 2008) that welfare under collusion does not dominate welfare under competition for values of trade costs below the prohibitive level, if the marginal costs of producing the two goods are identical and the goods are imperfect substitutes ($W^M(t, s = 0, \gamma < 1) \not\succ W^N(t, s = 0, \gamma < 1)$). Bond and Syropoulos (2008) find that, depending on the degree of product substitutability, welfare under maximal collusion in the case of free trade can be lower than at the prohibitive level of trade costs. Deltas et al. (2012) show that collusion can improve not only national welfare, but also consumer surplus in a model with horizontal differentiation à la Hotelling. In this framework, imperfect substitutability affects the cartel’s production decisions as the colluding firms internalize the pecuniary externality that they inflict upon each other.

Even when the assumption of unrelated goods is relaxed (i.e., $\gamma > 0$), welfare under

35In standard segmented-markets duopoly models with perfect substitutes welfare under monopoly can exceed welfare under competition if trade costs only take the form of transportation costs, but not of import tariffs. In this model, the possibility of welfare-enhancing collusion exists regardless of the type of trade costs considered, as shown in the text and in Appendix A.
collusion can still exceed welfare under competition for certain values of trade costs and the
cost differential. When trade costs take the form of transportation costs there always exists a
set of \((t, s)\) that ensures collusion welfare-dominates competition. In this case, region \(A\) from
Fig. 2 disappears as the degree of substitutability rises, but region \(A'\) persists, as shown in
Figs 3–4. Maximal collusion welfare-dominates competition for any value of \(\gamma \in (0, 1)\) and
any \(s \in (0, \alpha)\), as long as transportation costs are sufficiently higher than the cost differential.
Under collusion no trade takes place and both goods are produced only domestically. On
the other hand, under competition the duopolists trade either both goods \((t < t_y)\) or only
the cost-efficient good \((t_y \leq t < t_x)\) as they do not internalize the pecuniary externality
they inflict upon each other. Moreover, the higher the degree of product substitutability,
the stronger this negative price externality that producing more of one good exerts onto the
marginal revenue of the other and the lower the profits under competition. Thus, the cartel
generates efficiency gains, eliminates unnecessary transportation costs, and enjoys higher
profits, which more than compensate for the lower consumer surplus (due to cartel prices) at
any degree of substitutability. Figs 3–4 provide a graphical illustration of this interesting
result for \(\gamma = 0.4\) and \(\gamma = 0.9\).

Import Tariffs. In contrast, when trade costs take the form of tariffs, there exists a
threshold value of the degree of substitutability, \(\hat{\gamma} \in (0, 1)\), beyond which competition weakly
welfare-dominates collusion for all values of the cost heterogeneity and all tariff levels. As the
goods become closer substitutes, producing more of any one good inflicts a stronger negative
externality onto the the marginal revenue of the other. Therefore, profits, consumer surplus
and tariff revenues fall, with the decline in cartel profits and tariff revenues being much more
pronounced. The exact threshold value of \(\hat{\gamma}\) as well as a detailed analysis of welfare in the
presence of imperfect substitutes and import tariffs is provided in Appendix A.

To the best of my knowledge, this is the first paper to present a model, where multi-market
cartels can both improve national welfare and promote bilateral trade. This possibility
depends on the interaction between the trade costs and the cost differential, as well as on the
degree of product substitutability. In the next section, I analyze the impact of international cartels on trade empirically and offer evidence for the existence of trade-promoting cartels.

4 Empirical Analysis

The theoretical model offers a testable prediction about the impact of multi-product cartels on trade. *Proposition 1* stipulates that depending on the initial levels of trade costs and the cost differential, it is possible for collusion to enhance the value of trade of the cost-efficient good relative to competition as long as the goods are sufficiently distant substitutes. Specifically, when trade costs are lower than the cost heterogeneity and the goods are sufficiently unrelated, trade of the cost-efficient good under the collusive regime can be greater than trade of the same good under the competitive regime. In this case, the colluding firms do not produce the high-cost good in their respective home markets and do not export it at all, so the value of trade of that product would be greater under competition than under collusion. Therefore, the possibility that collusion enhances trade relative to competition depends crucially on the values of trade costs, the cost heterogeneity, and the degree of product substitutability. Thus, whether multi-product cartels promote bilateral trade or not is really an empirical question, which I seek to address in this part of the analysis. *Proposition 1* characterizes this result in more detail and translates into the following testable hypotheses:

**H1**: Multi-product cartels enhance the value of bilateral trade of their cost-efficient product relative to the absence of collusion.

**H2**: The positive effect of multi-product cartels on trade is more pronounced, the more distant substitutes the goods are.

In the empirical analysis, I focus on the realistic case of positive trade flows between country-cartel-members, which corresponds to the situation of trade costs being lower than the cost asymmetry in the theoretical model. The theory also predicts that when the above conditions
are satisfied, neither country exports its cost-inefficient product under collusion. Taking this result at face value implies that the data should contain only exports of cartel members’ competitive-advantage goods. Moreover, the empirical test of the hypotheses needs to be performed with data that: 1) include multi-product cartels; 2) focus on exports of cost-efficient goods, comparing changes over time in the presence and absence of collusion; 3) distinguish between unrelated and substitutable products. Next, I summarize the econometric approach and then offer empirical evidence that multi-product cartels indeed enhance trade between partners when the goods are sufficiently distant substitutes.

4.1 Econometric Strategy

To examine the relation between multi-product collusion and bilateral trade, I employ an augmented version of the gravity equation, which provides a systematic way of removing sources of unobserved heterogeneity:\(^36\)

\[
X_{ij,t}^{k,g} = \exp[\beta_1 CARTEL_{MLTPRD}^{k,g}_{ij,t} + \beta_2 CARTEL_{SNGLPRD}^{k,g}_{ij,t} + GRAV_{ij,t} + FES^g_{ij,t} + \epsilon_{ij,t}^{k,g}]
\]

where \(X_{ij,t}^{k,g}\) represents the nominal bilateral trade flows between \(i\) and \(j\) at time \(t\) for cartel \(k\) in product category \(g\). I test hypothesis \(H1\) by estimating equation (10), where the binary variable \(CARTEL_{MLTPRD}^{k,g}_{ij,t}\) takes the value of 1 when country \(i\) and country \(j\) both participate in a multi-product cartel \(k\), whereas the binary variable \(CARTEL_{SNGLPRD}^{k,g}_{ij,t}\) takes the value of 1 when country \(i\) and country \(j\) both participate in a single-product cartel \(k\) in sector \(g\). I expect the coefficient estimate of \(\beta_1\) to be positive and significant, but I have no \textit{ex-ante} expectations about the estimate of \(\beta_2\) based on the theoretical model.\(^37\)

\(GRAV_{ij,t}\) represents the vector of standard proxies for bilateral trade costs used in the gravity literature. These include the logarithm of bilateral distance between \(i\) and \(j\), \(DIST_{ij}\); a dummy variable for the existence of a common language between \(i\) and \(j\), \(LANG_{ij}\); another

\(^{36}\)Head and Mayer (2014) provide a detailed description of the theoretical foundations and the empirical applications of the gravity model.

\(^{37}\)Bond and Syropoulos (2008) build a single-product multi-market Cournot-duopoly model of collusion and show that in that case monopoly always hinders trade relative to competition.
dummy variable for the presence of a contiguous border between $i$ and $j$, $CNTG_{ij}$; a binary variable for the existence of colonial ties between $i$ and $j$, $CLNY_{ij}$. In addition, I also control for the existence of regional trade agreements between country $i$ and country $j$ also with a binary variable, $RTA_{ij,t}$. These variables do not vary across cartels and product categories.

$FES^g_{ij,t}$ denotes the vector of fixed effects that control for observed and unobserved heterogeneity. $\eta_{i,t}$ denotes time-varying source country dummies; $\delta_{j,t}$ denotes time-varying destination country dummies; $\mu_{g,t}$ denotes product-year fixed effects. Ideally, I need to include sector-exporter-time and sector-importer-time fixed effects as the theoretical gravity model indicates. However, due to the large number of countries, years and products, I encounter computational issues and therefore I experiment with various sets of fixed effects.

Next, I interact the above binary variables for multi-product and single-product collusion with proxies for the degree of product substitutability to test hypothesis $H2$. The theoretical model predicts that multi-product cartels can enhance trade as long as the degree of substitutability is sufficiently low. I employ Rauch’s (1999) classification of goods to proxy for the degree of product substitutability. More details about the data are provided in the following section. Building on (10), the next estimating equation becomes:

$$X^{k,g}_{ij,t} = \exp(\beta_1 CARTEL_{MLTPRD}^{k,g}_{ij,t} + \beta_2 CARTEL_{SNGLPRD}^{k,g}_{ij,t} + \beta_3 CARTEL_{MLTPRD}^{k,g}_{ij,t} \times SUBSTIT^g_{ij,t} + \beta_4 CARTEL_{SNGLPRD}^{k,g}_{ij,t} \times SUBSTIT^g_{ij,t} + \beta_5 SUBSTIT^g_{ij,t} + \text{GRAV}_{ij,t} \tilde{\theta} + FES^g_{ij,t}) + \epsilon^{k,g}_{ij,t}$$

The main coefficient of interest in specification (11) is $\beta_3$. The theory predicts that multi-product cartels enhance trade (of the cost-efficient good) as long as the degree of product substitutability is sufficiently low. That is, the higher the degree of substitutability between goods, the lower the positive impact of multi-product collusion on trade. Therefore, I expect

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38 Rauch (1999) divided goods into three categories - commodities, reference priced goods, and differentiated goods - based on whether they were traded on organized exchanges, were listed as having a reference price, or could not be priced by either of these means. More information about this proxy for product substitutability is provided in Appendix B.
the estimate of $\beta_3$ to be negative and significant, while the estimate of $\beta_4$ to be either insignificant or of lower order of magnitude. However, the interpretation of the estimate of $\beta_3$ (and the estimate of $\beta_4$) depends crucially on the particular proxy used for the elasticity of substitution. The additional variable $\text{SUBSTIT}^g$ denotes a proxy for the degree of substitutability and is included to avoid omitted variable bias.

To proxy for the degree of substitutability between products, I use Rauch’s (1999) categorical variable, which classifies goods into homogeneous, reference-priced, and differentiated. I create three separate binary variables based on Rauch’s classification of goods - one for homogeneous products, one for reference-priced products, and another one for differentiated products. Then, I interact each one of these variables with $\text{CARTEL}_{MLTPRD}^{k,g}_{ij,t}$. To avoid perfect collinearity, I omit the most substitutable category - the one for homogeneous goods. I compare the estimates of $\text{CARTEL}_{MLTPRD}^{k,g}_{ij,t} \times \text{SUBSTIT}^g$ for differentiated goods and for reference-priced products. Given the prediction of Proposition 1, the coefficient estimate on the interaction between $\text{CARTEL}_{MLTPRD}^{k,g}_{ij,t}$ and the dummy for differentiated products, $\text{SUBSTIT}_1^g$ should be positive and significant. In the sensitivity experiments, I employ the elasticity of substitution estimates of Broda and Weinstein (2006) as an alternative proxy for the degree of product substitutability.

As recommended by Santos-Silva and Tenreyro (2006), I estimate specifications (10)-(11) in multiplicative form using the Poisson Pseudo Maximum Likelihood (PPML) estimator, which accounts for both the heteroskedasticity present in the trade data and the existence of zero trade flows. OLS estimations of the log-linearized gravity equation produce inconsistent parameter estimates, while ignoring the zero trade flows will bias the estimated coefficients.

4.2 Data

The data used in this study combines several different datasets to study the link between international cartels and bilateral trade. First, the analysis uses information on 173 private discovered and prosecuted international cartels. Second, the cartel dataset is merged with
data on bilateral trade flows at the most disaggregated level readily available - *Harmonized System* 6-digit product level. The sample covers 34 OECD countries and spans the years between 1988 and 2012. The dataset includes information on the exact duration of each of the cartels, the countries of nationality of each of the cartel-members, as well as the 6-digit product code of each of the goods subject to collusive activities. In addition, the cartels and trade data are combined with data on various proxies for trade costs commonly used in the trade literature provided by the CEPII database. Lastly, I also use proxies for the degree of product substitutability created by Rauch (1999) and Broda and Weinstein (2006). Moreover, it is sensible to expect that the adjustment of trade in response to changes in firms’ behavior is not immediate. Therefore, following the recommendation of Cheng and Wall (2005), I employ 4-year intervals for the main specifications and experiment with 2-year, 3-year and 5-year intervals in the robustness checks. More details on all variables are provided in *Appendix B*.

Before I proceed, I need to point out that the analysis is subject to some caveats. First, the data cover only discovered and prosecuted international cartels and contain no information about collusive agreements that were never uncovered by the authorities. Therefore, the empirical results inherently suffer from a sample-selection bias. Second, the exact period of collusion as reported by the anti-trust authorities might not be absolutely accurate. The documents provided by the European Commission, for instance, on a number of occasions state “... cartel existed from as early as ...” or “... colluding at least from ...” Third, due to the lack of firm-level international trade data for such a large sample of countries, I resort to using the most product-disaggregated country-level data available. So, even though the theoretical model makes comparisons across firms, the empirical analysis is conducted at the

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39The data on private discovered and prosecuted international cartels also contain various cartel characteristics pertaining to the instruments of collusion (i.e., price-fixing, market-sharing, sales quotas, bid-rigging, etc.) as well as details about the practices adopted by the cartel members and the scope of collusion (i.e., market share controlled by the cartel). For more information, please refer to the Data Appendix of Agnosteva (2016).

40Cheng and Wall (2005) argue against the use of fixed effects with “... data pooled over consecutive years on the grounds that dependent and independent variables cannot fully adjust in a single year’s time.” (p.8).
country-product level. Keeping these caveats in mind, I next describe the main empirical results along with some robustness experiments.

4.3 Empirical Results

Table 1 presents the main results based on specification (11). In column (1), I first estimate the effects of standard gravity proxies for trade costs on bilateral trade using exporter fixed effects, importer fixed effect, product fixed effects and time fixed effects. Distance exerts a negative and significant impact on trade as it is standardly reported in the trade literature (Head and Mayer, 2014) and implies that a 10% increase in distance should be associated with a 6% decrease in trade. Further, as is common in the trade literature, the presence of contiguous borders between partners facilitates trade and so does membership to regional trade agreements. The estimates of the rest of the gravity variables are not significantly different from zero, which could be due to the fact that the data are at the most disaggregated level available or due to the restricted sample.

Next, in column (2), I examine the average partial effect of cartels on trade. The positive and significant estimate of the cartel binary variable (which takes the value of 1 if both country $i$ and country $j$ participate in cartel $k$ in sector $g$ at time $t$) indicates that the existence of collusion promotes bilateral trade between cartel-members. I further investigate this finding in column (3) by differentiating between multi-product and single-product cartels. Thus, I test hypothesis H1 described above. The results show that indeed multi-product collusion exerts a positive and significant impact on bilateral trade. The effect of single-product cartels is similarly positive and significant, but of lower magnitude. Moreover, a statistical test of the hypothesis that the two coefficients are equal, rejects the null with a p-value of 0.0105. In column (4), I experiment with the set of fixed effects and now use exporter-year, importer-year, and sector-year fixed effects instead of the previously included set of dummy variables. The results remain qualitatively unchanged with the impact of multi-product collusion still being positive and statistically significant. The estimates of
\( \hat{\beta}_{\text{CARTEL\_MLTPRD}} = 1.119 \) and \( \hat{\beta}_{\text{CARTEL\_SNGLPRD}} = 0.634 \) suggest that multi-product collusion increases trade by 62% more than single-product collusion. These interesting findings provide empirical support for the existence of trade-promoting international cartels.

In column (5), I test hypothesis \( H2 \) by estimating specification (11) using Rauch’s (1999) classification of goods to proxy for the degree of substitutability. As explained earlier, I employ Rauch’s (1999) categorical variable to create three binary indicators - for differentiated goods (\textit{SUBSTIT\_1}), for reference-priced goods (\textit{SUBSTIT\_2}), and for homogeneous goods (\textit{SUBSTIT\_3}). Then, I interact each of these variables with the multi-product and single-product binary indicators. To avoid perfect collinearity, I omit the interaction with the dummy for homogeneous goods, which, therefore, represents the reference group. The results show that the more differentiated the products are, the higher the positive impact of multi-market collusion on trade. The estimate on \( \text{CARTEL\_MLTPRD} \times \text{SUBSTIT\_1} \) is positive and significant. This finding is in line with the prediction of Proposition 1, which stipulates that multi-product cartels enhance trade relative to competition as long as the goods are sufficiently distant substitutes. Furthermore, the estimate on \( \text{CARTEL\_MLTPRD} \times \text{SUBSTIT\_2} \) is positive but insignificant, suggesting that multi-product cartels do not improve trade for reference-priced goods relative to homogeneous goods. Moreover, none of the interaction terms with the single-product cartel variable are statistically significant. Overall, the results based on specification (11) offer empirical support for the possibility that international cartels can enhance trade and are in line with the theoretical model.

Table 2 presents some additional tests, which ascertain the robustness of the aforementioned results. First, in column (1) of Table 2, I include country-pair time-invariant symmetric fixed effects, which control for all unobservable factors that might affect trade between an exporter and an importer symmetrically and also capture all bilateral time-invariant trade costs. Moreover, these bilateral fixed effects also account for any potential linkages between the cartel variables and the error term. The results remain qualitatively unchanged. The coefficient estimate on \( \text{CARTEL\_MLTPRD} \) is positive and significant and the estimate on
the interaction between this variable and the proxy for differentiated goods is also positive and significant. On the other hand, the coefficient on CARTEL_MLTPRD_SUBSTIT_2, the variable that captures the effect of multi-product collusion on trade for reference-priced goods, is positive, but not significantly different from zero. In addition, the interaction terms between CARTEL_SNGLPRD and the proxies for the degree of product substitutability are all insignificant. These results are in line with the main findings in Table 1 and show that multi-product cartels improve trade especially when the goods are relatively differentiated.

