Statistical Properties, Dynamic Conditional Correlation, Scaling Analysis of High-Frequency Intraday Stock Returns: Evidence from Dow-Jones and NASDAQ Indices[1]

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Abstract

This paper investigates statistical properties of high-frequency intraday stock returns across various frequencies. Both time series and panel data are employed to explore probability distribution properties, autocorrelations, dynamic conditional correlations, and scaling analysis in the Dow Jones Industrial Average (DJIA) and the NASDAQ intraday returns across 10-minute, 30-monute, 60-minute, 120-minute, and 390-minute frequencies from August 1, 1997, to December 31, 2003. The evidence shows that all of the statistical estimates are highly influenced by the opening returns that contain overnight and non-regular information. The stylized fact of high opening returns generates significant negative (in DJIA) and positive (in NASDAQ) autocorrelations. After excluding the opening intervals, DJIA exhibits a pattern similar to a random walk. While examining the AR(1)-GARCH (1, 1) pattern across both time and frequency variants, we find consistent negative AR(1) at 10-minute and 30-minute frequencies in the DJIA, positive AR(1) in the NASDAQ intraday returns, and no obvious pattern beyond the 30-minute intraday return series. By examining the dynamic conditional correlation coefficients between the DJIA and the NASDAQ at different frequencies, we find that the correlations are positive and fluctuate mainly in the range of 0.6 to 0.8. The variance of the correlation coefficients has been declining and appears to be stable for the post-2001 period. We then check the conditions for a stable Lévy distribution and find both the DJIA and the NASDAQ can converge to their systematic equilibriums after shocks, implying both systems are characterized by a self-stabilizing mechanism.

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Keywords: High Frequency, Probability Distribution, Financial Markets, Dynamic Conditional Correlation, Panel Data.

1. Introduction

Traditional analysis of stock returns relies heavily on economic fundamentals such as dividend yield, long-short interest rate spreads, risk, or book-price ratio (Fama and French, 1989, 1992; Campbell and Hamao, 1992; Avramov, 2002). The advantage of fundamentals-based analysis is that the underlying economic rationale can be tested and verified and the resulting empirical behavior can then be used for guiding investment decisions or monitoring market behavior by regulators or policy makers. The analysis of fundamentals thus appears to be relevant if it is applied to the time series data for investigating a longer-run phenomenon, including quarterly, monthly, or even daily data. However, using longer horizon factors may not be feasible for analyzing high-frequency data such as the intraday observations because the variations of return series are unlikely to be described by the economic fundamentals, since these data are often taken as datum or simply not available. For this reason, analysts need to explore alternative information and techniques to detect and derive the underlying empirical regularities being used to describe the market's behavior.

In this study, we apply modern time series techniques to detect the empirical regularities of high-frequency data for both the Dow Jones Industrial Average 30 (DJIA) and the NASDAQ stock indices. The reason for choosing these two indices stems from the fact that the former represents well-established and renowned firms in the US market, while the latter consists of high-tech and growth firms. These two indices thus represent not only the core of the US economy but also facilitate the menu for investors' choice in making the mean-variance investment decision. A successful empirical investigation emerging from this study is bound to provide an insight into understanding the behavioral relation of high-frequency data and its validity across different scales.

This study is motivated by the conventional approach that most empirical studies take: they focus on using a particular point of time series data to derive their empirical regularities. The resulting statistical analysis can be misleading if it is done without considering a broad selection of data points. For instance, in the study of daily observations, researchers often use closing price (returns) or average price (returns) without carefully constructing an appropriate measure, not even taking into consideration the impact of the opening observation. Taking different points of time from a particular trading day to represent a daily observation may inherently introduce some sort of anomaly into data construction. As a result, it may produce a biased estimate and statistical inference. Second, the conventional analysis of the empirical issue is usually based on a particular scale of measurement without concern for its scaling-variants. For instance, to examine the AR(1) process, daily data are frequently used. This approach pays no attention to frequency variations, such as the validity of 10-minute, 30-minute, 60-minute, 120-minute, or 390-minute horizons. Apparently, the derived empirical regularity is conditional on a particular time scale and lacks general implications. Third, although higher-frequency data have been analyzed in a number of research papers (Baillie and Bollerslev, 1991; Müller *et al*., 1990; Laux and Ng, 1993; Zhou, 1996; Andersen and Bollerslev, 1997; Ito *et al*., 1998), their focus is mainly on a single market, especially the foreign exchange market. The exceptions are Wood *et al*. (1985) and Abhyankar *et al*. (1997), who analyze the equity market. However, the dynamic relationship of intraday returns between two markets is mostly ignored in the empirical analysis of the market microstructure literature.

