The critical step for the bulk precipitation of calcium carbonate in water treated by PWT is the dissociation of the bicarbonate ions, Reaction 1.

\[
\text{HCO}_3^-(\text{aq}) \Leftrightarrow \text{OH}^-(\text{aq}) + \text{CO}_2(\text{aq}) \quad (\text{Reaction 1})
\]

Here we examine the possibility of applying the concept of heterogeneous catalysis to explain this dissociation reaction. The energy required to affect Reaction 1 will be compared with the energy that is attainable on a small scale as a result of electrostatic effects of surface irregularities (similar to the heterogeneous electrocatalysis). In heterogeneous catalysis, the reaction rate is increased without the catalyst being consumed by the reaction. In this case, we assume that the catalytic surface is the pipe wall. In this approach, ions are first diffused from the bulk water to the pipe wall, i.e., to the irregularities in the form of bumps at the wall of the pipe.

Consider a very small local bump, which serves as an electrode. The size of the local roughness ranges from 0.1 mm for a galvanized iron pipe to 0.0015 mm for a drawn tubing. The electric field around these local bumps (\(E_L\)) can be much larger than the space-averaged field within the pipe (\(E_0\)), which will be explained in detail next.

The average electric field produced by electrostatic interaction of charged particles at the surface is described as the charge of a particle divided by its radius of curvature squared:

\[
E_0 \propto \frac{q}{R^2} \quad (1)
\]

In the case of an infinitesimal bump irregularity of radius \(r\), the local electric field produced would be:

\[
E_L \propto \frac{q}{Rr} \quad (2)
\]
Dividing Eq. (1) by Eq. (2), we get:

\[
\frac{E_L}{E_0} = \frac{R}{r}
\]

(3)

The surface radius of curvature (R) in a pipe is on the order of 10 cm, and the radius of curvature of the irregularity (r) is on the order of 10^{-7} m. Thus, the electric field local to the bump irregularity is on the order of

\[
\frac{E_L}{E_0} = \frac{10^{-1}}{10^{-7}} = 10^{6}
\]

(4)

The local electric field (at the surface) is 10^6 times greater than that in the bulk solution. Thus, despite the fact that the space-averaged electric field (E_0) typically produced by a PWT device in the bulk is much too small (~ 1 V/m), the corresponding strength of the electric field local to the surface irregularities becomes

\[
E_L = E_0 \times 10^6 = 1 \times 10^6 \frac{V}{m}
\]

(4)

This value is of the same order of magnitude necessary to drive these reactions in a region local to these surface irregularities.

Knowing the strength of an induced electric field local to the pipe wall (E_L = 1 \times 10^6 V/m), we can take a careful look at a single positive charge (i.e., calcium ion) located at the surface of the wall. The calcium ion acts as a charge center, accelerating the dissociation process of bicarbonate ions, producing hydroxide ions OH^{-} and CO_2(gas) as the local electrostatic effect is strong enough to induce the rate-limiting Reaction 1. The OH^{-} ions are collected on the surface first. When enough OH^{-} ions are collected on the surface, Reactions 2 and 3 initiate, allowing carbonate ions to be formed (Reaction 2) which proceed to induce the formation or precipitation of calcium carbonate (Reaction 3).
\[
\text{OH}^-_{(aq)} + \text{HCO}_3^-_{(aq)} \leftrightarrow \text{CO}_3^{2-}_{(aq)} + \text{H}_2\text{O} \quad \text{(Reaction 2)}
\]

\[
\text{Ca}^{2+}_{(aq)} + \text{CO}_3^{2-}_{(aq)} \leftrightarrow \text{CaCO}_3(s) \quad \text{(Reaction 3)}
\]

The heterogeneous electrostatic catalysis process of PWT can generally be described as follows:

Step 1: Local roughness produces catalytic centers.
Step 2: Large local E field at surface
Step 3: Reaction 1 is catalyzed, producing OH\(^-\) ions at surface
Step 4: Reactions 2 and 3 follow in bulk as sufficient amounts of OH\(^-\) ions are collected, producing carbonate ions which subsequently combine with calcium ion.