Second, in column (2) of Table 2 I repeat the test of hypothesis H1, but now I include directional pair fixed effects. These time-invariant bilateral fixed effects capture all observable and unobservable sources of directional heterogeneity between any pair of exporters and importers. They also control for any unobservable heterogeneity in the cross-section that is correlated with the presence of cartels. The results in column (2) of Table 2 are not qualitatively different from the main findings in column (3) of Table 1 with the coefficient on the multi-product cartel variable being positive and significant and of larger magnitude than the coefficient on the single-product cartel variable. Note that the results in column (2) are obtained with the use of exporter, importer, sector and year fixed effects. Therefore, in column (3) of Table 2 I substitute these fixed effects with exporter-time, importer-time, and sector-time dummies, keeping the directional pair fixed effects. Again, the results remain qualitatively unchanged. Lastly, I test the robustness of specification (11) by substituting the standard gravity variables with directional pair fixed effects. Once again, the main results remain qualitatively the same. The coefficient on CARTEL_MLTPRD_SUBSTIT_1, the variable that captures the effect of multi-product collusion on trade for differentiated goods, is positive and significantly different from zero and so is the coefficient on the multi-product cartel variable, CARTEL_MLTPRD. These findings still support the theoretical model.

Third, as an alternative to Rauch’s (1999) classification of goods, I employ the elasticity of substitution estimates of Broda and Weinstein (2006). The authors provide estimates of the elasticity of import demand at the 3-digit HS level for 73 countries. Unfortunately, not
all of the OECD member countries are included in their sample. Nonetheless, given that in the theoretical model the degree of substitutability is not country specific, I use Broda and Weinstein’s (2006) data to create an average product elasticity across all importers. Then, I interact this proxy for product substitutability with the cartel indicator variables. The results are presented in columns (5)-(6) of Table 2. When using Broda and Weinstein’s (2006) proxy for the degree of product substitutability, the coefficient estimates should be interpreted differently. Namely, the higher the elasticity of substitution, the lower the positive effect of multi-product collusion on trade should be. Therefore, I expect the coefficient estimate of $\beta_3$ from specification (11) to be negative and significant. The results in column (5) show that the estimate of $\text{CARTEL}_{MLTPRD} \times \text{SUBSTIT}$ is actually negative and significant in accordance with the theory. Moreover, the coefficient on $\text{CARTEL}_{MLTPRD}$ remains positive and significant and even increases in magnitude. Lastly, in column (6) of Table 2 I test the robustness of these findings more stringently by including bilateral asymmetric fixed effects. The results are qualitatively similar to the ones presented in column (5) of Table 2. The coefficient on the interaction term between multi-product collusion and the elasticity of substitution, $\text{CARTEL}_{MLTPRD} \times \text{SUBSTIT}$, is negative and significant.

Lastly, I experiment with the sample size by using various time-intervals: 2-year, 3-year, and 5-year. The results are presented in columns (7)-(9) of Table 2. Overall, the main findings from column (5) of Table 1 remain robust to variations of the sample size. Again, both the coefficient on $\text{CARTEL}_{MLTPRD, \text{SUBSTIT}_1}$, the variable that captures the effect of multi-product collusion on trade for differentiated goods, is positive and significant and so is the coefficient on the multi-product cartel variable, $\text{CARTEL}_{MLTPRD}$. These findings suggest that multi-product cartels enhance trade and that this positive impact is even more pronounced when the goods are differentiated. The only substantial change is that the coefficient on $\text{CARTEL}_{SINGLEPRD, \text{SUBSTIT}_1}$, the variable that captures the effect of single-product collusion on trade for differentiated goods, also gains some significance.

\footnote{Belgium, Estonia, Israel, Luxembourg and Czech Republic are missing from Broda et al. (2006) data.}
when I use 2-year and 5-year intervals. The theoretical model, however, makes no specific prediction about the impact of single-product international cartels on trade.

Overall, the results presented in Table 2 remain in line with the main findings from Table 1 and are in support of the theoretical model. Thus, both theory and empirics suggest that it is possible for international cartels to enhance the value of bilateral trade. This positive and significant effect of multi-product cartels on trade is even more pronounced when the goods are differentiated as opposed to referenced-priced or homogeneous.

5 Concluding Remarks

This paper seeks to examine whether international cartels can promote bilateral trade and enhance national welfare relative to competition. To this end, I build a segmented-markets Cournot-duopoly model, where each firm produces two goods, but enjoys a competitive cost advantage in the production of one of them. The interaction between the difference in marginal production costs and the trade costs is central to the analysis.

First, I show that the opportunity for rationalization of production and trade under collusion can increase trade flows of the cost-efficient good relative to competition. This possibility arises when the production cost asymmetry is sufficiently greater than the trade costs and the goods are sufficiently distant substitutes. To the best of my knowledge, this is the first paper to present a model, where cartels promote trade between members. Second, using a novel dataset on international cartels, I test this prediction empirically. I find that the average effect of cartels on trade is positive and significant. Moreover, multi-product collusion exerts a positive and significant effect on bilateral trade and this effect is statistically stronger than the impact of single-product cartels. Further, the positive effect of multi-product cartels on trade becomes more pronounced, the more unrelated the goods are, in line with the theoretical model. Third, due to the cartel’s enhanced production efficiency, collusion can welfare-dominate competition. Specifically, there always exists a set of transportation costs
and cost differential values for which collusion provides a greater level of welfare relative to competition regardless of the degree of substitutability. In the case of import tariffs, which I describe in depth in Appendix A, this possibility arises only if the goods are sufficiently distant substitutes. Fourth, in Appendix A, I verify that in this framework maximal collusion is indeed a sustainable outcome of the repeated game and also show that trade liberalization (or a reduction in the cost heterogeneity) can enhance cartel stability and thus further strengthen the efficiency gains noted above. Thus, I demonstrate that international cartels should not necessarily be considered “a true scourge of the world economy.”

6 References


Figure 1: Production and Trade Decisions by Potential Cartel Members (γ = 0)

<table>
<thead>
<tr>
<th>Region</th>
<th>Cournot - Nash Competition</th>
<th>Maximal Collusion</th>
<th>Optimal Deviation</th>
</tr>
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Notes: This table describes the equilibrium output allocations of domestic production, \( x_k \), and imports from abroad, \( y_k \), of both goods \( k = 1, 2 \) under Cournot-Nash competition, maximal collusion and optimal deviation for all possible levels of trade costs, \( t \) and cost differentials, \( s \). The regions listed in the first column are depicted in Figure 1.
Figure 2: Welfare Comparison under Transportation Costs for Unrelated Goods ($\gamma = 0$)
Figure 3: Welfare Comparison under Transportation Costs for Imperfect Substitutes ($\gamma = 0.4$)

Figure 4: Welfare Comparison under Transportation Costs for Imperfect Substitutes ($\gamma = 0.9$)
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<td>(\hat{\beta}<em>{C,MLTPRD} = \hat{\beta}</em>{C, SNGLP} )</td>
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<td>Importer-Year</td>
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<td>312,207</td>
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Notes: This table reports estimates of the relation between multi-product cartels and bilateral trade obtained with the PPML estimator. Column (1) presents the estimates of the standard gravity variables. Column (2) tests the impact of collusion on trade. Column (3) focuses on the effect of multi-product cartels and single-product cartels on trade. Column (4) repeats the exercises from Column (3), but now using exporter-time, importer-time and sector-time fixed effects. Column (5) studies the impact of multi-product cartels on trade for differing degrees of product substitutability, using Rauch’s (1999) classification of goods. The estimates of the fixed effects are omitted for brevity. See text for further details. Clustered standard errors in parentheses. * p < 0.10, * p < 0.05, ** p < 0.01
Table 2: Multi-Product Cartels and International Trade - Robustness Checks

<table>
<thead>
<tr>
<th>SYMM</th>
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<th>ASYMM</th>
<th>ASYMM</th>
<th>B&amp;W</th>
<th>B&amp;W</th>
<th>2-Year</th>
<th>3-Year</th>
<th>5-Year</th>
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<td>-0.664**</td>
<td>-0.722**</td>
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<td>(0.066)</td>
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<td>(0.065)</td>
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<td>(0.132)</td>
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<td>0.098</td>
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<td>(0.098)</td>
<td>(0.100)</td>
<td>(0.100)</td>
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<td>-0.151</td>
<td>(0.098)</td>
<td>(0.191)</td>
<td>(0.114)</td>
<td>(0.112)</td>
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<td>1.337**</td>
<td>0.434+</td>
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<td>(0.156)</td>
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<td>0.946**</td>
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<td>(0.166)</td>
<td>(0.189)</td>
<td>(0.268)</td>
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<td>1.100**</td>
<td>1.097**</td>
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<td>(0.228)</td>
<td>(0.210)</td>
<td>(0.221)</td>
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<td>(0.237)</td>
<td>(0.236)</td>
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<td>(0.287)</td>
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<td>-0.035</td>
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<td>(0.264)</td>
<td>(0.264)</td>
<td>(0.275)</td>
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<tr>
<td>CARTEL-MLTPRD_SUBSTIT</td>
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<td>-0.287**</td>
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<td>(0.080)</td>
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</table>

Fixed Effects:

| Exporter | No | Yes | No | No | No | No | No | No | No | No | No |
| Importer | No | Yes | No | No | No | No | No | No | No | No | No |
| Sector   | No | Yes | No | No | No | No | No | No | No | No | No |
| Year     | No | Yes | No | No | No | No | No | No | No | No | No |
| Exporter-Year | Yes | No | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| Importer-Year | Yes | No | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| Sector-Year | Yes | No | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| Country-Pair | Yes | Yes | Yes | No | Yes | Yes | No | No | No | No | No |

Notes: This table reports estimates of the relation between multi-product cartels and bilateral trade obtained with the PPML estimator. Column (1) reproduces the results from column (5) of Table 1 with symmetric pair fixed effects. Columns (2)-(4) employ asymmetric fixed effects and reestimate columns (3)-(5) of Table 1. Column (5) and Column (6) employ Broda and Weinstein's (2006) elasticity estimates as proxy for the degree of product substitutability. In Column (5), I use an average product elasticity across importers. In Column (6), I test the results from column (5) more stringently by including bilateral asymmetric fixed effects. In Columns (7)-(9), I use 2-year, 3-year, and 5-year intervals in the data, respectively. The estimates of the fixed effects are omitted for brevity. See text for further details. Clustered standard errors in parentheses. + p < 0.10, * p < 0.05, ** p < 0.01.
Appendix A: Theory

Derivation of the Cournot-Nash Equilibrium Output Levels: Here, I provide solutions of the optimal output allocations under Cournot-Nash competition. First, when \( t \leq \bar{t}_y \) and \( s \leq \bar{s}_y \), the Cournot-Nash equilibrium output levels are given by: 
\[
x_1^N = \frac{(1-\gamma)\alpha-s(2+\gamma) \pm t(1-\gamma)}{2(1-\gamma)^2}, \quad x_2^N = \frac{(1-\gamma)\alpha-s(2+\gamma) \mp t(1-\gamma)}{2(1-\gamma)^2}, \quad y_1^N = \frac{(1-\gamma)\alpha-s(2+\gamma) - 2t(1-\gamma)}{3(1-\gamma)^2}, \quad y_2^N = \frac{(1-\gamma)\alpha+s(1+2\gamma) - 2t(1-\gamma)}{3(1-\gamma)^2}.
\]
When the initial level of trade costs or the cost differential exceeds the prohibitive values of \( t \) or \( s \), equation (5) needs to be re-optimized for all possible cases to re-derive the optimal output allocations:

**Part a)** \( t \in [\bar{t}_y, \bar{t}_x] \): First, the value of \( t \) such that \( y_1^N = 0 \) is obtained from equation (4b), which gives \( \bar{t}_y \equiv t = \frac{\alpha(1-\gamma) - s(2+\gamma)}{2(1-\gamma)^2} \). Then, taking into consideration the fact that \( y_1^N = 0 \) for \( t \in [\bar{t}_y, \bar{t}_x] \), re-optimization of the Cournot-Nash problem delivers equilibrium outputs for intermediate levels of trade costs, \( t \in [\bar{t}_y, \bar{t}_x] \): 
\[
x_1^N = \frac{(1-\gamma)\alpha-s(2+\gamma) \pm (1+2\gamma) s \mp t(1-\gamma)}{2(1-\gamma)^2}, \quad x_2^N = \frac{(1-\gamma)\alpha-s(2+\gamma) \mp (1+2\gamma) s \pm t(1-\gamma)}{2(1-\gamma)^2}, \quad y_1^N = \frac{(1-\gamma)\alpha-s(2+\gamma) \mp t(1-\gamma)}{2(1-\gamma)^2}, \quad y_2^N = \frac{(1-\gamma)\alpha-s(2+\gamma) \pm t(1-\gamma)}{2(1-\gamma)^2}.
\]

**Part b)** \( t \in [\bar{t}_x, \alpha] \): Next, using the optimal output allocations from Part a) above, the value of \( t \) such that \( y_2^N = \frac{1}{3}(\alpha + s - 2t) = 0 \) is given by \( \bar{t}_x = \frac{\alpha - s}{2} \). Then, re-optimization of the Cournot-Nash problem for sufficiently high levels of trade costs, such that \( t \in [\bar{t}_x, \infty) \) delivers: 
\[
x_1^N = \frac{(1-\gamma)\alpha-s(2+\gamma) \pm (1+2\gamma) s \mp (1+2\gamma) t}{2(1-\gamma)^2}, \quad x_2^N = \frac{(1-\gamma)\alpha-s(2+\gamma) \mp (1+2\gamma) s \pm (1+2\gamma) t}{2(1-\gamma)^2}, \quad y_1^N = \frac{(1-\gamma)\alpha-s(2+\gamma) \pm (1+2\gamma) t}{2(1-\gamma)^2}.
\]

**Part c)** \( s \in [\bar{s}_y, \bar{s}_x] \): Similarly, the value of \( s \) such that \( y_1^N = 0 \) is obtained from equation (4b), which gives \( \bar{s}_y \equiv s = \frac{(1-\gamma)(\alpha-2t)}{2(1-\gamma)^2} \). For this interval of cost asymmetries, \( y_1^N = 0 \) and the optimization problem coincides with the one in Part a) above with the optimal outputs given by: 
\[
x_1^N = \frac{(1-\gamma)\alpha-s(2+\gamma) \pm (1+2\gamma) s \mp t(1-\gamma)}{2(1-\gamma)^2}, \quad x_2^N = \frac{(1-\gamma)\alpha-s(2+\gamma) \mp (1+2\gamma) s \pm t(1-\gamma)}{2(1-\gamma)^2}, \quad y_2^N = \frac{1}{3}(\alpha + s - 2t).
\]

**Part d)** \( s \in [\bar{s}_x, \alpha] \): Lastly, using the optimal output allocations from Part c) above, solve for the value of \( s \) such that the cost-inefficient good is no longer produced domestically, \( x_2^N = \frac{(1-\gamma)(1-\gamma) + 2(1-\gamma)^2 - s(1-\gamma)^2}{4(1-\gamma)^2} = 0 \) to obtain \( \bar{s}_x \equiv s = \frac{(1-\gamma)(1-\gamma) + 2(1-\gamma)^2}{(1-\gamma)^2} \). Then, the equilibrium output allocations for sufficiently high levels of the cost differential, such that \( s \in [\bar{s}_x, \alpha] \), are given by: 
\[
x_1^N = \frac{(1-\gamma)\alpha-s(2+\gamma) \pm (1+2\gamma) s \mp t(1-\gamma)}{2(1-\gamma)^2}, \quad y_2^N = \frac{(1-\gamma)(1-\gamma) + 2(1-\gamma)^2 - s(1-\gamma)^2}{4(1-\gamma)^2}, \quad y_1^N = x_2^N = 0.
\]
From the last equation, the value of \( t \) such that the cost-efficient good will not be traded: \( \bar{t}_0 = \alpha(1-\gamma)^2 \).

![Figure 5: Production and Trade Decisions by Potential Cartel Members (\( \gamma = 0.4 \))](image-url)
Fig. 5 illustrates how changes in the degree of substitutability affect the prohibitive levels of $t$ and $s$. As the goods become closer substitutes, the regions, which allow for positive domestic production of the cost-inefficient good and trade of either of the goods, shrink. This is due to the fact that the higher the degree of product substitutability, the stronger the negative price externality that producing more of one good inflicts onto the marginal revenue of the other.

**Proof of Lemma 1** The proof follows readily by using the definition of $\Pi^N$ in equation (5), evaluating $\Pi^N$ at the relevant points and then comparing them.

a) If $t \in [0, \tilde{t}_y)$, then $\frac{\partial \Pi^N(t,s,\gamma)}{\partial t} = -\frac{2(2\alpha-s-10t)}{9(1+\gamma)} \leq 0$ with $t_{min}^N = \frac{\alpha}{8} - \frac{s}{10}$. $\frac{\partial^2 \Pi^N(t,s,\gamma)}{\partial t^2} = \frac{20}{9(1+\gamma)} > 0$. If $t \in [\tilde{t}_y, \tilde{t}_x)$, then $\frac{\partial \Pi^N(t,s,\gamma)}{\partial t} = \frac{(2\alpha-8s-10t)}{9} \leq 0$ with $t_{min}^N = \frac{\alpha}{8} + \frac{4s}{5}$. $\frac{\partial^2 \Pi^N(t,s,\gamma)}{\partial t^2} = \frac{10}{9} > 0$. It is possible for $\Pi^N(t,s,\gamma)$ to have two local minima for any degree of product substitutability when the cost differential takes on intermediate values, $s \in \left[\frac{\alpha(1-\gamma)}{12(1-\gamma^2)}, \frac{2\alpha(1-\gamma)}{9(1-\gamma^2)}\right]$.

b) If $s \in [0, \tilde{s}_y)$, then $\frac{\partial \Pi^N(t,s,\gamma)}{\partial s} = -\frac{2(2\alpha(1-\gamma)-s(10+8\gamma)-t(1-\gamma))}{9(1+\gamma)} \leq 0$ with $s_{min}^N = \frac{2\alpha(1-\gamma)-t(1-\gamma)}{2(1+\gamma)}$. $\frac{\partial^2 \Pi^N(t,s,\gamma)}{\partial s^2} = \frac{2(10+8\gamma)}{9(1+\gamma)} > 0$. If $s \in [\tilde{s}_y, \tilde{s}_x)$, then $\frac{\partial \Pi^N(t,s,\gamma)}{\partial s} = \frac{\alpha(1-\gamma)(4-\gamma)+16(1-\gamma^2)-s(20-11\gamma^2)}{18(1-\gamma)} \leq 0$ with $s_{min} = \frac{\alpha(1-\gamma)(4-\gamma)+16(1-\gamma^2)}{20-11\gamma^2}$, $\frac{\partial^2 \Pi^N(t,s,\gamma)}{\partial s^2} = \frac{20-11\gamma^2}{18(1-\gamma^2)} > 0$. It is possible for $\Pi^N(t,s,\gamma)$ to have two local minima for any degree of product substitutability when trade costs are intermediate, $t \in \left[\frac{\alpha(1-\gamma)(2-\gamma)}{12+8\gamma-\gamma^2}, \frac{2\alpha(1-\gamma)}{9(1-\gamma^2)}\right]$.