This paper differs from the existing literature in the following ways. First, in addition to exploring the time series properties involving high-frequency data, this study extends the conventional analysis to include the scaling dimension, since time series analysis can capture only limited information in terms of a particular time horizon. With the addition of the frequency-varying dimension, we will have more complete knowledge of the test relation, ranging from short-span to long-horizon data. Second, most of the empirical literature employs only time series data to investigate the autocorrelation, without screening out the significance of opening intervals. Our study shows that the outcome of estimating the autocorrelation is dominated by the significance of the opening intervals. Hence, our research, by reshaping the time series data into cross-section panel data, allows us to compare the panel autocorrelations across different time frequencies without worrying about the interference of the opening intervals. Third, although a dynamic conditional correlation (DCC) technique has been employed to investigate the leads and lags across different markets, very few attempts have been geared to the analysis of intraday returns with different frequencies. Our evidence shows that using different scaling would lead to cross-correlation variations, suggesting that the validity of dynamic correlations between two time series is conditional on a particular time scale.

Finally, we analyze scaling behaviors of the time series on returns to probe the stability of time series distributions. Following the scheme proposed by Mantegna and Stanley (1995, 2000), we perform scaling analyses on DJIA and NASDAQ changes with various time intervals. Both exhibit well-behaved scaling and belong to a stable distribution based on the criterion of Lévy's α stable distribution condition (Voit, 2003).

This paper is organized as follows. Section 2 briefly outlines the data and the construction of series frequency with various intervals for the intraday data of the DJIA and NASDAQ indices. Section 3 presents some summary statistics of returns and volatility for both indices. Section 4 investigates the time series and panel autocorrelations between the DJIA and the NASDAQ. Section 5 discusses the dynamic conditional correlation between the DJIA and the NASDAQ based on different time frequencies. Section 6 presents probability distribution and scaling analysis. Section 7 contains concluding remarks.

2. Data and Stock Returns across Different Frequencies

This study is based on the intraday data on the DJIA and the NASDAQ from the Trade and Quotation (TAQ) database. The TAQ data files contain continuously recorded information on the trades and quotations of securities. The DJIA stocks are the most actively traded securities; the capital size of the firms in the DJIA also helps to ensure a high degree of liquidity. Alternatively, the stocks listed on the NASDAQ exchange are characterized by high-tech growing firms, which are associated with higher price volatility.

The intraday 10-minute scale values for both the DJIA and the NASDAQ span the period from August 1, 1997, through December 31, 2003, including 1,543 days covering the whole trading day of six-and-a-half hours starting from 9:30 to 15:50 EST. The overnight (or over-weekend) period constitutes an unusual time period, since it involves an interval much longer than 10 minutes. Therefore, the value of a stock index at opening prices presented an anomaly when compared with other data points. Following the analysis in Andersen and Bollerslev (1997), we constructed 10-minute returns with the daily transaction record extending from 9:30 to 15:50 (EST), a total of 39 10-minute returns for each day.[1] The 10-minute horizon is short enough that the accuracy of the continuous records of realized returns and volatility can be measured well; it is also long enough that the confounding influences from market microstructure frictions are negligible.

<Insert Figure 1 about here>

Figure 1 shows time series paths of the DJIA and NASDAQ indices sampled at 10-minute frequencies. As mentioned above, the trading time is defined in a continuous fashion, starting from the opening of the day to the closing, and this process repeats on the following trading days. As shown in Figure 1, although both the DJIA and the NASDAQ follow a similar path over time because they may be influenced by common economic fundamentals, their short-run variations are somehow different, exhibiting different degrees of correlations and speeds of change. In particular, DJIA returns display relatively stable movement, whereas the NASDAQ has experienced a dramatic structure change, showing a speedy upward trend around mid-2000, followed by a sharp downward trend thereafter.[2] This difference may be rooted in the nature of the businesses they list in that the DJIA comprises well-established companies, while the NASDAQ is made up of growing, high-tech firms. The latter is viewed to have a higher return that compensates for a higher risk. To further explore the underlying characteristics of these two series, it is convenient to start with the investigation of the basic statistics for the returns of these two indices.