**Proof of Lemma 2** The proof follows readily from the definition of $\Pi^M$ in equation (7):

a) If $t \leq \min (s, \alpha(1-\gamma))$, then $\frac{\partial \Pi^M(t)}{\partial t} = -\frac{\alpha(1-\gamma)+t}{2(1-\gamma)} < 0$ and $\frac{\partial^2 \Pi^M(t)}{\partial t^2} = \frac{1}{2(1-\gamma)} > 0$. From here, I can easily see that $arg\ min_t \Pi^M(t) \rightarrow \min (s, \alpha(1-\gamma))$ and $arg\ max_t \Pi^M(t) = 0$. If $t \in [\alpha(1-\gamma), s)$, then $\frac{\partial \Pi^M(t)}{\partial t} = 0$.

b) If $s \leq \min (t, \alpha(1-\gamma))$, then $\frac{\partial \Pi^M(s)}{\partial s} = -\frac{\alpha(1-\gamma)+s}{2(1-\gamma^2)} < 0$ and $\frac{\partial^2 \Pi^M(s)}{\partial s^2} = \frac{1}{2(1-\gamma^2)} > 0$. From here, I can easily see that $arg\ min_s \Pi^M(s) \rightarrow \min (t, \alpha(1-\gamma))$ and $arg\ max_s \Pi^M(s) = 0$. If $s \in [\alpha(1-\gamma), t)$, then $\frac{\partial \Pi^M(s)}{\partial s} = 0$.

**Proof of Proposition 1** I compare the volume of trade under competition, $y^N_k$, and the volume of trade under monopoly, $y^M_k$, when trade costs are such that $t < \min(s, \alpha(1-\gamma))$, so that trade actually occurs under collusion. In this case, the cartel produces good 1 domesticaly and imports good 2 from the other market. Therefore, I compare the volume of trade of good 2, the cost-efficient good, for home under monopoly and under competition. If trade costs are below the prohibitive level for the cost-efficient good, $t < \tilde{t}_y$, and if $t < \min(s, \alpha(1-\gamma))$:

$$y^M_2 - y^N_2 = \frac{\alpha(1-\gamma)-t}{2(1-\gamma^2)} - \frac{(1-\gamma)(\alpha+s(2\gamma+1)-2t(1-\gamma))}{3(1-\gamma^2)} \geq 0 \quad (12)$$

$$y^M_2 - y^N_2 = \frac{\alpha(1-\gamma)-s(2\gamma+1)+t(1-4\gamma)}{(1-\gamma^2)} \geq 0 \quad (13)$$

Whether the above expression is positive, for given trade costs, depends on the degree of substitutability and on the cost differential. If the goods are completely unrelated, $\gamma = 0$, and the cost

---

42Fig. 5 presents the constraints for Cournot-Nash competition, maximal collusion, and deviation.
differential is not too high, \( s < \frac{\alpha+t}{2} \), the volume of trade under monopoly will exceed the volume of trade under competition for the cost-efficient good: \( y_2^M - y_2^N > 0 \). For intermediate values of \( \gamma \), such that the goods are imperfect substitutes, the above expression is positive only if \( \gamma < \gamma^* = \frac{\alpha+t-2t}{\alpha+t+2t} \in [0,1) \) and if the cost differential is not too high, \( s < \frac{\alpha+t}{2} \).

Next, I compare the value of home’s imports (or foreign’s exports) of good 2, the cost-efficient good, under monopoly and competition. Recall that:

\[
p_2^N = (\alpha - (x_2^N + y_2^N)) = \frac{1}{3}(\alpha + t + s) \tag{14}
\]

\[
p_2^M = (\alpha - (x_2^M + y_2^M)) = \frac{1}{2}(\alpha + t) \tag{15}
\]

Using the price equations along with the equilibrium outputs, the value of trade under monopoly, \( V^M = p_2^M y_2^M \), and the value of trade under competition, \( V^N = p_2^N y_2^N \), are given by:

\[
V^N = \frac{(\alpha + s + t)(\alpha(1-\gamma) - 2t(1-\gamma) + s(1+2\gamma))}{9(1-\gamma^2)} \tag{16}
\]

\[
V^M = \frac{(\alpha + t)(\alpha(1-\gamma) - t)}{4(1-\gamma^2)} \tag{17}
\]

From the above expressions it follows that, \( V^M > V^N \) as long as the cost differential is not too high, \( s < \frac{\alpha+t}{2} \), and the goods are sufficiently unrelated, \( \gamma < \gamma^* = \frac{5\alpha+2s-8t(\alpha-2s+4)}{5\alpha^2+5\alpha+13s+8s^2+16st+8s^3+4t^2+16st+16s^2+4t^2} \in [0,1) \). It is also straightforward to verify that the competitive firms no longer trade the cost-inefficient good and the volume of trade of the cost-efficient good equals \( y_2^N = \frac{1}{3}(\alpha + s - 2t) \).

### Collusion and Welfare, Part I: Transportation Costs

**Proof of Lemma**

If trade costs take the form of transportation costs:

\[
W^N = \begin{cases} 
\alpha(Q_2^N + Q_2^M) - \frac{1}{4}(Q_2^N)^2 - \frac{1}{4}(Q_2^M)^2 - \gamma(Q_2^N)(Q_2^M) - s(Q_2^N) - t(y_1^N + y_2^N) \quad & \text{if } t < \tau_y \\
\alpha(x_1^N + x_2^N) - \frac{1}{4}(x_1^N)^2 - \frac{1}{4}(x_2^N)^2 - \gamma(x_1^N)(x_2^N) - s(x_2^N) - t(y_1^N + y_2^N) \quad & \text{if } \tau_y \leq t < \tau_x \\
\alpha(x_1^N + x_2^N) - \frac{1}{4}(x_1^N)^2 - \frac{1}{4}(x_2^N)^2 - \gamma(x_1^N)(x_2^N) - s(y_1^N + y_2^N) \quad & \text{if } t \geq \tau_x \quad s < \alpha(1-\gamma) \\
\alpha(x_1^N + x_2^N) - \frac{1}{4}(x_1^N)^2 - \frac{1}{4}(x_2^N)^2 - \gamma(x_1^N)(x_2^N) - s(y_1^N + y_2^N) \quad & \text{if } t \geq \tau_x \quad s \geq \alpha(1-\gamma) \\
\alpha(x_1^N + x_2^N) - \frac{1}{4}(x_1^N)^2 - \frac{1}{4}(x_2^N)^2 \quad & \text{if } s \leq \sigma_y \quad t < \alpha(1-\gamma) \\
\alpha(x_1^N + x_2^N) - \frac{1}{4}(x_1^N)^2 - \frac{1}{4}(x_2^N)^2 \quad & \text{if } s \geq \sigma_x \quad t \geq \alpha(1-\gamma) \\
\end{cases}
\]

Thus, after rearranging terms and simplifying the expressions,

\[
W^N = \begin{cases} 
\frac{8\alpha(1-\gamma)(\alpha-s-t)+(1-\gamma)(11t+4s)+2^2(7\gamma+11)}{9(1-\gamma)^2} \quad & \text{if } t < \tau_y \\
\frac{8\alpha(1-\gamma)^2}{3\alpha^2} \quad & \text{if } \tau_y \leq t < \tau_x \\
\frac{8\alpha(1-\gamma)(\alpha-s-t)+(1-\gamma)(11t+4s)+2^2(7\gamma+11)}{9(1-\gamma)^2} \quad & \text{if } t \geq \tau_x \quad s < \alpha(1-\gamma) \\
\frac{8\alpha(1-\gamma)^2}{3\alpha^2} \quad & \text{if } t \geq \tau_x \quad s \geq \alpha(1-\gamma) \\
\frac{8\alpha(1-\gamma)(\alpha-s-t)+(1-\gamma)(11t+4s)+2^2(7\gamma+11)}{9(1-\gamma)^2} \quad & \text{if } s \leq \sigma_y \quad t < \alpha(1-\gamma) \\
\frac{8\alpha(1-\gamma)^2}{3\alpha^2} \quad & \text{if } s \geq \sigma_x \quad t \geq \alpha(1-\gamma) \\
\end{cases}
\]

(18)
Part (a). If $t \in [0, \tilde{t}_y)$, then $\frac{\partial W^N}{\partial t} = \frac{2(-4a+2s+11t)}{9(1+\gamma)} \leq 0$ and $t^N_{\min} = \frac{4a-2s}{11}$. However, $t^N_{\min} \in [0, \tilde{t}_y)$, only if $s < \frac{\alpha(1-\gamma)}{12}$. This can be easily verified by comparing $t^N_{\min}$ and $\tilde{t}_y$ and finding the values of $s$, which satisfy $t^N_{\min} \in [0, \tilde{t}_y)$.

If $t \in [\tilde{t}_y, \tilde{t}_x)$, then $\frac{\partial W^N}{\partial t} = -\frac{4a-7s+11t}{9(1+\gamma)} \leq 0$ and $t^N_{\min} = \frac{4a+7s}{11}$. However, $t^N_{\min} \in [\tilde{t}_y, \tilde{t}_x)$, only if $s \geq \frac{\alpha(1-\gamma)}{12}$. This can be easily verified by finding the values of $s$, which satisfy $t^N_{\min} \in [\tilde{t}_y, \tilde{t}_x)$.

Part (b). Again, from the definition of $W^N(t, s, \gamma)$, it easily follows that if $s \in (0, \bar{s}_y)$, then

$$ \frac{\partial W^N}{\partial s} = \frac{2((-4a+2t)(1-\gamma)+s(1+\gamma))}{9(1-\gamma^2)} \leq 0 \quad \text{with} \quad s^N_{\min} \in (0, \bar{s}_y) \text{ if } t \in [0, \frac{\alpha(1+\gamma)}{2(2\gamma+3)}]. $$

If $s \in [\bar{s}_y, \bar{s}_x)$, then

$$ \frac{\partial W^N}{\partial s} = \frac{\alpha(16-11\gamma)(1-\gamma)+s(17\gamma^2-44+28t(1-\gamma^2))}{36(1-\gamma^2)} \leq 0 \quad \text{with} \quad s^N_{\min} \in [\bar{s}_y, \bar{s}_x) \text{ if } t \in \left[\frac{\alpha(1-\gamma)^2}{2(1-\gamma)}, \frac{\alpha(1-\gamma)}{4(1-\gamma)^2}\right]. $$

Part (c). First, note that $W^N(0, s, \gamma) = \frac{8a^2(1-\gamma)-8s(1-\gamma)+s(1+\gamma)}{9(1-\gamma^2)}$ and also $W^N(\bar{t}_x, s, \gamma) = \frac{3(2a^2-2\alpha^2-s^2+2sa-2s^2)}{8(1-\gamma)}$. Comparing these two expressions, one can easily verify that $W^N(0, s, \gamma) > W^N(\bar{t}_x, s, \gamma)$.

**Proof of Lemma 4** From the definition of welfare, one can show, after rearranging terms, that welfare under maximal collusion under transportation costs is given by

$$ W^M = \begin{cases} 
3\frac{[2\alpha(1-\gamma)(\alpha-t)+t^2]}{8(1-\gamma^2)} & \text{if } t < \min(s, \alpha(1-\gamma)) \\
\frac{3}{4}\alpha^2 & \text{if } t \geq \alpha(1-\gamma), s \\
3\frac{2\alpha(1-\gamma)(\alpha-s+s^2)}{8(1-\gamma^2)} & \text{if } s \leq \min(t, \alpha(1-\gamma)) \\
\frac{3}{8}\alpha^2 & \text{if } s \geq \alpha(1-\gamma), t
\end{cases} \quad (19) $$

a) If $t < \min(s, \alpha(1-\gamma))$, then

$$ \frac{\partial W^M}{\partial t} = \frac{3(-\alpha(1-\gamma)+t)}{4(1-\gamma)} < 0 \quad \text{and} \quad \frac{\partial^2 W^M}{\partial t^2} = \frac{3}{4(1-\gamma^2)} > 0, \quad \text{with} \quad \arg\min_t W^M(t) \rightarrow \min(s, \alpha(1-\gamma)) \text{ and } \arg\max_t W^M(t) = 0. $$

b) If $s \leq \min(t, \alpha(1-\gamma))$, then

$$ \frac{\partial W^M}{\partial s} = \frac{3(-\alpha(1-\gamma)+s)}{4(1-\gamma)} < 0 \quad \text{and} \quad \frac{\partial^2 W^M}{\partial s^2} = \frac{3}{4(1-\gamma^2)} > 0, \quad \text{with} \quad \arg\min_s W^M(s) \rightarrow \min(t, \alpha(1-\gamma)) \text{ and } \arg\max_s W^M(s) = 0. $$

**Proof of Proposition 2** In the case of transportation costs, welfare under Nash competition is defined in (18), while welfare under maximal collusion is defined in (19).

1) Suppose $t < \tilde{t}_y$, $t < \min(s, \alpha(1-\gamma))$, i.e. trade costs and the cost differential are such that they allow for trade in both goods under competition and for trade in the cost-efficient good under collusion. In this case, the difference in welfare levels, $\Delta W \equiv W^N(t, s, \gamma) - W^M(t, \gamma)$, is given by:

$$ \Delta W \equiv W^N(t, s, \gamma) - W^M(t, \gamma) = \frac{s^2(88+56\gamma) - t^2(88\gamma - 61) + (1-\gamma)(32t - \alpha(10t+64s)+10\alpha^2)}{72(1-\gamma^2)} \quad (20) $$

Equation (20) is a quadratic function in ($t, s$) and in order to examine its shape, I need to first make sure that it is a non-degenerate, i.e. its Hessian is invertible. Henceforth, I denote $W_s$ the partial derivative of the difference $\Delta W \equiv W^N(t, s, \gamma) - W^M(t, \gamma)$ with respect to variable $z$.

$$ W_t = -\frac{-5\alpha(1-\gamma) + 16s(1-\gamma) + t(61-88\gamma)}{36(1-\gamma^2)} \quad (21) $$
Then, using the second-order derivatives of $\Delta W$ with respect to $t$ and $s$, the Hessian is given by:

$$H_W = \begin{bmatrix}
\frac{61(1-\gamma^2)}{9(1-\gamma^4)} & \frac{16(1-\gamma)}{9(1-\gamma^2)} \\
\frac{16(1-\gamma)}{9(1-\gamma^2)} & \frac{36(1-\gamma^2)}{9(1-\gamma^4)}
\end{bmatrix}$$

and is invertible for values of $\gamma \in \left[0, \frac{-53+\sqrt{23257}}{144}\right]$. The critical point is given by: $t_0 = \frac{(11\gamma-1)(\gamma-1)\alpha}{12\gamma^3+53\gamma-71}$ and $s_0 = -2\frac{\alpha(t-13)(\gamma-1)}{12\gamma^4-53\gamma+71}$. For $\gamma \in \left(\frac{-53+\sqrt{23257}}{144}, 1\right)$, $(t_0, s_0)$ is actually a saddle point as the determinant of $H_W$ becomes negative. Using the sign of the determinant of $H_W$, I can determine the shape of the quadratic function (20). Thus, for values of $\gamma \in \left[0, \frac{-53+\sqrt{23257}}{144}\right]$, (20) will take the shape of an ellipse, while for $\gamma \in \left(\frac{-53+\sqrt{23257}}{144}, 1\right)$, (20) will be a hyperbola.

When $t = 0$ (and $\gamma = 0$), $W = \frac{1}{36} (\alpha - 2s) (5\alpha - 22s)$ and therefore, the $s$-intercepts are $s = \frac{\alpha}{2}$ and $s = \frac{5\alpha}{22}$. On the other hand, at $s = 0$ (and $\gamma = 0$), $W = \frac{5\alpha^2}{36} - \frac{5\alpha t}{9} + \frac{61t^2}{72}$ and the equation has not real solutions, suggesting that the ellipse never crosses the $t$-axis. Moreover, when $t = 0$ (and $\gamma = 0$), welfare under monopoly is greater than welfare under competition as long as $s \in \left(\frac{5\alpha}{22}, \frac{\alpha}{2}\right)$. Further, for $t < \bar{t}_y$, $t < \min(s, \alpha(1-\gamma))$, and when $\gamma = 0$, $W_M(t, 0) > W_N(t, s, 0)$ for

$$s \in \left(\frac{\alpha - \sqrt{\alpha^2 - 4\alpha - 142t^2}}{\sqrt{\alpha^2 - 4\alpha - 142t^2}}, \frac{\alpha + 2t + \sqrt{\alpha^2 - 4\alpha - 142t^2}}{2\sqrt{\alpha^2 - 4\alpha - 142t^2}}\right) \quad \text{and} \quad t \in \left(\frac{2\alpha + 4s}{\sqrt{\alpha^2 - 4\alpha - 142t^2}}, \frac{2\alpha + 4s}{\sqrt{\alpha^2 - 4\alpha - 142t^2}}\right)$$

Notice that if the cost differential is sufficiently high, then $W_M(t, 0) > W_N(t, s, 0)$ for the entire relevant range of trade costs, $t \in (0, t_2)$.

The same result holds true even for a positive degree of substitutability between good 1 and good 2, as long as $\gamma < \hat{\gamma}$. To find this threshold value $\hat{\gamma}$, I need to find the value of $\gamma$ such that the curve (20) is exactly tangent to the $s$-axis. Thus, I evaluate equation (20) at $t = 0$, find the value of $s$ that maximizes it, $s^*$, and then evaluate (20) at $t = 0$ and $s = s^*$. Thus, (20) at $t = 0$ and $s = s^* = \frac{4\alpha(1-\gamma)}{(11+7\gamma)}$ is equal to

$$\Delta W|_{t=0,s=s^*} = \frac{\alpha^2(11\gamma - 1)}{2(1+\gamma)(11+7\gamma)}$$

Therefore, $\hat{\gamma} = \frac{1}{11}$ and $W_M(t, \gamma) > W_N(t, s, \gamma)$ when $t < s$, $t < \min(\alpha(1-\gamma), \bar{t}_y)$ and $\gamma \in (0, \hat{\gamma})$. Moreover, as $\gamma$ increases and approaches $\hat{\gamma}$, the set of values of the cost differential and transportation costs, $(t, s)$, for which $W_M(t, \gamma) > W_N(t, s, \gamma)$ shrinks and vanishes when $\gamma \geq \hat{\gamma}$.

2) Next, I consider the interval of trade costs such that trade of the cost-efficient good persists under monopoly, while under competition the cost-inefficient good is produced domestically, but not traded: $t < \min(s, \alpha(1-\gamma))$ and $s \in [\bar{s}_y, \bar{s}_x]$. In this case, the difference in welfare levels, $\Delta W \equiv W_N(t, s, \gamma) - W_M(t, \gamma)$, is given by:

$$\Delta W \equiv W_N(t, s, \gamma) - W_M(t, \gamma) = \frac{s^2(17\gamma^2 - 44) - t^2(17 - 44\gamma^2) - \alpha^2(54 - 22\gamma) + \alpha\gamma t(5 - 32\gamma) + (1 - \gamma^2)(-5\alpha^2 + 56st) + \alpha(32s - 22t)}{72(1 - \gamma^2)}$$

Equation (25) is a quadratic function in $(t, s)$ and in order to examine its shape, I need to first make sure that it is a non-degenerate, i.e. its Hessian is invertible.

$$W_t = \frac{16\alpha \gamma^2 + 28\gamma^2 s - 44\gamma^2 t - 27\alpha \gamma + 11\alpha - 28s + 17t}{36(1 - \gamma^2)}$$
\[
W_s = -\frac{11\alpha \gamma^2 + 17 \gamma^2 s - 28 \gamma^2 t - 27 \alpha \gamma + 16 \alpha - 44 s + 28 t}{36 (1 - \gamma^2)} \tag{27}
\]

Then, using the second-order derivatives of \(\Delta W\) with respect to \(t\) and \(s\), the Hessian is given by:

\[
H_W = \begin{bmatrix}
\frac{17 - 44 \gamma^2}{36(1 - \gamma^2)} & -\frac{7}{9} \\
-\frac{7}{9} & \frac{44 - 12 \gamma^2}{36(1 - \gamma^2)}
\end{bmatrix}
\]

and the determinant of \(H_W\) is negative for all values of \(\gamma \in [0, 1)\). The saddle point is given by \(t_0 = \frac{\alpha(\gamma - 1)(\gamma - 2)(4\gamma^2 - 21\gamma + 2)}{4\gamma^4 + 73\gamma^2 + 4}\) and \(s_0 = \frac{\alpha(\gamma - 1)(\gamma - 1)(2\gamma^2 - 21\gamma + 4)}{4\gamma^4 + 73\gamma^2 + 4}\). Moreover, as the determinant of \(H_W\) is negative, \(25\) will take the form of a hyperbola for all \(\gamma \in [0, 1)\).

When \(t = 0\) (and \(\gamma = 0\)), \(W = \frac{\alpha}{12} (\alpha - 2s) (5\alpha - 22s)\) and therefore, the \(s\)-intercepts are \(s_1 = \frac{\alpha}{22}\) and \(s_2 = \frac{5\alpha}{22}\). On the other hand, at \(s = 0\) (and \(\gamma = 0\)), \(W = \frac{1}{12} (5\alpha + 17t)(\alpha + t)\) and the \(t\)-intercepts are \(t_1 = -\alpha\) and \(t_2 = -\frac{5\alpha}{17}\). When \(t = 0\) (and \(\gamma = 0\)), welfare under monopoly is greater than welfare under competition as long as \(s \in \left[\frac{5\alpha}{22}, \frac{\alpha}{2}\right]\). More generally, for \(t < s < s^*\) and \(s \in \left[\bar{s}_y, \bar{s}_x\right]\), and when \(\gamma = 0\), \(W^M(t, 0) > W^N(t, s, 0)\) for

\[
\left\{ \begin{array}{ll}
\bar{s}_y < s < \bar{s}_x & \text{if } t \in \left(\frac{2}{17}\alpha, \alpha\right) \\
\bar{s}_y < s < \bar{s}_x & \text{if } t \in \left(0, \frac{2}{17}\alpha\right)
\end{array} \right.
\tag{28}
\]

Notice that if trade costs are sufficiently small, then \(W^M(t, 0) > W^N(t, s, 0)\) for the entire relevant range of the cost differential, \(s \in \left[\bar{s}_y, \bar{s}_x\right]\).

The same result holds true even for a positive degree of substitutability between good 1 and good 2, as long as \(\gamma < \bar{\gamma}\). To find this threshold value \(\bar{\gamma}\), I need to find the value of \(\gamma\) such that the hyperbola is exactly tangent to the \(t_y\)-line. Thus, I evaluate equation \(25\) at \(t = t_0\), find the value of \(s\) that maximizes it, \(s^*\), and then evaluate \(25\) at \(t = t_y\) and \(s = s^*\). Thus, \(25\) at \(t = t_y\) and \(s = s^*\) is equal to

\[
\Delta W|_{t = t_y, s = s^*} = \frac{\alpha^2 (25 \gamma^3 - 155 \gamma^2 + 104 \gamma - 4)}{8(\gamma + 1) (8 \gamma^3 - 43 \gamma^2 - 44 \gamma + 52)} \tag{29}
\]

Given the non-linearity of equation \(29\), I study how the hyperbola \(25\) changes as \(\gamma\) changes from 0 to 1. First, the saddle point of the hyperbola is located in the I quadrant of the \((t, s)\) space, i.e. \(t_0 \in [0, \alpha]\) and \(s_0 \in [0, \alpha]\), for values of \(\gamma\) in the neighborhood of 0. However, as \(\gamma\) increases slightly, the saddle point moves to the II quadrant with \(t_0\) taking a negative value. As \(\gamma\) raises further and approaches 1, the saddle point of the hyperbola \(25\) shifts down to the III quadrant of the \((t, s)\) space with both its coordinates being negative. Second, the two curves of the hyperbola get closer and closer to each other as \(\gamma\) increases from 0 to 1. Lastly, the value of \(\gamma\), above which competition weakly welfare-dominates collusion within the relevant range of trade costs and cost differential values is \(\bar{\gamma} = 0.041\). \(W^M(t, \gamma) \leq W^N(t, s, \gamma)\) when \(t < s < \alpha(1 - \gamma)\) and \(s \in \left[\bar{s}_y, \bar{s}_x\right]\) and \(\gamma \in [\bar{\gamma}, 1)\). As \(\gamma\) increases and approaches \(\bar{\gamma}\), the set of values of the cost differential and transportation costs, \((t, s)\), for which \(W^M(t, \gamma) > W^N(t, s, \gamma)\) shrinks and vanishes when \(\gamma \geq \bar{\gamma}\).

3) Now, I focus on the case of no trade in any good under collusion, while trade persists in both goods under competition: \(t \geq s\), \(s < \min(\alpha(1 - \gamma), \bar{s}_y)\). The difference in the welfare functions is:

\[
\Delta W \equiv W^N(t, s, 0) - W^M(s, 0) = \frac{(1 - \gamma)(88t^2 + 32st + 10\alpha^2 - 10\alpha s - 64\alpha t) + s^2(61 + 56\gamma)}{72(1 - \gamma^2)} \tag{30}
\]

Equation \(30\) is a quadratic function in \((t, s)\) and in order to examine its shape, I need to first...
make sure that it is a non-degenerate, i.e. its Hessian is invertible.

\[
W_t = -\frac{2(4\alpha - 2s - 11t)}{9(\gamma + 1)} \tag{31}
\]

\[
W_s = \frac{5\alpha \gamma + 56\gamma s - 16\gamma t - 5\alpha + 61s + 16t}{36(1 - \gamma^2)} \tag{32}
\]

Then, using the second-order derivatives of \(\Delta W\) with respect to \(t\) and \(s\), the Hessian is given by:

\[
H_W = \begin{bmatrix}
\frac{22}{9(\gamma + 1)} & \frac{4}{9(\gamma + 1)} & \frac{49\gamma + 11}{9(\gamma + 1)} \\
\frac{4}{9(\gamma + 1)} & \frac{56\gamma + 61}{9(\gamma + 1)} & \frac{56\gamma + 61}{9(\gamma + 1)} \\
\frac{49\gamma + 11}{9(\gamma + 1)} & \frac{56\gamma + 61}{9(\gamma + 1)} & \frac{56\gamma + 61}{9(\gamma + 1)} \\
\end{bmatrix}
\]

and the determinant of \(H_W\) is positive for all values of \(\gamma \in [0, 1]\). Thus, the critical point is given by \(t_0 = \frac{2\alpha(\gamma + 1)}{22\gamma + 11}\) and \(s_0 = \frac{\alpha(\gamma + 1)}{22\gamma + 11}\). Further, as the determinant of \(H_W\) is positive, \(\Delta W\) will take the form of an ellipse for all \(\gamma \in [0, 1]\).

When \(t = 0\) (and \(\gamma = 0\)), \(W = \frac{5\alpha^2}{9\gamma} - \frac{5\alpha s}{9\gamma} + \frac{61s^2}{2\gamma^2}\) and the function has no real solutions, suggesting that the ellipse does not cross the \(s\)-axis. On the other hand, at \(s = 0\) (and \(\gamma = 0\)), \(W = \frac{1}{36}(\alpha - 2t)(5\alpha - 22t)\) and the \(t\)-intercepts are \(t_1 = \frac{\alpha}{2}\) and \(t_2 = \frac{5\alpha}{22}\). When \(s = 0\) (and \(\gamma = 0\)), welfare under monopoly is greater than welfare under competition as long as \(t \in (\frac{5\alpha}{22}, \frac{\alpha}{2})\). More generally, for \(t \geq s, s < \min(\alpha(1 - \gamma), \bar{s}_y)\), and when \(\gamma = 0\), maximal collusion is welfare-superior to Cournot-Nash competition, \(W^M(s, 0) > W^N(t, s, 0)\), for

\[
\begin{cases}
 t \in \left(\frac{4\alpha - 2s - 3\sqrt{4\alpha^2 - 4\alpha s - 142s^2}}{11}, \frac{4\alpha - 2s + 3\sqrt{4\alpha^2 - 4\alpha s - 142s^2}}{11}\right) \\
 t \in \left[\frac{4\alpha - 2s - 3\sqrt{4\alpha^2 - 4\alpha s - 142s^2}}{11}, \frac{4\alpha - 2s + 3\sqrt{4\alpha^2 - 4\alpha s - 142s^2}}{11}\right] \\
\end{cases}
\]

The same result holds true even for a positive degree of substitutability between good 1 and good 2, as long as \(\gamma < \frac{\gamma^*}{2}\). To find this threshold value \(\frac{\gamma^*}{2}\), I need to find the value of \(\gamma\) such that the hyperbola is exactly tangent to the \(t\)-axis. Thus, I evaluate equation \(\Delta W\) at \(s = 0\), find the value of \(t\) that maximizes it, \(t^*\), and then evaluate \(\Delta W\) at \(s = 0\) and \(t = t^*\). Thus, \(\Delta W\) at \(s = 0\) and \(t = t^* = \frac{4}{11}\alpha\) is equal to

\[
\Delta W_{|s=0, t=t^*} = -\frac{\alpha^2 (2 + \gamma) (2 \gamma^2 - 33 \gamma + 22) (26 \gamma^3 + 61 \gamma^2 + 220 \gamma - 172)}{36 (8 \gamma^3 - 43 \gamma^2 - 44 \gamma + 52)^2 (\gamma + 1)} \tag{34}
\]

Therefore, \(W^M(s, \gamma) > W^N(t, s, \gamma)\) when \(t \geq s, s < \min(\alpha(1 - \gamma), \bar{s}_y)\) and \(\gamma \in [0, 0.638) \cup (0.696, 1)\).

4) Next, I consider the case of no trade in the cost-inefficient good under competition and no trade in any good under collusion: \(t \geq s, s < \alpha(1 - \gamma)\) and \(t \in [\bar{t}_y, \bar{t}_x]\). Then,

\[
\Delta W = W^N(t, s, \gamma) - W^M(s, \gamma) = \frac{1}{72} (5\alpha - 22t + 17s) (\alpha - 2t + s) \tag{35}
\]

Equation \(\Delta W\) is a quadratic function in \((t, s)\) and in order to examine its shape, I need to first make sure that it is a non-degenerate, i.e. its Hessian is invertible.

\[
W_t = \frac{11t}{9} - \frac{4\alpha}{9} - \frac{7s}{9} \tag{36}
\]

\[
W_s = -\frac{7t}{9} + \frac{11\alpha}{36} + \frac{17s}{36} \tag{37}
\]

Then, using the second-order derivatives of \(\Delta W\) with respect to \(t\) and \(s\), the Hessian is given by:
$H_W = \begin{bmatrix} \frac{11}{9} & -\frac{7}{9} \\ -\frac{7}{9} & \frac{17}{9} \end{bmatrix}$ and the determinant of $H_W$ is negative and does not depend on the values of $\gamma$.

Thus, there is a saddle point given by $t_0 = \alpha$ and $s_0 = \alpha$. In addition, given that the determinant of $H_W$ is negative, (33) will take the form of a hyperbola regardless of the value of $\gamma \in [0, 1)$.

When $t = 0$ (and $\gamma = 0$), $W = \frac{(5\alpha+17s)(\alpha+s)}{72}$ and the $s$-intercepts are given by $s_1 = -\frac{5s}{17}$ and $s_2 = -\alpha$. On the other hand, at $s = 0$ (and $\gamma = 0$), $W = \frac{(-2t+\alpha)(-2t+5\alpha)}{72}$ and the $t$-intercepts are $t_1 = \frac{\alpha}{2}$ and $t_2 = \frac{5\alpha}{22}$. When $s = 0$ (and $\gamma = 0$), welfare under monopoly is greater than welfare under competition as long as $t \in \left(\frac{5\gamma}{22}, \frac{9}{2}\right)$. More generally, for $t \geq s$, $t < \alpha(1-\gamma)$ and $t \in \left[\bar{t}_y, \bar{t}_x\right)$, maximal collusion is welfare-superior to Cournot-Nash competition, $W^M(s, \gamma) > W^N(t, s, \gamma)$, for any degree of product substitutability, $\gamma$, for

$$\begin{cases} t \in \left(\frac{5\alpha+17s}{22}, \bar{t}_x\right) & \& s \in \left(0, \frac{2}{17}\alpha\right) \\ t \in \left(\bar{t}_y, \bar{t}_x\right) & \& s \in \left(0, \frac{1}{17}\alpha\right) \end{cases}$$

(38)

Note that this is true for any value of $\gamma$. That is, within this range of trade costs and cost differential values, collusion welfare-dominates competition for any degree of product substitutability.

5) I move on to the case of the cost-inefficient good not being traded under collusion, but still being produced domestically, while under competition the cost-inefficient good is not produced in the domestic market at all, nor is it imported: $t \geq s$ and $s \in [\bar{s}_x, \alpha(1-\gamma)]$. Then,

$$\Delta W \equiv W^N(t, s, \gamma) - W^M(s, \gamma) = \frac{\alpha(\alpha-t)(24-16\gamma - 2s(1-\gamma)) + t^2(12-\gamma^2)}{2(\gamma-2)^2(\gamma + 2)^2} - \frac{3(1-\gamma)2\alpha(\alpha-s) + s^2}{8(1-\gamma^2)}$$

(39)

Equation (39) is a quadratic function in $(t, s)$ and in order to examine its shape, I need to first make sure that it is a non-degenerate, i.e. its Hessian is invertible.

$$W_t = -\frac{\alpha \gamma^3 - \alpha \gamma^2 + \gamma^2 t - 8 \alpha \gamma + 12 \alpha - 12 t}{(\gamma-2)^2(\gamma + 2)^2}$$

(40)

$$W_s = \frac{3 \alpha (1-\gamma) - s}{4(1-\gamma^2)}$$

(41)

Then, using the second-order derivatives of $\Delta W$ with respect to $t$ and $s$, the Hessian is given by:

$$H_W = \begin{bmatrix} \frac{12-\gamma^2}{(\gamma-2)^2(\gamma + 2)^2} & 0 \\ 0 & -\frac{3}{4(1-\gamma^2)} \end{bmatrix}$$

and the determinant of $H_W$ is negative for all values of $\gamma \in [0, 1)$.

Thus, there is a saddle point given by $t_0 = \frac{\alpha(3+\gamma)(\gamma-2)^2}{12-\gamma^2}$ and $s_0 = \alpha(1-\gamma)$. Furthermore, as the determinant of $H_W$ is negative, (33) will take the form of a hyperbola for any value of $\gamma \in [0, 1)$.

When $t = 0$ (and $\gamma = 0$), $W = \frac{3s(2\alpha-s)}{8}$ and the $s$-intercepts are given by $s_1 = 0$ and $s_2 = 2\alpha$. On the other hand, at $s = 0$ (and $\gamma = 0$), $W = -\frac{3t(2\alpha-t)}{8}$ and the $t$-intercepts are $t_1 = 0$ and $t_2 = 2\alpha$. Moreover, for $t \geq s$ and $s \in [\bar{s}_x, \alpha(1-\gamma)]$, maximal collusion is welfare-superior to Cournot-Nash competition, $W^M(s, \gamma) > W^N(t, s, \gamma)$, for

$$\begin{cases} t \in \left(M, \frac{2(1-\gamma)(\alpha \gamma + \gamma s - \alpha + 2s)}{2(1-\gamma^2)}\right) & \& s \in \left(\frac{(\alpha(1-\gamma))(9-2\gamma)}{9+2\gamma}, \alpha(1-\gamma)\right) & \& \gamma \in (0, 1) \end{cases}$$

(42)

where

$$M = \frac{2\alpha(\gamma^2 - 9 \gamma^2 - 9 \gamma + 8) - 24 \alpha + \sqrt{(\gamma - 1)(\gamma + 1)(\gamma - 2)^2(\gamma - 2)^2(4 \gamma - 1 - (\gamma - 4) \gamma^2 + 34 \gamma^2 + 34 \gamma^2 - 34 \gamma^2 + 72 \gamma - 36)\alpha^2 + 6 \alpha(\gamma - 1)(\gamma - 1)(\gamma - 1)(\gamma - 1)(\gamma - 1)(\gamma - 1)(\gamma - 1)(\gamma - 1)(\gamma - 1)(\gamma - 1)(\gamma - 1)(\gamma - 1)(\gamma - 1)(\gamma - 1)(\gamma - 1)} + 3 \alpha^2(\gamma^2 - 12)}{(\gamma - 1)(\gamma + 1)(\gamma - 2)^2(\gamma - 2)^2(4 \gamma - 1 - (\gamma - 4) \gamma^2 + 34 \gamma^2 + 34 \gamma^2 - 34 \gamma^2 + 72 \gamma - 36)\alpha^2 + 6 \alpha(\gamma - 1)(\gamma - 1)(\gamma - 1)(\gamma - 1)(\gamma - 1)(\gamma - 1)(\gamma - 1)(\gamma - 1)(\gamma - 1)(\gamma - 1)(\gamma - 1)(\gamma - 1)(\gamma - 1)(\gamma - 1)(\gamma - 1)} \right)$$

Within this range of trade costs and cost differential values, collusion welfare-dominates competition as long as good 1 and good 2 are imperfect substitutes.