3. Summary Statistics of the DJIA and the NASDAQ

3.1 Basic Statistics

Presented in Table 1 are the summary statistics reporting intraday returns from 10 minutes to 390 minutes, and interday returns from one day to five days and one week to five weeks. As shown in Table 1, the DJIA's average return at a 10-minute interval is 0.000006 with a standard deviation of 0.0019. The distribution is slightly right-skewed (with a skewness of 0.17) and has a high narrow peak (with a kurtosis of 23.37), implying positive returns occur more often than negative returns in a 10-minute interval series. With respect to the NASDAQ, both average returns (0.000013) and the standard deviation (0.0039) are higher than those of the DJIA. The return series is more right-skewed (0.25), associated with a higher and narrower peak (29.15). The feature of higher returns accompanied by higher risk is more apparent when we compare the 10-minute stock returns between the DJIA and the NASDAQ.

<Insert Table 1 about here>

By checking the average returns at different time scaling, it is apparent that the larger scale returns are equal to 10-minute returns times a multiple of a 10-minute scale. For instance, the 390-minute return is seen to be 0.000006×39 = 0.000234. The standard deviation, however, was growing at a rate almost proportional to the square root of the sampling frequency. This result is consistent with that of Andersen and Bollerslev (1997), who investigated foreign exchange (FX) market intraday returns. This also implies that high-frequency returns carry some common features between stock and FX markets. In general, both returns and volatility (standard deviation) are increasing with increasing scaling from 10-minute to 390-minute intervals. However, with interday data scaling from one day to five days or one week to five weeks, higher average returns and volatility are seen to increase with increasing scaling, but the standard deviation does not grow at a rate proportional to the square root of the sampling frequency as intraday frequency does. These observations are different from those reported by Silva *et al*. (2004). Silva *et al*. (2004) studied the distribution of stock returns of four individual large American companies at mesoscopic time lags for the period 1993 to 1999. There were no major market disturbances during this period, and Silva et al. found a linear relation between the square of variance and the time lag from five minutes to a month, after introducing an effective overnight time lag. However, we do not observe this feature for interday data even after the introduction of overnight time lags. More specifically, we did not find a single overnight time lag that is global for different time intervals from interday to weekly data. Since our empirical data are based on market indices from 1997 to 2003, they cover several macroeconomic events such as the collapse of the dot-com companies and the 9/11 attack. Whether market disturbances violate market efficiency (Shiller,1989) and, hence, the time-lag-variance proportionality may deserve further investigation.

By comparing the skewness, we find that most of the intraday returns have a positive sign, whereas the interday returns display a negative sign. This indicates that most of the daily or weekly interday returns are negatively skewed, meaning negative returns occur more often than positive returns, since the distribution has a longer left tail. This phenomenon occurs in both DJIA and NASDAQ markets.[3]

With respect to the kurtosis, all of the intraday returns show narrower peaks than normal, since the kurtosis is larger than 3. However, the kurtoses are declining (from more than 20 to almost 3) with increasing time intervals (from 10 minutes to five weeks) in both indices. Particularly, the kurtosis of the 10-minute interval returns reaches the highest peak among all intraday interval measures, and it is decreasing with increasing scales to 130-minute, daily, and weekly statistics.