6) Next, I consider the case of no trade and no domestic production of the cost-inefficient
good under collusion, while under competition the cost-inefficient good is traded, but not produced domestically: \( s > \alpha(1 - \gamma) \), \( s \in [\bar{s}_x, \alpha) \) and \( t \in (\alpha(1 - \gamma), \alpha(1 - \frac{1}{2}\gamma)) \). Then,

\[
\Delta W \equiv W^N(t, s, \gamma) - W^M(s, \gamma) = \frac{(2\alpha - \alpha\gamma - 2t)(3\alpha\gamma^3 - 2\alpha\gamma^2 + 2\gamma^2t - 20\alpha\gamma + 24\alpha - 24t)}{8(\gamma - 2)^2(\gamma + 2)^2} \tag{43}
\]

Equation (43) is a quadratic function in \( t \) only and that this case is only relevant when \( \gamma \neq 0 \). If \( \gamma = 0 \), then the case of \( t > s \), \( s > \alpha(1 - \gamma) \) and \( s \in (\bar{t}_x, \alpha) \) is non-existent as evident in Fig. (2).

For \( s > \alpha(1 - \gamma) \), \( s \in [\bar{s}_x, \alpha) \) and \( t \in (\alpha(1 - \gamma), \alpha(1 - \frac{1}{2}\gamma)) \), maximal collusion is welfare-superior to Cournot-Nash competition, \( W^M(s, \gamma) > W^N(t, s, \gamma) \), for

\[
\left\{ \begin{array}{l}
\frac{\alpha(3\gamma^3 - 2\gamma^2 - 20\gamma + 2)}{2(12 - \gamma^2)}, \alpha \left(1 - \frac{1}{2}\gamma\right) \\
\frac{\alpha(3\gamma^3 - 2\gamma^2 - 8\gamma + 12 + t^2(12 - \gamma^2)}{2(\gamma - 2)^2(2 + \gamma)^2} - \frac{32\alpha(1 - \gamma)(\alpha - t) + t^2}{8(1 - \gamma^2)}
\end{array} \right. \tag{44}
\]

Within this range of trade costs and cost differential values, collusion welfare-dominates competition regardless of the degree of product substitutability between good 1 and good 2.

7) In the case of the cost differential being prohibitively high for the cost-inefficient good to even be produced domestically under competition, while trade costs are sufficiently low and allow for trade of the cost-efficient good under maximal collusion, i.e. \( t < s, t < \alpha(1 - \gamma) \), and \( s \in [\bar{s}_x, \alpha) \), I find that the difference in welfare levels if given by

\[
\Delta W \equiv W^N(t, s, \gamma) - W^M(s, \gamma) = \frac{2\alpha(\alpha - t)(3\gamma^3 - 2\gamma^2 - 8\gamma + 12 + t^2(12 - \gamma^2)}{2(\gamma - 2)^2(2 + \gamma)^2} - \frac{32\alpha(1 - \gamma)(\alpha - t) + t^2}{8(1 - \gamma^2)} \tag{45}
\]

Clearly, expression (45) is a quadratic function in \( t \) only and, again, this case is relevant for \( \gamma > 0 \). If \( \gamma = 0 \), then the case of \( t < s, t < \alpha(1 - \gamma) \), and \( s \in (\bar{t}_x, \alpha) \) is non-existent as evident in Fig. (2). For this range of values of the cost differential and the transportation costs, competition welfare-dominates collusion for any degree of substitutability. That is, when \( t < s, t < \alpha(1 - \gamma) \), and \( s \in (\bar{s}_x, \alpha) \), \( W^N(t, s, \gamma) > W^M(t, \gamma) \) for any \( \gamma \in (0, 1) \) and any \( (t, s) \) within the relevant range.

Lastly, when \( t \geq \bar{t}_x \) and \( t \geq s < \alpha(1 - \gamma) \) under both competition and maximal collusion no trade in either of the goods takes place and each firm is a monopolist in its own market, producing both goods for domestic consumption only. Therefore, the levels of welfare under these types of market structure are identical. Furthermore, in the case of \( t > \alpha(1 - \frac{1}{2}\gamma) \) and \( s > \alpha(1 - \gamma) \), only the cost-efficient good is produced in the domestic market and no trade takes place under both types of market structure. Each firm is a monopolist in its own country and the welfare levels coincide.

Collusion and Welfare, Part II: Import Tariffs

In this section of Appendix A I study the welfare implications of collusion in the case of import tariffs. First, I define welfare when trade costs take the form of tariffs and then characterize it for both Cournot-Nash competition and maximal collusion. Second, I compare the levels of welfare under duopoly and monopoly when the goods are completely unrelated and show that collusion can again welfare-dominate competition. Then, I allow for the presence of imperfect substitutes and re-examine the validity of the results.

If trade costs take the form of import tariffs, welfare in the domestic country is given by

\[
V = CS + \Pi_1 + \Pi_2 + t_y_1 + t_y_2, \quad \text{where} \quad CS \text{ and } \Pi_k \text{ capture consumer surplus and the global profit from good } k, \text{ respectively. It is easy to verify that } CS = \frac{1}{2}(Q_1^2 + Q_2^2) + \gamma Q_1 Q_2. \quad \text{Utilizing these observations in the definition of welfare and simplifying gives the welfare of the domestic country:}
\]

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\[ V = \alpha (Q_1 + Q_2) - \frac{1}{2} (Q_1^2 + Q_2^2) - \gamma (Q_1) (Q_2) - s (Q_2) \]  

(46)

**Lemma A1.** (Nash Equilibrium) Under Cournot-Nash competition, reciprocal reductions in import tariffs (“trade liberalization”) enhance both countries’ welfare (i.e., \( \partial V^N / \partial (-t) > 0 \)). Furthermore,

- a) \( \arg \min_s V^N (t, s, \gamma) = \frac{(8\alpha - t)(1-\gamma)}{2(22 + 14\gamma)} \in (0, \bar{s}_y) \) if \( t \in \left( 0, \frac{2\alpha (1-\gamma)}{(14 + 9\gamma)} \right) \) and \( \arg \min_s V^N (t, s, \gamma) = \frac{[\alpha (16 - 11\gamma) + 16(1 + \gamma)](1-\gamma)}{(44 - 17\gamma)^2} \in [\bar{s}_y, \bar{s}_x] \) if \( t \in \left[ \frac{\alpha (2 + \gamma - \gamma^2)}{(20 + 8\gamma - 3\gamma^2)}, \alpha \right] \);

- b) \( V^N (0, s, \gamma) > V^N (\bar{t}_x, s, \gamma) \);

- c) \( V^N (t, s, \gamma) > W^N (t, s, \gamma) \) for all \( t \in (0, \bar{t}_x) \).

Trade liberalization improves welfare, \( V^N \), as it leads to an expansion of domestic output \( x^N_k + y^N_k \) for \( k = 1, 2 \). Welfare is non-monotonic in the production cost differential for \( 0 \leq s \leq \bar{s}_y \) and \( \bar{s}_y \leq s < \bar{s}_x \). Increases in the cost heterogeneity lead to higher production of the efficient good and lower production of the inefficient good, with the effect on total output being negative. As the cost asymmetry expands, consumer surplus and tariff revenue fall, while profits fall (rise) if the initial level of \( s \) is low (high), hence the non-monotonicity of \( V^N \) in \( s \).

**Proof of Lemma A1:** If trade costs take the form of import tariffs:

\[
V^N = \begin{cases} 
\alpha (Q_1 + Q_2)^N - \frac{1}{2} (Q_1^N + Q_2^N)^2 - \frac{1}{2} (Q_1^N)^2 - \gamma (Q_1^N) (Q_2^N) - s (Q_2^N) & \text{if } t < \bar{t}_y \\
\alpha (x_1^N + x_2^N)^2 - \frac{1}{2} (x_1^N)^2 - \frac{1}{2} (x_2^N)^2 - \gamma (x_1^N) (x_2^N) - s (x_2^N) & \text{if } t \geq \bar{t}_x \text{ and } s \leq \alpha (1-\gamma) \\
\alpha (x_1^N + y_2^N)^2 - \frac{1}{2} (x_1^N)^2 - \frac{1}{2} (y_2^N)^2 - \gamma (x_1^N) (y_2^N) & \text{if } s \geq \bar{s}_x \text{ and } t < \alpha (1-\gamma) \\
\alpha (x_1^N)^2 - \frac{1}{2} (x_1^N)^2 & \text{if } s \geq \bar{s}_x \text{ and } t \geq \alpha (1-\gamma) \\
\alpha (x_1^N)^2 - \frac{1}{2} (x_1^N)^2 & \text{if } s \geq \bar{s}_x \text{ and } t \geq \alpha (1-\gamma) \\
\end{cases}
\]

Thus, after rearranging terms and simplifying the expressions,

\[
\begin{align*}
\text{if } t < \bar{t}_y & \\
\text{if } \bar{t}_y \leq t \leq \bar{t}_x & \\
\text{if } t \geq \bar{t}_x & \text{ and } s \leq \alpha (1-\gamma) \\
\text{if } s \leq \bar{s}_y \text{ and } \bar{s}_y \leq s < \bar{s}_x & \\
\text{if } s \geq \bar{s}_x & \text{ and } t < \alpha (1-\gamma) \\
\text{if } s \geq \bar{s}_x \text{ and } t \geq \alpha (1-\gamma) \\
\end{align*}
\]

(47)

**Part (a).** From the definition of \( V^N (t, s, \gamma) \), it easily follows that if \( s \in (0, \bar{s}_y) \), then \( \frac{\partial V^N}{\partial s} = \frac{-8\alpha (1-\gamma) + t(1-\gamma) + 2s(11+\gamma)}{9(1-\gamma)^2} \leq 0 \) and \( s^N_{\min} = \frac{(8\alpha - t)(1-\gamma)}{2(22 + 14\gamma)} \) with \( s^N_{\min} \in (0, \bar{s}_y) \) if \( t \in \left[ 0, \frac{2\alpha (1-\gamma)}{(14 + 9\gamma)} \right] \).

If \( s \in [\bar{s}_y, \bar{s}_x] \), \( \frac{\partial V^N}{\partial s} = \frac{(16 - 11\gamma)(1-\gamma) + s(117-44) + 16(1-\gamma)^2}{36(1-\gamma)^2} \leq 0 \) and \( s^N_{\min} = \frac{(\alpha (16 - 11\gamma) + 16(1+\gamma))(1-\gamma)}{(44 - 17\gamma)^2} \) with \( s^N_{\min} \in [\bar{s}_y, \bar{s}_x] \) if \( t \in \left[ \frac{\alpha (2 + \gamma - \gamma^2)}{(20 + 8\gamma - 3\gamma^2)}, \alpha \right] \).
Part (b). First, note that $V^N(0, s, \gamma) = \frac{8a^2(1-\gamma)-8as(1-\gamma)+s^2(1+7\gamma)}{9(1-\gamma)}$ and also $V^N(\tilde{t}_x, s, \gamma) = \frac{3(2\alpha^2\gamma-2\alpha\gamma s-2\alpha^2+2\alpha s-s^2)}{8(1-\gamma)}$. Comparing these two expressions, one can easily verify that $V^N(0, s, \gamma) > V^N(\tilde{t}_x, s, \gamma)$.

Part (c). From the definitions of $W^N(t, s, \gamma)$ and $V^N(t, s, \gamma)$ it easily follows that $V^N(t, s, \gamma) - W^N(t, s, \gamma) = \frac{t(2\alpha s-4t)}{1+\gamma} = ty^N_1 + y^N_2 > 0$ for $t \in [0, \tilde{t}_y)$. Also, for $t \in [\tilde{t}_y, \tilde{t}_x)$, $V^N(t, s, \gamma) - W^N(t, s, \gamma) = \frac{t(\alpha s-2t)}{3} = ty^N_2 > 0$.

**Lemma A2. (Maximal Collusion) Under maximal collusion, national welfare in the case of import tariffs, $V^M$, has the following properties:**

a) If $t \leq \min(s, \alpha(1-\gamma))$, then $\frac{\partial V^M(t)}{\partial t} < 0$ and $\frac{\partial^2 V^M(t)}{\partial t^2} < 0$, with $\arg\max_t V^M(t) \to 0$ and $\arg\min_t V^M(t) \to \min(s, \alpha(1-\gamma))$. If $t \in [\alpha(1-\gamma), s)$, then $\frac{\partial V^M(t)}{\partial t} = 0$.

b) If $s \leq \min(t, \alpha(1-\gamma))$, then $V^M(s) \equiv W^M(s)$. Moreover, $\lim_{s \to t} V^M(s) = \begin{cases} \frac{3(2\alpha(1-\gamma)(\alpha-t)+t^2)}{8(1-\gamma)} & \text{if } t < \min(s, \alpha(1-\gamma)) \\ \frac{2(1-\gamma)(3\alpha^2-\alpha t-t^2)}{8(1-\gamma^2)} & \text{if } t \geq \alpha(1-\gamma), s \\ \frac{3(2\alpha(1-\gamma)(\alpha-s)+s^2)}{8(1-\gamma^2)} & \text{if } s \leq \min(t, \alpha(1-\gamma)) \\ \frac{3\alpha^2}{8} & \text{if } s \geq \alpha(1-\gamma), t \end{cases}$ (48)

a) If $t < \min(s, \alpha(1-\gamma))$, then $\frac{\partial V^M(t)}{\partial t} = -\frac{\alpha(1-\gamma)+t}{4(1-\gamma^2)} < 0$ and $\frac{\partial^2 V^M(t)}{\partial t^2} = \frac{1}{4(1-\gamma^2)} < 0$, with $\arg\max_t V^M(t) \to 0$ and $\arg\min_t V^M(t) \to \min(s, \alpha(1-\gamma))$. If $t \in [\alpha(1-\gamma), s)$, then $\frac{\partial V^M(t)}{\partial t} = 0$.

b) If $s \leq \min(t, \alpha(1-\gamma))$, then $V^M(s) \equiv W^M(s)$ as evident from (19) and (18). Moreover, $\lim_{s \to t} V^M(s) = \frac{3(2\alpha(1-\gamma)(\alpha-t)+t^2)}{8(1-\gamma^2)} \neq V^M(t) = \frac{2(1-\gamma)(3\alpha^2-t^2)}{8(1-\gamma^2)}$, suggesting $V^M$ is discontinuous at $t = s$.

**Proposition A1. (Welfare Comparison – Tariffs) Suppose the two goods are completely unrelated (i.e., $\gamma = 0$) and trade costs take the form of tariffs. Then, for any $t \in [0, \alpha)$, there exists a range of values for the cost differential $s$ such that $0 \leq t < s < \tilde{s}_s$ that imply collusion welfare-dominates competition ($V^M(t, s, 0) > V^N(t, s, 0)$) for all $s$ in this range. In this case, under collusion, firms specialize completely in the good they produce most efficiently and trade it internationally, whereas under competition firms may produce and trade their inefficient good.
Fig. (6) provides a graphical illustration of Proposition A1. In the case of tariffs, maximal collusion might welfare-dominate Cournot competition for any tariff level as long as the cost heterogeneity is sufficiently greater than the heterogeneity in trade policy. As evident from region A of Fig. (6), this result holds true both when the duopolists trade both goods and when they trade only the less costly good, as long as the cost differential is considerably large. The monopolist specializes completely in the production of the low-cost good in each market and imports the other one, generates efficiency gains, and enjoys larger profits. Two additional effects contribute to this novel result: 1) when the goods are completely unrelated, shipping more of one good will not inflict a negative price externality onto the other; 2) as long as the initial level of trade costs is not infinitesimal, the cartel enjoys greater tariff revenues than the competitive firms. Thus, the increase in profits and tariff revenues due to the switch from competition to collusion more than offsets the reduction in consumer surplus and maximal collusion welfare-dominates Cournot competition.

As Fig. (6) shows, in the case of import tariffs, the symmetry between \( t \) and \( s \) with respect to the 45°-line disappears. Welfare under collusion in the case of tariffs is discontinuous in \( s \) and in \( t \) at \( s = t \). This discontinuity is due to the presence (resp., absence) of tariff revenues for \( t \leq s \) (resp., \( t > s \)). Specifically, for any \( t \leq s \), the cartel imports the cost-inefficient good and accumulates positive tariff revenues. However, for any \( t > s \), there is no trade between the two countries under unconstrained collusion, the goods are produced only domestically, and the cartel no longer enjoys tariff revenues. Welfare under collusion is decreasing in \( s \) as both profits and consumer surplus fall when the cost heterogeneity expands. Under Cournot competition, however, the duopolists still trade either both goods (\( t < \bar{t}_y \)) or only the cost-efficient good (\( \bar{t}_y \leq t < \bar{t}_x \)). In this case, the higher monopoly profits fail to compensate for the decrease in consumer surplus and the absence of tariff revenues. Therefore, welfare under competition exceeds welfare under collusion, as shown in region B of Fig. (6). Further, for prohibitively high tariffs, \( t \in [\bar{t}_x, \alpha) \), or cost differential values, \( s \in [\bar{s}_x, \alpha) \), welfare under maximal collusion coincides with welfare under competition.

Next, I relax the assumption of unrelated goods and allow for \( \gamma \in (0,1) \). Collusion can still welfare-dominate competition for certain values of trade costs and the cost differential. However, when trade costs take the form of tariffs, there exists a threshold value for \( \gamma > 0 \) beyond which the cartel fails to enhance national welfare for all relevant \((t, s)\) values. The following proposition provides a description of these possibilities.
Proposition A2: (Welfare and Product Substitutability) The higher the degree of substitutability between goods $\gamma$, the smaller the set of $(t, s)$ under which collusion welfare-dominates competition. In particular, if trade costs takes the form of tariffs, there exists a threshold $\hat{\gamma} \in (0, 1)$, such that competition welfare-dominates monopoly for all $(t, s)$ if $\gamma \in (\hat{\gamma}, 1)$.