Several regularities can be drawn from in this section: First, low-frequency returns in multiples of higher-frequency returns happen only in intraday returns; they are not significant in interday returns. Second, the standard deviation of intraday returns is shown growing at a rate almost proportional to the square root of the sampling frequency; however, interday returns do not show a similar pattern. Third, most daily or weekly interday returns are negatively skewed, meaning negative returns occur more often than positive; however, most intraday returns are positively skewed. Fourth, all of the values of kurtosis of the intraday returns are greater than 3; however, this kurtosis is declining with an increasing scale. Fifth, the intraday return series does not necessarily exhibit the best fit for normal distribution. Instead, daily returns (one-day series) show a better fit for normal distribution than those of other frequencies based on both skewness and kurtosis estimates, although they are still not perfect.[4] Figure 2 provides the time series plots of stock returns for the intraday DJIA data sampled by 10, 30, 130, and 390 minutes as well as interday data.[5]

<Insert Figure 2 about here>

The volatility from Figure 2 is seen to be getting larger from 1998 to 1999 across all various frequencies, including both intraday and interday data. Moreover, this volatility is increasing with the increasing scales. Checking the events during that period suggests that the Asian financial crisis appeared to be a significant one, implying that the volatility spillover occurs not only from big to small markets but also from small markets to big ones.

3.2 Panel Intraday Returns and Volatility

To investigate the intraday behaviors of the return series among all 10-minute intervals from 9:30 to 15:50, we reshape the time series data into panel data with 39

10-minute intervals in every trading day across all 1,543 days. Both 10-minute intraday returns and volatilities from 9:30 to 15:50 for the DJIA and the NASDAQ are presented in Figures 3.1.a and 3.1.b, respectively. Both indices exhibit the highest return in the opening interval of 9:30 a.m. and follow a similar pattern across 10-minute intervals. An especially high return in the opening interval reflects pronounced adjustments to the information accumulated overnight. This opening interval tends to produce a much higher return and volatility than any 10-minute interval. A parallel pattern, however, with a relatively moderate magnitude, is displayed in the market closures.

The NASDAQ in general has a higher return than the DJIA, and this becomes more apparent in both the opening and closing intervals. The higher returns are associated with higher volatilities over the entire trading intervals. As shown in Figure 3.1.b, the volatility against the 10-minute trading interval displays a smile curve. This smile curve is consistent with the shape presented in equity (Wood *et al.*, 1985) and derivative markets (Goodhart and O'Hera, 1997), implying a common feature associated with the volatility among various high frequencies financial assets.

<Insert Figures 3.1.a - 3.1.b about here>

To provide more information of the returns and volatilities for the 10-mimute trading interval, we present boxplots of the returns and volatilities. Figures 3.2.a1 and 3.2.a2 are boxplots of the DJIA and the NASDAQ returns for the 10-mimute frequency, respectively. Figures 3.2.b1 and 3.2.b2 are the boxplots for the DJIA and the NASDAQ volatility of the 10-mimute frequency. Each boxplot contains values of the maximum, minimum, and interquartile range to the median. Obviously, the plots of median values in Figures 3.2 exhibit a similar pattern as that of mean values in

Figures 3.1. However, the boxplots display a broader spectrum of the information content of the intraday statistics.

<Insert Figures 3.2.a1-3.2.b2 about here>

By increasing the time scale from 10 minutes to 30 minutes, as shown in Figures 4a and 4b, we continue to find the smile curve for both the DJIA and the NASDAQ. Yet, as shown in Figures 4c and 4d, the curvature for both indices has been increasing with the increasing time scale to 30 minutes. Some common patterns exhibited in both indices can be seen. First, the opening interval always shows the highest volatility; the lunch intervals display the lowest volatility for both 10- and 30-minute frequencies. Second, regardless of whether the frequency is 10 or 30 minutes, the NASDAQ always exhibits a greater curvature than the DJIA across all intervals over the entire day. Evidently, larger scale not only creates a higher volatility but it is also accompanied by a higher speed of change in volatility that results in a deeper curvature of the smile.

<Insert Figures 4a-4d about here>

The finding of a smile curve is consistent with social behavior during daily operations.[6] In the morning of each trading day, investors, in reacting to institutional arrangements for trading hours, tend to rack up voluminous transactions based on the information accumulated overnight, creating excessive volatility in the opening interval. Trading activity then slows down as investors collect news and process information over the course of the day. It reaches bottom around the lunch hour.[7] In projecting the closing hour, the accumulated trading activity rises and then accelerates before the market closes. To provide a rationale, recent studies (Admati and Pfeiderer, 1988; Foster and Viswanathan, 1990; Slezak, 1994) argue that this observed intraday

U-shape pattern in intraday stock market volatility is mainly attributable to the strategic interaction of traders around market openings and closings.