![Figure 7: Welfare Comparison under Tariffs for Imperfect Substitutes ($\gamma = 0.091$)](image)

![Figure 8: Welfare Comparison under Tariffs for Imperfect Substitutes ($\gamma = 0.103$)](image)

I refer to Figs (7) - (8) to convey the intuition behind Proposition A2. When the assumption of unrelated goods is relaxed, region A from Fig. (6) shrinks. As the goods become closer substitutes, producing more of any one good inflicts a stronger negative externality onto the the marginal revenue of the other. Profits, consumer surplus and tariff revenues fall, with the decline in cartel profits and tariff revenue being more pronounced. As $\gamma$ surpasses the threshold level $\hat{\gamma}$, region A from Fig. (6) vanishes completely. If the degree of substitutability is sufficiently large,
then competition weakly welfare-dominates unconstrained collusion for all values of the cost heterogeneity and all tariff levels, as illustrated in Figs [7] – [8]. When \( \gamma = 0.001 \) collusion can no longer offer a greater level of welfare for any combinations of \((t, s) \in [0, \tilde{t}_y) \times (0, \tilde{s}_y)\), but can still welfare-dominate competition for some values of \((t, s) \in [\tilde{t}_y, s) \times [\tilde{s}_y, \tilde{s}_x)\). However, when the degree of product substitutability surpasses \( \gamma = 0.103 \equiv \hat{\gamma} \), competition is welfare-superior for all \((t, s)\).

**Proof of Proposition A1 & Proposition A2:** Using the definitions of welfare under Cournot-Nash competition and maximal collusion, I examine the shape and properties of the welfare functions for all possible values of \( s \) and \( t \).

1) First, I focus on the case when tariffs are sufficiently small so that both goods are traded under competition, while under collusion only the cost-efficient good is traded, i.e. \( t < s, t < \min(\alpha(1 - \gamma), \tilde{t}_y) \). Then, the difference \( V \equiv V^N(t, s, \gamma) - V^M(t, \gamma) \) is equal to

\[
\Delta V \equiv V^N(t, s, \gamma) - V^M(t, \gamma) = \frac{(1 - \gamma)(10\alpha^2 + 2\alpha t - 64\alpha s + 8st) + s^2(88 + 56\gamma) + t^2(1 + 8\gamma)}{72(1 - \gamma^2)}
\]

Equation (49) is also a quadratic function in \((t, s)\) and in order to examine its shape, I need to first make sure that it is a non-degenerate, i.e. its Hessian is invertible. Henceforth, I denote \( V_z \) the partial derivative of the difference \( \Delta V \equiv V^N(t, s, \gamma) - V^M(t, \gamma) \) with respect to variable \( z \).

\[
V_t = -\frac{(1 - \gamma)(\alpha - 4s) - t(1 + 8\gamma)}{36(1 - \gamma^2)}
\]

\[
V_s = \frac{(1 - \gamma)(-8\alpha + t) + (14\gamma + 22)s}{9(1 - \gamma^2)}
\]

Then, using the second-order derivatives of \( \Delta W \) with respect to \( t \) and \( s \), the Hessian is given by:

\[
H_W = \begin{bmatrix}
\frac{(1 + 8\gamma)}{36(1 - \gamma^2)} & \frac{1}{9(1 + \gamma)} \\
\frac{1}{9(1 + \gamma)} & \frac{2}{9(1 + \gamma^2)}
\end{bmatrix}
\]

and is invertible for all values of \( \gamma \in [0, 1) \). The critical point is given by: \( t_0 = -\frac{\alpha(\gamma - 1)(\gamma - 5)}{6\gamma^2 + 11\gamma + 1} \) and \( s_0 = \frac{(7\gamma + 1)(1 - \gamma)\alpha}{2(6\gamma^2 + 11\gamma + 1)} \). Furthermore, as the determinant of \( H_W \) is positive, (49) will take the form of an ellipse for all admissible values of \( \gamma \in [0, 1) \).

When \( t = 0 \) and \( \gamma = 0 \), \( V = \frac{1}{36} (\alpha - 2s) (5\alpha - 22s) \) and therefore, the \( s \)-intercepts are \( s = \frac{\alpha}{2} \) and \( s = \frac{5\alpha}{22} \). On the other hand, at \( s = 0 \) and \( \gamma = 0 \), \( V = \frac{5\alpha^2}{36} + \frac{1}{36} \alpha t + \frac{t^2}{72} \) and the equation has no real solutions, suggesting that the ellipse never crosses the \( t \)-axis. When \( t = 0 \) and \( \gamma = 0 \), welfare under monopoly is greater than welfare under competition as long as \( s \in \left(\frac{5\alpha}{22}, \frac{\alpha}{2}\right] \). The possibility that \( V^M(t, 0) > V^N(t, s, 0) \) when \( t < s, t < \min(\alpha(1 - \gamma), \tilde{t}_y) \) and \( \gamma = 0 \) arises for

\[
\left\{ s \in \left(\frac{4}{17}\alpha - \frac{1}{22}t - \frac{3}{44}\sqrt{4\alpha^2 - 12\alpha t - 2t^2}, \tilde{s}_y\right) \quad \& \quad t \in [0, \frac{2}{3}\alpha) \right\}
\]

The same result holds true even for a positive degree of substitutability between good 1 and good 2, as long as \( \gamma < \hat{\gamma} \). To find this threshold value \( \hat{\gamma} \), I need to find the value of \( \gamma \)

\footnote{Given that trade costs are assumed to equal 0 in this case, the differences in welfare under tariffs and transportation costs are exactly the same. Hence, the values of the cost differential, for which monopoly welfare-dominates competition, are also the same.}
such that the ellipse is exactly tangent to the \( s \)-axis. Thus, I evaluate equation (49) at \( t = 0 \), find the value of \( s \) that maximizes it, \( s^* \), and then evaluate (49) at \( t = 0 \) and \( s = s^* \). Thus, (49) at \( t = 0 \) and \( s = s^* = \frac{4\alpha(1-\gamma)}{(11+7\gamma)} \) is equal to

\[
\Delta V|_{t=0,s=s^*} = \frac{\alpha^2(11\gamma - 1)}{4(1+\gamma)(11+7\gamma)}
\]

Therefore, \( \hat{\gamma} = \frac{1}{11} \) and \( V^M(t, \gamma) > V^N(t, s, \gamma) \) when \( t < s, t < \min(\alpha(1-\gamma), \bar{t}_y) \) and \( \gamma \in [0, \hat{\gamma}) \). Moreover, as \( \gamma \) increases and approaches \( \hat{\gamma} \), the set of values of the cost differential and tariff levels, \((t, s)\), for which \( V^M(t, \gamma) > V^N(t, s, \gamma) \) shrinks and vanishes when \( \gamma \geq \hat{\gamma} \).

2) Now, I move on to the case of no trade of any of the goods under collusion, while trade in both goods persists under competition: \( t \geq s, s < \min(\alpha(1-\gamma), \bar{s}_y) \). The difference in the two welfare functions is then given by:

\[
\Delta V = V^N(t, s, \gamma) - V^M(s, \gamma) = \frac{(1-\gamma)(10\alpha^2 + 16\alpha t + 10\alpha s - 8st + 8t^2) - s^2(61 + 56\gamma)}{72(1-\gamma^2)}
\]

Equation (54) is also a quadratic function in \((t, s)\) and in order to examine its shape, I first make sure that it is a non-degenerate.

\[
V_t = -\frac{2(\alpha + t) - s}{9(1+\gamma)} \quad V_s = \frac{5\alpha \gamma + 56s\gamma - 4t\gamma - 5\alpha + 61s + 4t}{36(1-\gamma^2)}
\]

Then, using the second-order derivatives of \( \Delta W \) with respect to \( t \) and \( s \), the Hessian is given by: \( H_W = \begin{bmatrix} \frac{-2}{9(1+\gamma)} & \frac{1}{9(1+\gamma)} \\ \frac{9+5\gamma}{61+56\gamma} & \frac{36(1-\gamma^2)}{36(1-\gamma^2)} \end{bmatrix} \) and its determinant is negative for all values of \( \gamma \in [0, 1) \). Thus, there is a saddle point given by: \( t_0 = -\frac{4\alpha(\gamma+1)}{3\gamma+5} \) and \( s_0 = -\frac{2\alpha(\gamma-1)}{3\gamma+5} \). Further, given that the determinant of \( H_W \) is negative, (54) will take the form of a hyperbola for any \( \gamma \in [0, 1) \).

When \( t \geq s, s < \min(\alpha(1-\gamma), \bar{s}_y) \), \( V^M(t, \gamma) < V^N(t, s, \gamma) \) for all possible values of the cost differential and all tariff levels. Within this range of values of the cost differential and tariff levels, competition always welfare-dominates collusion.

3) Next, I consider the case of trade of the cost-efficient good under collusion, while there is no trade in the cost-inefficient good under competition: \( t < \min(s, \alpha(1-\gamma)) \) and \( s \in [\bar{s}_y, \bar{s}_x] \). The difference in the two welfare functions is then given by:

\[
\Delta V = V^N(t, s, \gamma) - V^M(t, \gamma) = \frac{-(1-\gamma^2)(-5\alpha^2 + 32st) - t^2(5 + 4\gamma^2) - s^2(17\gamma - 44) + \alpha s(32 - 54\gamma) - \alpha t(10 + 18\gamma) + \alpha^2(22s - 8t)}{72(1-\gamma^2)}
\]

Equation (57) is also a quadratic function in \((t, s)\) and in order to examine its shape, I
need to first make sure that it is a non-degenerate, i.e. its Hessian is invertible.

\[
V_t = \frac{4\alpha \gamma^2 + 16\gamma^2s + 4\gamma^2t - 9\alpha \gamma + 5\alpha - 16s + 5t}{36(1-\gamma^2)} \tag{58}
\]

\[
V_s = -\frac{11\alpha \gamma^2 + 17\gamma^2s - 16\gamma^2t - 27\alpha \gamma + 16\alpha - 44s + 16t}{36(1-\gamma^2)} \tag{59}
\]

Then, using the second-order derivatives of \(\Delta W\) with respect to \(t\) and \(s\), the Hessian is given by: 
\[
H_W = \begin{bmatrix}
\frac{5+4\gamma^2}{36(1-\gamma^2)} & -\frac{4}{9} \\
-\frac{4}{9} & \frac{17\gamma^2-44}{36(1-\gamma^2)}
\end{bmatrix}
\]
and its determinant is negative for all values of \(\gamma \in [0, 0.249]\).

Thus, there is a saddle point given by: 
\[
t_0 = -\frac{4\alpha(\gamma+1)}{3\gamma+5} \quad \text{and} \quad s_0 = -\frac{2\alpha(\gamma-1)}{3\gamma+5}\quad \text{when} \quad \gamma \in [0, 0.249).
\]

As the determinant of \(H_W\) is negative for \(\gamma \in [0, 0.249]\), \(\eqref{57}\) will take the form of a hyperbola, while for \(\gamma \in [0, 0.249]\) \(\eqref{57}\) will be an ellipse.

Monopoly can again welfare-dominate competition, \(V^M(t, 0) > V^N(t, s, 0)\), when \(t < \min(s, \alpha(1 - \gamma))\) and \(s \in [\bar{s}_y, \bar{s}_x]\) and \(\gamma = 0\)

\[
\left\{ \begin{array}{cl}
s \in [\bar{s}_y, \bar{s}_x) & \quad t \in [0, \frac{2}{9}\alpha] \\
s \in (\frac{5}{27}(\alpha + t), \bar{s}_x) & \quad t \in (\frac{5}{27}\alpha, \frac{5}{17}\alpha] \\
s \in (t, \bar{s}_x) & \quad t \in (\frac{5}{17}\alpha, \alpha)
\end{array} \right. \tag{60}
\]

The same result holds true even for a positive degree of substitutability between good 1 and good 2, as long as \(\gamma < \hat{\gamma}\). To find this threshold value \(\hat{\gamma}\), I find the value of \(\gamma\) such that the ellipse is exactly tangent to the 45\(^o\)-line. Thus, I evaluate equation \(\eqref{57}\) at \(t = s\), find the value of \(s\) that maximizes it, \(s^*\), and then evaluate \(\eqref{57}\) at \(t = s\) and \(s = s^*\). Thus, \(\eqref{57}\) at \(t = s\) and \(s = s^* = \frac{\alpha(1 - \gamma)(11 - 7\gamma)}{19\gamma^2 + 17}\) is equal to

\[
\Delta V|_{t=s, s=s^*} = \frac{\alpha^2(4\gamma^3 - 3\gamma^2 + 10\gamma - 1)}{2(1 + \gamma)(19\gamma^2 + 17)} \tag{61}
\]

Therefore, \(\hat{\gamma} = 0.103\) and \(V^M(t, \gamma) > V^N(t, s, \gamma)\) when \(t < s\), \(t < \alpha(1 - \gamma)\) and \(s \in [\bar{s}_y, \bar{s}_x]\) and \(\gamma \in [0, \hat{\gamma})\). Moreover, as \(\gamma\) increases and approaches \(\hat{\gamma}\), the set of values of the cost differential and tariff levels, \((t, s)\), for which \(V^M(t, \gamma) > V^N(t, s, \gamma)\) shrinks and vanishes when \(\gamma \geq \hat{\gamma}\).

4) Now, I focus on the case of no trade of any of the goods under collusion and no trade of the cost-inefficient good under competition: \(s < \min(t, \alpha(1 - \gamma))\) and \(t \in [\bar{t}_y, \bar{t}_x]\). The difference in the two welfare functions is then given by:

\[
\Delta V \equiv V^N(t, s, \gamma) - V^M(s, \gamma) = \frac{1}{72}(\alpha + s - 2t)(5\alpha + 2t + 17s) \tag{62}
\]

Equation \(\eqref{62}\) is also a quadratic function in \((t, s)\) and in order to examine its shape, I need to first make sure that it is a non-degenerate, i.e. its Hessian is invertible.

\[
V_t = -\frac{1}{9}(\alpha - 4s - t) \tag{63}
\]
Then, using the second-order derivatives of $\Delta W$ with respect to $t$ and $s$, the Hessian is given by: $H_W = \begin{bmatrix} \frac{4-3\gamma^2}{(\gamma-2)^2(\gamma+2)^2} & 0 \\ 0 & \frac{3(\alpha(1-\gamma)-s)}{4(1-\gamma^2)} \end{bmatrix}$ and its determinant is positive for all values of $\gamma \in [0, 1)$. The critical point is given by: $t_0 = \frac{3\alpha}{3\gamma^2-4}$ and $s_0 = \alpha(1-\gamma)$. As the determinant of $H_W$ is positive, (65) will take the shape of an ellipse for all $\gamma \in [0, 1)$. For this range of tariff levels and cost differential values, $t > s$, $t < \alpha(1-\gamma)$ and $t \in [\bar{t}_y, \bar{t}_x)$, Cournot-Nash competition always provides a higher level of welfare than maximal collusion. When $t > s$, $t < \alpha(1-\gamma)$ and $t \in [\bar{t}_y, \bar{t}_x)$, $V^N(t, s, \gamma) > V^M(s, \gamma)$. 

5) Two more regions become relevant for welfare comparisons when $\gamma \neq 0$. When there is no trade of any of the goods under unconstrained collusion, but the cost-inefficient good is still produced domestically, while under competition the cost-inefficient good is not produced in the domestic market any longer, but is still traded: $t > s$ and $s \in [\bar{s}_x, \alpha(1-\gamma))$. The difference in the two welfare functions is then given by:

$$\Delta V \equiv V^N(t, s, \gamma) - V^M(s, \gamma) = \frac{-2\alpha(t-2)^2 + \alpha^2(24 - 16\gamma) + t^2(3\gamma^2 - 4) - 3\alpha(1-\gamma)(\alpha - s) + s^2}{2(\gamma-2)^2(\gamma+2)^2} \quad (65)$$

Equation (65) is also a quadratic function in $(t, s)$ and in order to examine its shape, I need to first make sure that it is a non-degenerate, i.e. its Hessian is invertible.

$$V_t = \frac{-4\alpha(1-\gamma) + \alpha \gamma^2 + t(4 - 3\gamma^2)}{(\gamma - 2)^2(\gamma + 2)^2} \quad (66)$$

$$V_s = \frac{1}{36}(11\alpha + 17s - 16t) \quad (67)$$

Then, using the second-order derivatives of $\Delta W$ with respect to $t$ and $s$, the Hessian is given by: $H_W = \begin{bmatrix} \frac{4-3\gamma^2}{(\gamma-2)^2(\gamma+2)^2} & 0 \\ 0 & \frac{3(\alpha(1-\gamma)-s)}{4(1-\gamma^2)} \end{bmatrix}$ and its determinant of the Hessian is positive for all values of $\gamma \in [0, 1)$. The critical point is given by: $t_0 = \frac{\alpha(\gamma-2)^2}{3\gamma^2-4}$ and $s_0 = \alpha(1-\gamma)$. As the determinant of $H_W$ is positive, (65) will take the shape of an ellipse for all $\gamma \in [0, 1)$. For this range of tariff levels and cost differential values, $t > s$ and $s \in [\bar{s}_x, \alpha(1-\gamma))$, competition always provides a higher level of welfare than collusion. When $t > s$ and $s \in [\bar{s}_x, \alpha(1-\gamma))$, $V^N(t, s, \gamma) > V^M(s, \gamma)$. 

6) Naturally, the next case to consider is when under maximal collusion the cost-inefficient good is neither produced domestically nor traded, while under competition the cost-inefficient good is not produced in the domestic market any longer, but is still traded: $s > \alpha(1-\gamma)$, $s \in [\bar{s}_x, \alpha)$ and $t \in [\alpha(1-\gamma), \alpha(1-\frac{1}{2}\gamma))$. The difference in the two welfare functions is then given by:

$$\Delta V \equiv V^N(t, s, \gamma) - V^M(s, \gamma) = \frac{2\alpha^2\gamma^3 - 2\alpha^2\gamma^2 - 2\alpha \gamma^2 t + 3\gamma^2 t^2 - 16\alpha^2 \gamma + 8\alpha \gamma t + 24\alpha^2 - 8 \alpha t - 4t^2}{2(\gamma-2)^2(2+\gamma)^2} - \frac{3}{8}\alpha^2 \quad (68)$$

60
Equation (68) is also a quadratic function in $t$ only with

$$V_t = -\frac{4\alpha(1 - \gamma) + \alpha \gamma^2 + t(4 - 3\gamma^2)}{(\gamma - 2)^2(\gamma + 2)^2}$$

(69)

$$V_{tt} = -\frac{4 - 3\gamma^2}{(\gamma - 2)^2(\gamma + 2)^2}$$

(70)

For this range of tariff levels and cost differential values, $t > \alpha(1 - \gamma), s > \alpha(1 - \gamma), s \in [\bar{s}_x, \alpha)$ and $t < \alpha(1 - \frac{1}{2}\gamma)$, competition always provides a higher level of welfare than collusion for any degree of product substitutability. When $s > \alpha(1 - \gamma), s \in [\bar{s}_x, \alpha)$ and $t \in [\alpha(1 - \gamma), \alpha(1 - \frac{1}{2}\gamma)], V^N(t, s, \gamma) > V^M(s, \gamma)$ for any value of $\gamma \in (0, 1)$.