4. Autocorrelations of Time Series

4.1. Time Series Estimates of AR(1)

Since autocorrelation plays a central role in evaluating market efficiency, the recent literature has used AR(1) to detect feedback trading behavior (Sentana and Wadhwani, 1992; Antoniou *et al.,* 2005). Thus, it is of interest for us to investigate the sign of autocorrelation in order to understand more about investors' trading behaviors in both markets. In this section, we first consider time series autocorrelation models in our estimations. In expression, we write:

$$R_{\tau,t} = \delta_\tau + \phi_\tau R_{\tau,t-1} + \varepsilon_{\tau,t}\,, \qquad (1)$$

where $R_{\tau,t}$ is a vector of returns applied to the DJIA and the NASDAQ series; δ_τ is a vector of constant; ϕ_τ is a 2x2 coefficient matrix with off-diagonals of zero; the subscript τ is a scale index; and $\varepsilon_{\tau,t}$ is a vector of random error terms. The AR(1) term included in equation (1) accounts for autocorrelation arising from non-synchronous trading, price limitations, slow price adjustments, market frictions, or feedback trading (see Lo and MacKinlay, 1990; Amihud and Mendelson,1987; Fama and French, 1988; Sentana and Wadhwani, 1992; Damodaran, 1993; Harvey, 1995; Scholes and Williams, 1977; Koutmos, 1998, 1999).

Estimations are made on the return series by setting τ = 10-minute, 20-minute,…, and 390-minute frequencies. In this time series estimation, the observations are arranged in the time sequence, including the lengthy opening interval. The estimates of the AR(1) coefficients for each τ frequency are reported in Table 2.

<Insert Table 2 about here>

The evidence in Table 2 shows that AR(1) coefficients on both the DJIA and the NASDAQ present mixed signs and lack of statistical significance. The exceptions are the coefficients for the 120-minute intervals for the DJIA and the 390-minute intervals for the NASDAQ. These significant statistics do not seem to have a consistent pattern. It appears to us that the unsatisfactory results may be attributable to the inclusion of the opening data point or, simply, to the misspecification of the model, or both.

4.2 AR(1)-GARCH(1, 1) Model

As documented by Laux and Ng (1993) and Andersen and Bollerslev (1997), since the high-frequency return volatility, such as that of the exchange rate and S&P 500 futures, displays a changing intraday pattern, we are led to consider the point that estimations based on equation (1) could be misspecified. Following the conventional approach, the conditional variance for high-frequency returns is assumed to follow a GARCH(1, 1) process as given by:

$$\sigma_{\tau,t}^2 = \omega_\tau + \alpha_\tau \varepsilon_{\tau,t-1}^2 + \beta_\tau \sigma_{\tau,t-1}^2 \qquad (2)$$

where $\sigma_{\tau,t}^2$ is the conditional variance for frequency τ. Since the conditional volatility is time-varying, the unconditional returns distributions generated by a normal GARCH model will have fat tails. This is especially true for the high-frequency data. In this perspective, a leptokurtic distribution, such as a student t-distribution (Bollerslev, 1987) or the generalized error distribution (GED) (Nelson, 1991), is usually assumed for the error process in the conditional mean equation. In this paper, we follow Nelson (1991) by using the GED.[8] The estimates based on the log-maximum likelihood method are reported in Table 3.

<Insert Table 3 about here>

The evidence presented in Table 3 is quite consistent with respect to the sign and other statistical results. Specifically, AR(1) coefficients are negative for the DJIA returns and positive for the NASDAQ returns. The p-values suggest that these coefficients are statistically significant for the 10-minute and 30-minite frequencies, whether or not the data on opening returns are included in the estimations. The diverse signs of AR(1) coefficients reflect two distinct trading behaviors associated with investors involved in the DJIA and NASDAQ markets. Theory (Sentana and Wadhwani, 1992; Antoniou, *et al*., 2005) suggests that the presence of positive feedback trading leads to negatively autocorrelated stock returns, while negative feedback trading tends to produce positively autocorrelated stock returns. Our evidence suggests that investors in the DJIA market have been dominated by the group of positive feedback traders, buying (selling) stocks after prices rise (fall), while investors in the NASDAQ market are mainly governed by a negative feedback group, buying (selling) after prices decline (rise).[9]