7) Next, I look at the possibility of the cost differential being prohibitively high so that the cost-inefficient good is not produced in the domestic market under Cournot-Nash competition, while under collusion tariff levels allow for trade and complete specialization: $t < s, t < \alpha(1 - \gamma)$, and $s \in [\bar{s}_x, \alpha)$. The difference in the two welfare functions is then:

$$\Delta V \equiv V^N(t, s, \gamma) - V^M(s, \gamma) = \frac{2\alpha^2\gamma^3 - 2\alpha^2\gamma^2 - 2\alpha \gamma^2 t + 3\gamma^2 t^2 - 16 \alpha^2 \gamma + 8 \alpha t + 24 \alpha^2 - 8 \alpha t - 4t^2}{2(\gamma - 2)^2(2 + \gamma)^2}$$

$$-\frac{2(1 - \gamma)(3\alpha^2 - \alpha t) - t^2}{8(1 - \gamma^2)}$$

(71)

Equation (71) is also a quadratic function in $t$ only with

$$V_t = -\frac{\gamma^2 (\alpha \gamma^3 + 11 t\gamma^2 - 5 \alpha \gamma^2 + 8 \alpha \gamma - 20 t - 4 \alpha)}{4(\gamma - 2)^2(\gamma + 2)^2(1 - \gamma^2)}$$

(72)

$$V_{tt} = \frac{\gamma^2 (20 - 11 \gamma^2)}{4(\gamma - 2)^2(\gamma + 2)^2(1 - \gamma^2)}$$

(73)

For this range of tariff levels and cost differential values, $t < s, t < \alpha(1 - \gamma)$, and $s \in [\bar{s}_x, \alpha)$, competition is welfare-superior to collusion for any degree of product substitutability. When $t < s, t < \alpha(1 - \gamma)$, and $s \in [\bar{s}_x, \alpha), V^N(t, s, \gamma) > V^M(s, \gamma)$ for any value of $\gamma \in (0, 1)$.

When $t \geq \bar{t}_x$ and $t \geq s < \alpha(1 - \gamma)$ under both competition and collusion no trade in either of the goods takes place and each firm is a monopolist in its own market, producing both goods for domestic consumption only. The levels of welfare under competition and under collusion are identical. Moreover, when $t > \alpha(1 - \frac{1}{2}\gamma)$ and $s > \alpha(1 - \gamma)$, only the cost-efficient good is produced in the domestic market and no trade takes place under both types of market structure. In this case, each firm is a monopolist in its own country and the welfare levels coincide.

Stability of Maximal Collusion

In this section I analyze the sustainability of unconstrained collusion, its dependence on trade costs and the cost differential, and verify that the aforementioned findings hold true.

Multi-market collusion arises in the model as a result of repeated firm interactions over
the infinite time horizon and in multiple markets. The literature on game theory has already established that repeated interactions over time allow firms to sustain collusion by acting cooperatively until one of them deviates and retaliation ensues afterwards. It is also well known from Bernheim and Whinston (1990) that if firms meet in more than one isolated market, then any deviation by a cartel member in a given market will extend to all markets and will be followed by punishment in all markets. Multi-market interactions can facilitate cartel agreements by allowing members to pool their incentive constraints across markets and thus relaxing a binding incentive-compatibility constraint.

Let \( \delta < 1 \) denote the firms’ common discount factor. Following Bernheim and Whinston (1990), I assume multi-market collusion requires allocating to each firm a pair \((x, y)\), which denotes the output vector of good 1 and good 2 targeted for domestic production and imports, respectively. Collusion is sustained through a grim-trigger strategy, stipulating continued adherence to the prescribed output \((x, y)\) and reversion to the Cournot competitive equilibrium upon violation of the cartel agreement. Collusion will be stable as long as the present value of profits along the collusive path outweighs the one-period gain from deviation plus the discounted profit loss from the ensuing punishment. Therefore, the incentive-compatibility constraint, which ensures a sustainable cartel agreement is given by:

\[
\frac{1}{1 - \delta} \Pi^C(x, y, t, s) \geq \Pi^D(x, y, t, s) + \frac{\delta}{1 - \delta} \Pi^N(t, s) \tag{74}
\]

The no-deviation constraint, (74), is pooled both across markets and across products. I let the no-deviation constraint (74) bind and solve for \( \delta^M \), the minimum discount factor capable of sustaining maximal collusion:

\[
\delta^M(t, s) = \frac{\Pi^D(x^M, y^M, t, s) - \Pi^M(x^M, y^M, t, s)}{\Pi^D(x^M, y^M, t, s) - \Pi^N(t, s)} \tag{75}
\]

As outlined in the earlier analysis, the monopoly output levels depend on the interaction between the cost heterogeneity and the heterogeneity in geography. Recall that if trade costs are below the cost asymmetry, \( t < s \), complete specialization is established under maximal collusion and each good is produced only in the country that has a competitive advantage in it and exported to the other market. On the other hand, if trade costs exceed the cost heterogeneity, \( t > s \), there is no trade and both goods are produced domestically in both markets. And, in case of trade costs being equal to the cost asymmetry, the cartel is indifferent between importing the cost-inefficient good and producing it domestically. Therefore, any combination of outputs satisfying \( x_2^M + y_2^M = \alpha - s \) maximizes the monopolist’s profits, when \( t = s \). The optimal deviation payoffs, however, are minimized when both cartel members share equally each market due to the strict convexity of \( \Pi^D \) in \((x, y)\). Thus, when \( t = s \), each firm will choose to maintain presence in its rival’s market and \( x_2^M = y_2^M = \alpha - s \). Lastly, we are already familiar with the dependence of profits under monopoly, optimal deviation and competition on \( t \) and \( s \), outlined in the previous section. The minimum discount factor capable of sustaining maximal collusion exhibits the following properties:

**Proposition A3** (Minimum Discount Factor) The minimum discount factor that is capable of sustaining maximal collusion, \( \delta^M(t, s) \), has the following properties:
a) if \( t < s \),
   i) \( \delta^M(0, s) = \frac{9(\alpha-2s)}{3s+2s^2} \), \( \delta^M(\bar{t}_y, s) = \frac{9(\alpha-2s)}{3s+2s^2} \), \( \delta^M(t, \bar{t}_y^{-1}, 0) = \frac{9s}{8s-19t} \)
   ii) \( \exists t_{\delta_1} \in [0, \bar{t}_y) \) s.t. \( \frac{\partial \delta^M}{\partial t} \leq 0 \) for \( t \leq t_{\delta_1} \) and \( \frac{\partial \delta^M}{\partial t} > 0 \) for \( t \in [\bar{t}_y, s) \)
   iii) \( \exists s_{\delta_1} \in (t, \bar{s}_y) \) s.t. \( \frac{\partial \delta^M}{\partial s} \leq 0 \) for \( s \leq s_{\delta_1} \) and \( \exists s_{\delta_2} \in [\bar{s}_y, \bar{t}_x) \) s.t. \( \frac{\partial \delta^M}{\partial s} \leq 0 \) for \( s \leq s_{\delta_2} \)

b) if \( t > s \),
   i) \( \delta^M(t, 0) = \frac{9(\alpha-2t)}{3s+2s^2} \), \( \delta^M(\bar{t}_y, s) = \frac{9s}{8s-19t} \), \( \delta^M(t, \bar{t}_x^{-1}) = \frac{9(\alpha-2t)}{3s+2s^2} \)
   ii) \( \exists t_{\delta_2} \in (s, \bar{t}_y) \) s.t. \( \frac{\partial \delta^M}{\partial t} \leq 0 \) for \( t \leq t_{\delta_2} \) and \( \exists t_{\delta_3} \in [\bar{t}_y, \bar{t}_x) \) s.t. \( \frac{\partial \delta^M}{\partial t} \leq 0 \) for \( t \leq t_{\delta_3} \)
   iii) \( \exists s_{\delta_3} \in (0, \bar{s}_y) \) s.t. \( \frac{\partial \delta^M}{\partial s} \geq 0 \) for \( s \geq s_{\delta_3} \) and \( \frac{\partial \delta^M}{\partial s} > 0 \) for \( s \in [\bar{s}_y, t) \)

c) if \( t = s \), then \( \delta^M(s, s) = \frac{3(\alpha^2-6\alpha+11s^2)}{19\alpha^2-34\alpha+34s^2} \), \( \Pi^N = \lim_{t \to s} \delta^M = \frac{2\alpha^2-10\alpha+17s^2}{10\alpha^2-18\alpha+21s^2-16H^N} \), where \( \Pi^N \) is given in equation (5).

Proposition A3 part a) focuses on the case of trade being present under collusion. Part a.i) evaluates the minimum discount factor at the extremes of free trade (\( t = 0 \)) and no trade in the cost-inefficient good (\( t \geq \bar{t}_y \) and \( s \geq \bar{s}_y \)). Parts a.ii) and a.iii) show the dependence of the discount factor on trade costs and the cost differential for all relevant intervals. Interestingly, the relation between \( \delta^M \) and \( t \) (resp., \( s \)), holding \( s \) (resp., \( t \)) constant, is non-monotonic. When \( t < s \), \( \Pi^M \) is decreasing in \( t \) and independent of \( s \), \( \Pi^D \) is decreasing in \( (t, s) \), while \( \Pi^N \) is non-monotonic in \( (t, s) \). Increases in trade costs raise cartel’s exporting costs and reduce profits. As \( t \) rises, profits under optimal deviation fall due to the higher costs of trading, which enhances collusive stability. The punishment profits, on the other hand, depend non-monotonically on \( t \): for low values of \( t \), \( \partial \Pi^N / \partial t < 0 \), while for high values of \( t \), \( \partial \Pi^N / \partial t > 0 \). Combining these effects of \( t \) on all profit functions explains the non-monotonicity of the minimum discount factor in trade costs. The link between \( \delta^M \) and \( s \) is also non-monotonic due to the non-monotonicity of \( \Pi^N \) in \( s \), which dominates the negative effect of the cost differential on the deviation profits for high initial values of \( s \).

![Figure 9: Minimum Discount Factor and Trade Costs](image-url)
One interesting result deserves more emphasis: when the inefficient good is no longer traded under competition, trade liberalization improves collusive sustainability. That is, for values of \( t \in [\bar{t}_y, s) \), the minimum discount factor is increasing in trade costs, \( \partial \delta^M / \partial t > 0 \), due to the fact that within this range of trade costs, \( \partial \Pi^N / \partial t < 0 \). This possibility is illustrated by the red dotted line in the second panel of Fig. (9) and stands in sharp contrast to the previous literature, which finds the discount factor to be decreasing in trade costs (e.g., Bond and Syropoulos, 2008). Moreover, as already shown, for trade costs within this range of values \( t \in [\bar{t}_y, s) \), maximal collusion can welfare-dominate competition if the cost heterogeneity is sufficiently high. Combining these results, one can argue that by enhancing cartel’s stability trade liberalization can further contribute to national welfare.

![Figure 10: Minimum Discount Factor and Cost Asymmetries](image)

Next, I consider the case of no trade under maximal collusion, discussed in part b) of Proposition A3. Part b.i) evaluates the minimum discount factor at the extremes of free trade \( (t = 0) \) and no trade in the inefficient good \( (t \geq \bar{t}_y \text{ and } s \geq \bar{s}_y) \). Then, parts b.ii) and b.iii) study the link between the discount factor and trade costs and between the minimum discount factor and the cost differential for all relevant intervals. Since in this case \( \Pi^M \) is decreasing in \( s \) and independent of \( t \), the effects of trade costs on the minimum discount factor are channeled through \( \Pi^D \) and \( \Pi^N \). The negative effect of \( t \) on \( \Pi^D \) becomes second order at high levels of \( t \) and is dominated by the non-monotonicity of the punishment payoffs in \( t \). Analogously, the non-monotonicity of \( \Pi^N \) in \( s \) also prevails over the negative effect of \( s \) on \( \Pi^D \) and \( \Pi^M \) for high initial values of \( s \). Interestingly, for intermediate values of the cost heterogeneity, \( s \in [\bar{t}_y^{-1}, t) \), the minimum discount factor is increasing in \( s \), \( \partial \delta^M / \partial s > 0 \). Therefore, as the difference in marginal costs shrinks, the cartel’s stability is improved. Moreover, for the same range of values of \( s \), when transportation costs are sufficiently high, \( 44 \) or for \( t \in [\bar{t}_2, t_a) \) in the case of transportation costs, where \( t_a < s \) and is defined in equation (60). \( 45 \) This possibility is illustrated by the red dotted line in the second panel of Fig. (10) and is driven by the fact that when the goods are completely unrelated \( \partial \Pi^N / \partial s < 0 \) for \( s \in [\bar{t}_y^{-1}, t) \).

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44 Or for \( t \in [\bar{t}_2, t_a) \) in the case of transportation costs, where \( t_a < s \) and is defined in equation (60).
45 This possibility is illustrated by the red dotted line in the second panel of Fig. (10) and is driven by the fact that when the goods are completely unrelated \( \partial \Pi^N / \partial s < 0 \) for \( s \in [\bar{t}_y^{-1}, t) \).
maximal collusion can provide a greater level of welfare than Cournot competition. Thus, reductions in the cost heterogeneity by strengthening the collusive agreement can further sustain the gains in efficiency and improve national welfare. Figs. 9–10 illustrate the non-monotonicity of the minimum discount factor and its dependence on trade costs (for a given level of s) and the cost differential (for a given level of t). The red curves capture part a) of Proposition A3 with trade being present under collusion, while the blue curves correspond to part b) of Proposition A3 with no trade under monopoly.

Lastly, part c) of Proposition A3 shows that when trade costs are equal to the cost differential, the minimum discount factor is discontinuous. This discontinuity is due to the strict convexity of the deviation payoffs, which implies that cartel members’ incentives to deviate are minimized when they share equally both markets: \( x_2^M = y_2^M = \frac{1}{2} \frac{a(1-\gamma)-s}{2(1-\gamma^2)} \). If trade costs either exceed or fall short of the cost differential, the cartel strictly prefers either to cease trade or to specialize completely and import the cost-inefficient good.

In this section, I demonstrate that collusion is a stable outcome of the repeated game for values of the discount factor greater than \( \delta^M \). To verify that the trade and welfare comparisons from the previous sections are valid, I take the values of t and s, which ensure that 1) trade of the cost-efficient good is greater under collusion than under competition; 2) welfare is higher under collusion than under competition in the case of transportation costs and tariffs; and substitute them in the expression for \( \delta^M \) in (75). I perform this exercise step by step for each of the possible parameter values, assuming that the goods are completely unrelated (\( \gamma = 0 \)), for simplicity. In all cases, I obtain plausible values of \( \delta^M \) below 1, which confirms the validity of the main results. Namely, I establish that not only can collusion promote bilateral trade and improve national welfare relative to competition, but that such cartel agreements are stable contracts as long as firms value profits sufficiently.

**Proof of Proposition A3:** To derive expressions for \( \delta^M \), I substitute the relevant profit functions in the ICC in equation (74), set the ICC equal to 0 and solve for \( \delta \equiv \delta^M \). Recall from the analysis in the text that the value of both \( \Pi^M \) and \( \Pi^D \) depends on the initial levels of t and s. More specifically, \( \Pi^M \) and \( \Pi^D \) take on different values depending on whether \( t < s \) or \( t > s \). If \( t < s \), then \( \Pi^D = \frac{5}{8} \alpha^2 - \frac{5}{8} \alpha t - \frac{5}{8} \alpha^2 + \frac{1}{2} t s + \frac{5}{8} s t + \frac{9}{16} t^2 \), while if \( t > s \), \( \Pi^D = \frac{5}{8} \alpha^2 - \frac{5}{8} \alpha s - \frac{1}{2} \alpha t + \frac{5}{8} s^2 + \frac{1}{4} s t + \frac{9}{16} t^2 \). Lastly, if \( t = s \), \( \Pi^D = \frac{19}{32} \alpha^2 - \frac{7}{8} \alpha s - \frac{1}{2} \alpha t + \frac{13}{32} s^2 + \frac{3}{8} s t + \frac{1}{2} t^2 \). The value of the punishment payoffs, \( \Pi^N \), is also different depending on the range of trade costs and cost differentials as shown in equation (5). Below, I present the expressions for \( \delta^M \) for the aforementioned cases as a function of \( \Pi^N \):

\[
\delta^M(t, s, 0) = \begin{cases} 
\frac{2 \alpha^2 - 8 s t - 10 \alpha t + 9 s^2 + 14 s t + 9 t^2 - 16 \Pi^N}{10 \alpha^2 - 8 s t + 10 \alpha t + 9 s^2 + 14 s t + 9 t^2} & \text{if } t < \min(s, \alpha) \\
\frac{2 \alpha^2 - 8 a - 8 s t + 5 s^2 + s t + 8 t^2}{10 \alpha^2 - 10 \alpha t + 9 s^2 + 14 s t + 8 t^2} & \text{if } s < \min(t, \alpha) \\
\frac{3 \alpha^2 + 2 s t - 20 \alpha t + 13 s^2 + 12 s t + 16 t^2}{19 \alpha^2 - 14 a s - 20 \alpha t + 13 s^2 + 12 s t + 16 t^2 - 32 \Pi^N} & \text{if } t = s 
\end{cases}
\]

**Part (a):** \( t < s \)

**Part (i):** The proof of part (a.i) follows readily by evaluating (76) at \( t = 0, t = T_y \), and \( s = T_y^{-1} \). It is also straightforward to show \( \delta^M(0, s, 0) > \delta^M(T_y, s, 0) \) if \( s \in (\frac{a}{2}, \alpha) \) or if \( s \in (0, \frac{a}{38}) \).

\[46\] The analysis of these parameter values in all relevant cases is available upon request.
Part (a.ii)

Suppose \( t \in [0, \bar{t}_y) \). Then, to derive the full expression for \( \delta^M(t, s, 0) \), I substitute the relevant value of \( \Pi^N \) from (5) and simplify: \( \delta^M(t, s, 0) = \frac{9(2 \alpha^2 - 8 \alpha s - 2 \alpha t + 8 s^2 + 4 st + 5 t^2)}{26 \alpha^2 - 26 \alpha s - 26 \alpha t - 88 s^2 + 4 st - 79 t^2} \). It follows that \( \frac{\partial \delta^M}{\partial t} = \frac{-432(2 \alpha^3 - 12 \alpha^2 s - 28 \alpha s^2 + 24 \alpha t + 8 s^2 - 8 t^2)}{(26 \alpha^2 - 8 \alpha s - 26 \alpha t - 88 s^2 + 4 st - 79 t^2)^2} \). Setting this derivative equal to 0, gives the value of \( t \), which minimizes the discount factor. Let \( \arg\min_t \delta^M = t_{s_1} \equiv - \frac{3 \alpha^2 + 4 \alpha s - 4 s^2 + 3 \sqrt{2} \sqrt{(2 \alpha^2 - 4 \alpha s + s^2)(-\alpha - 2 s)^2}}{-1 s + 6 \alpha} \in (0, \bar{t}_y) \) and
\[
\frac{\partial \delta^M}{\partial t} > 0 \text{ if } \begin{cases} \ t \in (t_{s_1}, s) \quad \text{and} \quad s \in (0, \frac{\alpha}{4}) \\
\ t \in (t_{s_1}, \bar{t}_y) \quad \text{and} \quad s \in (0, \frac{\alpha}{4}) \\
\end{cases}
\]

Otherwise, \( \frac{\partial \delta^M}{\partial s} \leq 0 \) if \( t \leq t_{s_1} \).

If \( t \in [\bar{t}_y, \bar{t}_x) \), then \( \delta^M(t, s, 0) = \frac{9(2 \alpha^2 - 8 \alpha s - 2 \alpha t + 8 s^2 + 4 st + 5 t^2)}{22 \alpha^2 - 40 \alpha s - 58 \alpha t - 88 s^2 + 164 st + 1 t^2} \) and \( \frac{\partial \delta^M}{\partial t} = \frac{216(3 \alpha^2 - 26 \alpha s + 9 t^2 + 68 \alpha s^2 - 16 \alpha t - 12 t^2 - 56 \alpha s^2 - 4 t^2 + 34 t^2)}{(22 \alpha^2 - 40 \alpha s - 58 \alpha t - 88 s^2 + 164 st + 1 t^2)^2} \). This derivative turns out to be positive for all values of \( t \) in the relevant range, namely \( \frac{\partial \delta^M}{\partial t} > 0 \) for any \( t \in [\bar{t}_y, s) \).

Part (a.iii)

If \( s \in [t, \bar{t}_y) \), then \( \delta^M(t, s, 0) = \frac{9(2 \alpha^2 - 8 \alpha s - 2 \alpha t + 8 s^2 + 4 st + 5 t^2)}{26 \alpha^2 - 8 \alpha s - 26 \alpha t - 88 s^2 + 4 st - 79 t^2} \) and \( \frac{\partial \delta^M}{\partial s} = \frac{-432(4 \alpha^3 - 16 \alpha s - 6 t^2 + 16 \alpha s^2 + 16 \alpha t - 12 t^2 - 8 t^2 - 8 s^2 + 8 t^2 + 7 t^3)}{(26 \alpha^2 - 8 \alpha s - 26 \alpha t - 88 s^2 + 4 st - 79 t^2)^2} \). Similarly, setting the derivative equal to 0 and solving for \( s \) gives the value of \( s \) that minimizes \( \delta^M(t, s, 0) \). Let \( \arg\min_s \delta^M = s_{s_1} \equiv \frac{-3 \alpha^2 + 4 \alpha s + 2 t^2 + 3 \sqrt{2} \sqrt{(t^2 - 2 \alpha^2 + 4 \alpha s)^2}}{4(2 \alpha - t)} \in [t, \bar{t}_y) \). Thus,
\[
\frac{\partial \delta^M}{\partial s} > 0 \text{ if } \begin{cases} \ s \in (s_{s_1}, \bar{t}_y) \quad \text{and} \quad t \in (0, \alpha) \\
\ s \in (t, \bar{t}_y) \quad \text{and} \quad t \in (0, \frac{\alpha}{4}) \\
\end{cases}
\]

Otherwise, \( \frac{\partial \delta^M}{\partial s} \leq 0 \) if \( s \leq s_{s_1} \).

If \( s \in [\bar{t}_y, \bar{t}_x) \), then \( \delta^M(t, s, 0) = \frac{9(2 \alpha^2 - 8 \alpha s - 2 \alpha t + 8 s^2 + 4 st + 5 t^2)}{22 \alpha^2 - 40 \alpha s - 58 \alpha t - 88 s^2 + 164 st + 4 t^2} \) and \( \frac{\partial \delta^M}{\partial s} = \frac{-432(2 \alpha^3 - 8 \alpha s^2 - 3 t^2 + 8 \alpha s^2 + 20 \alpha t - 6 t^2 - 28 t^2 - 2 t^2 + 7 t^3)}{(22 \alpha^2 - 40 \alpha s - 58 \alpha t - 88 s^2 + 164 st + 4 t^2)^2} \). To find the value of \( s \) that minimizes \( \delta^M(t, s, 0) \), I set this derivative equal to 0, again, and solve for \( s \): \( \arg\min_s \delta^M = s_{s_2} \equiv \frac{-3 \alpha^2 + 10 \alpha t - t^2 + 3 \sqrt{(2 \alpha^2 - 36 \alpha t + 53 t^2)^2}}{4(2 \alpha - t)} \in [\bar{t}_y, \bar{t}_x) \). Thus,
\[
\frac{\partial \delta^M}{\partial s} \geq 0 \text{ for any } s \geq s_{s_2} \text{ and } t \in (0, \alpha).
\]

Part (b): \( t > s \)

Part (b.1) The proof of part (b.1) follows readily by evaluating (76) at \( s = 0, t = \bar{t}_y \), and \( s = \bar{t}_y \).

Part (b.ii)

If \( t \in [s, \bar{t}_y) \), then \( \delta^M(t, s, 0) = \frac{9(2 \alpha^2 - 2 \alpha s - 8 \alpha t + 5 s^2 + 4 st + 8 t^2)}{26 \alpha^2 - 26 \alpha s - 8 \alpha t - 79 s^2 + 4 st - 88 t^2} \) and
\[
\frac{\partial \delta^M}{\partial t} = \frac{-432(4 \alpha^3 - 6 \alpha s^2 - 16 t^2 - 12 \alpha s^2 + 16 t^2 + 16 t^2 + 7 \alpha s^2 + 8 t^2 - 8 t^2)}{(26 \alpha^2 - 26 \alpha s - 8 \alpha t - 79 s^2 + 4 st - 88 t^2)^2} \]. I find that \( \arg\min_t \delta^M = t_{s_1} \equiv 0 \).
\[-(4\alpha^2+4\alpha s+2s^2+3\sqrt{2}s^2(2\alpha^2-2\alpha s+s^2))\]

\[
\frac{\partial \delta^M}{\partial t} > 0 \quad \text{if} \quad \begin{cases} 
    t \in (t_{\delta_2}, \bar{t}_y) \quad \text{and} \quad s \in (0, \alpha) \\
    t \in (s, \bar{t}_y) \quad \text{and} \quad s \in (0, \frac{\alpha}{4})
\end{cases}
\]

Otherwise, \(\frac{\partial \delta^M}{\partial s} \leq 0\) if \(t \leq t_{\delta_2}\).

If \(t \in [\bar{t}_y, \bar{t}_x]\), then \(\delta^M(t, s, 0) = \frac{9(2\alpha^2-2\alpha s-8\alpha t+5s^2+4st+8t^2)}{22\alpha^2-58\alpha s-40\alpha t+s^2+164st-81t^2}\) and

\[
\frac{\partial \delta^M}{\partial t} = -\frac{432(2\alpha^3-3\alpha^2s-8\alpha t^2-6\alpha s^2+20\alpha s+8t^2+17s^3-22t^2-28t^2s)}{(22\alpha^2-58\alpha s-40t+s^2+164ts-81t^2)^2}.
\]

Solving for the value of \(t\) that minimizes \(\delta^M(t, s, 0)\) in this case gives

\[
\arg\min_t \delta^M(t, s, 0) = t_{\delta_3} \equiv -\frac{4\sqrt{8\alpha^2-36\alpha s+53s^2}}{4(2\alpha^2-7s)} \in [\bar{t}_y, \bar{t}_x].
\]

Thus, \(\frac{\partial \delta^M}{\partial t} \geq 0\) for any \(t \geq t_{\delta_3}\) and \(s \in (0, \alpha)\).

Part (b.iii)

If \(s \in [0, \bar{s}_y]\), then \(\delta^M(t, s, 0) = \frac{9(2\alpha^2-2\alpha s-8\alpha t+5s^2+4st+8t^2)}{26\alpha^2-26\alpha s-8\alpha t-79s^2+4st-88t^2}\) and

\[
\frac{\partial \delta^M}{\partial s} = \frac{432(12\alpha^2-2\alpha^2s-2\alpha s^2-28\alpha s+8t^2+7s^2+8t^2s-8t^4)}{(-88t^2+4+4t+26\alpha^2-26\alpha s-79s^2+4st-88t^2)^2}.
\]

Following the same procedure, let \(\arg\min_s \delta^M = s_{\delta_3} \equiv -\frac{6\alpha^2+14\alpha t-4t^2+3\sqrt{2}(t^2-2\alpha^2+2\alpha^2s^2-4t^2s-8t^4)}{6\alpha-7t} \in [0, \bar{s}_y].\)

Thus, \(\frac{\partial \delta^M}{\partial s} \geq 0\) for any \(s \geq s_{\delta_3}\) and \(t \in (0, \frac{\alpha}{2})\).

If \(s \in [\bar{s}_y, \bar{s}_x]\), then \(\delta^M(t, s, 0) = \frac{9(2\alpha^2-2\alpha s-8\alpha t+5s^2+4st+8t^2)}{22\alpha^2-58\alpha s-40\alpha t+s^2+164st-81t^2}\) and

\[
\frac{\partial \delta^M}{\partial s} = \frac{216(3\alpha^3+9\alpha^2s-26\alpha^2-12\alpha s^2-16\alpha t+8t^2+34t^2s-4t^2s-56t^3)}{(22\alpha^2-58\alpha s-40t+s^2+164ts-81t^2)^2}.
\]

In this case, the derivative is positive for any value of \(s\) in the relevant range, namely, \(\frac{\partial \delta^M}{\partial s} > 0\) for any \(s \in [\bar{s}_x, t]\).

Part (c): \(t = s\)

When \(t = s\), the monopolist is indifferent between shutting down trade and producing both goods in both countries or specializing completely in the production of the cost-efficient good and exporting it. In this case the optimal solution to the cartel problem is a correspondence: any combination of outputs satisfying \(x_2^M + y_2^M = \frac{\alpha(1-\gamma)-s}{2(1-\gamma)}\) maximizes the monopolist’s profits, when \(t = s\). The optimal deviation payoffs, however, are minimized when both cartel members share equally each market due to the strict convexity of \(\Pi^D\) in \((x_k, y_k)\) for \(k = 1, 2\). Thus, when \(t = s\), each firm will choose to maintain a presence in its rival’s market and \(x_2^M = y_2^M = \frac{\alpha(1-\gamma)-s}{2(1-\gamma)}\). In this case, \(\Pi^M(s, s, 0) = \frac{1}{4} (2\alpha^2 - 2\alpha s + s^2)\) and \(\Pi^D(s, s, 0) = \frac{1}{32} (19\alpha^2 - 34\alpha s + 41s^2)\). Therefore, it follows

\[
\delta^M(s, s, 0) = \frac{3(\alpha^2-6\alpha s+11s^2)}{19\alpha^2-34\alpha s+41s^2-32\Pi^D} \neq \lim_{t \to s} \delta^M(t, s) = \frac{2\alpha^2-10\alpha s+17s^2}{10\alpha^2-15\alpha s+21s^2-16\Pi^D}.
\]

To obtain all possible values of \(\delta^M\) at \(t = s\), one needs to substitute for \(\Pi^M\) for the different ranges of \(t\) and \(s\) as described in equation (5).
Appendix B: Data

The data employed in this study have been carefully developed to allow for a closer examination of the relation between international cartels and bilateral trade and have been used in Agnosteva (2016) and in Agnosteva, Syropoulos and Yotov (2016) as well. The dataset focuses on the 34 member countries of the Organization for Economic Co-operation and Development (OECD) for the period between 1988 and 2012. Due to the lack of readily available international trade data at the firm level, I use the most disaggregated international trade data available, namely 6-digit Harmonized System (HS) level. Trade data at this level of aggregation are not available prior to 1988, which determines the span of the sample. The decision to analyze OECD countries only is due to several factors: 1) trade data at such a high level of disaggregation are likely to be more reliable for more developed economies; 2) trade between OECD members represents about two-thirds of the total world trade; 3) 165 of the 173 cartels in my sample are comprised of participants exclusively from OECD countries. Next, I describe in more details the main covariates used in the empirical analysis along with the corresponding data sources.

International Cartels. The dataset covers 173 private discovered and prosecuted international cartels defined by Connor (2006) as “... a conspiracy in restraint of trade that has or is alleged to have one or more corporate or individual participants with headquarters, residency, or nationality outside the jurisdiction of the investigating antitrust authority.” The earliest cartel in the data dates from 1958, while the latest discovered and prosecuted cartel ceased to exist in 2010. A total of 48 different countries have participated in at least one of the cartels in my sample with 28 of these countries being OECD members. In order to match the cartel data with the trade data, I focus only on cartels that were functional after 1988 (which amounts to about 98% of the cartels in the original data). Moreover, any international cartels, which operated in services sectors (such as banking, insurance, cargo shipping, transportation, etc.) have been excluded due to the lack of reliable and highly disaggregated services trade data.

In order to fit the purpose of this study, the data have been tailored to include the countries of nationality, residence, or headquarters of the firms and/or individuals that have participated in these cartels. Furthermore, 37% of the cartels in the final dataset are multi-product cartels, meaning that firms’ collusive practices actually extended to several different lines of goods. Another key feature of the dataset is that it also contains the specific duration of each of the international cartels as specified in the available sources. The main sources of information include (but are not limited to) the Department of Justice, the European Commission, the Canadian Competition Bureau, U.K.’s Office of Fair Trading, books, different journal articles and/or newspaper articles.

Another important detail about the data is that I have also coded up the 6-digit HS product code corresponding to each of the cartelized goods. Mapping the description of the products subject to collusion with the relevant HS codes turned out to be straightforward in some cases and rather challenging in others. For instance, in the Hydrogen Peroxide and Perborate cartel the 6-digit

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47 The initial data on private international cartels have been kindly provided by John M. Connor and Jeffrey E. Zimmerman. Substantial modifications have been made to the dataset by both expanding the list of international cartels, adding new variables, and verifying the existing information.

48 Table lists all countries cartel-members present in my dataset.

49 Which suggests that there are 6 OECD countries in my sample, which have not participated in any of the 173 discovered and prosecuted international cartels considered in this study.

50 Following Levenstein and Suslow (2010), I use nationality of the parent company to identify the country of origin for each cartel member, unless any foreign subsidiaries were convicted of violating antitrust laws.

HS codes corresponding to hydrogen peroxide and perborate are, respectively, 284700 and 284030. On the other hand, the HS codes linked to the products of the Carbon Electrodes cartel\(^{52}\) are both 854511 and 854519 (one for the kind used for furnaces and one for the kind used for electrolytic purposes, respectively). Thus, in some cases a single cartel is linked to more than one HS code depending on the number and the variety of the cartelized products. In a few instances, firms colluded in the production or distribution of an entire line of products and I had to use a more aggregated HS product category (such as 2-digit HS product codes or 4-digit HS product codes). Some examples include the Automobile Parts cartel (HS code - 8708), the Ferrosilicon cartel (HS code - 7202), the Haberdashery Products cartel (HS codes - 7319, 9606, 9607), and a few more\(^{53}\).

### Bilateral Trade Flows

Data on bilateral trade flows at the 6-digit HS level for the period between 1988 and 2012 come from the United Nation’s COMTRADE database. I employ cif imports, which is the theoretically correct trade variable, as the base for the empirical analysis. Import data are available for 42% of the observations. However, in order to improve the number of non-missing observations, I use a mirror procedure to map bilateral exports to imports. This further increases the percentage of data coverage to 53%.

### Bilateral Trade Costs

Following the theory and the extensive empirical gravity literature, I proxy for bilateral trade costs with the logarithm of bilateral distance between trading partners \(i\) and \(j\), the presence of a contiguous border between trading partners \(i\) and \(j\), the existence of colonial ties as well as of a common language between trading partners \(i\) and \(j\). Data on bilateral distance and the rest of the standard gravity proxies are from the CEPII database. Moreover, I control for the presence of regional trade agreements between exporter \(i\) and importer \(j\) using data carefully compiled and generously provided by Mario Larch.

### Product Substitutability

Rauch (1999) develops a classification of SITC Rev 2 goods into differentiated, reference-priced, and those that trade on organized exchanges (homogeneous goods). Rauch (1999) argues that homogeneous goods are the ones sold on organized markets such as corn, wheat, etc. On the other hand, products not sold on organized markets, but whose benchmark prices are listed in trade journals and industry guides he classifies as referenced-priced goods. Such goods are more likely to exhibit some unique attributes, but are somewhat substitutable. The remaining goods he considers differentiated. Rauch’s (1999) classification offers several advantages - 1) it is at a relatively disaggregated level, 4-digit SITC, which could be matched to the 6-digit HS codes; 2) the classification of commodities into the aforementioned three categories is quite intuitive and generally conforms to the economic definition of product substitutability; 3) the classification is comprehensive. Alternatively, I employ the elasticity of substitution estimates of Broda et al. (2006). The authors provide estimates of the elasticity of import demand at the 3-digit HS level for 73 countries for the period 1994-2003. Unfortunately, not all of the OECD member countries are included in their sample. Belgium, Estonia, Israel, Luxembourg and the Czech Republic are the ones missing from Broda et al. (2006) data. For those five countries for which elasticity estimates are unavailable, I experiment by 1) obtaining the average product elasticity across exporters; 2) dropping these countries from the sample.


\(^{53}\)Only for the Toys and Games cartel, I had to use the 2-digit HS product code as no specifics regarding the products were provided by the U.K.’s Office of Fair Trading. Insufficient details regarding the specific type and characteristics of the cartelized products prevents me from using 6-digit HS codes.
Table 3: Summary Statistics

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Notes: This table reports the summary statistics for the variables used in the main specifications.
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<td>Switzerland</td>
<td>Yes</td>
</tr>
<tr>
<td>Taiwan</td>
<td>No</td>
</tr>
<tr>
<td>Togo</td>
<td>No</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>Yes</td>
</tr>
<tr>
<td>United States</td>
<td>Yes</td>
</tr>
</tbody>
</table>

**Notes:** This table lists all countries present in the International Cartels Data and also specifies whether they are OECD members or not.