Another point that emerges from the empirical evidence in Table 3 is that the coefficients of the GARCH components are all highly significant, justifying the fact that stock return volatilities are characterized by a heteroscedastic process. Note that with the exception of the 30-minute interval in the DJIA, $\hat{\alpha}_\tau + \hat{\beta}_\tau$ is very close to unity, indicating a high degree of persistence of volatility.

5. Dynamic Conditional Correlation between the DJIA and NASDAQ

It is generally recognized that financial markets are highly integrated and efficient; price movements in one market are likely to spill over to another market instantaneously. Empirical evidence about stock return correlations abounds, ranging from individual stocks and mutual funds to stock indices for national markets.[10] For this reason the analysis of stock returns should not be restricted to a single market. Rather, in a general equilibrium apparatus, the interrelation between asset returns often carries some information content. One simple way to explore the relation of two returns is to calculate the correlation coefficient. However, a textbook type of correlation coefficient is usually assumed to be constant throughout a given window width. This approach apparently fails to capture the dynamic nature of financial markets, which are continuously subjected to ongoing shocks due to endogenous changes or innovations (Longin and Solnik, 1995). For this reason, we specify a multivariate model, which is capable of computing dynamic conditional correlation (DCC) coefficients that are capable of capturing ongoing market elements and shocks.

Following Engle (2002) and Chiang *et al*. (2007), the mean equation is assumed to be represented by equation (1); the multivariate conditional variance is given by:

$$H_{\tau,t} = D_{\tau,t} V_{\tau,t} D_{\tau,t}, \tag{3}$$

where $V_{\tau,t}$ is a symmetric conditional correlation matrix of ε_t, $D_{\tau,t}$ is a (2x2) matrix with the conditional variances $h_{\tau,ii,t}$ for two stock returns (where i = DJIA and NASDAQ) on the diagonal. That is, $D_{\tau,t} = diag[\sqrt{\sigma^2_{\tau,ii,t}}]_{(2,2)}$. Equation (3) suggests that the dynamic properties of the covariance matrix $H_{\tau,t}$ are determined by

$D_{\tau,t}$ and $V_{\tau,t}$ for a given τ. The DCC model proposed by Engle (2002) involves two-stage estimations of the conditional covariance matrix H_t in equation (3). In the first stage, univariate volatility models are fitted for each of the stock returns and estimates of $\sqrt{\sigma^2_{\tau,ii,t}}$ (i =1 and 2) are obtained by using equation (4). In the second stage, stock-return residuals are transformed by their estimated standard deviations from the first stage. That is $\eta_{\tau,i,t} = \varepsilon_{\tau,i,t} / \sqrt{\sigma^2_{\tau,ii,t}}$, where $\eta_{\tau,i,t}$ is then used to estimate the parameters of the conditional correlation. The evolution of the correlation in the DCC model is given by equation (5):

$$\sigma^2_{\tau,ii,t} = c_{\tau,i} + \alpha_{\tau,i}\varepsilon^2_{\tau,i,t-1} + \beta_{\tau,i}\sigma^2_{\tau,ii,t-1}, \quad i = 1, 2 \tag{4}$$

$$Q_{\tau,t} = (1 - \alpha_{\tau,i} - \beta_{\tau,i})\overline{Q}_\tau + \alpha_{\tau,i}\, \eta_{\tau,i,t-1}\eta'_{\tau,i,t-1} + \beta_{\tau,i}Q_{\tau,t-1}, \tag{5}$$

where $Q_{\tau,t}$ = $(q_{\tau,ij,t})$ is the 2×2 time-varying covariance matrix of $\eta_{\tau,i,t}$, $\overline{Q}_\tau = E[\eta_{\tau,i,t}\eta'_{\tau,i,t}]$ is the 2×2 unconditional variance matrix of $\eta_{\tau,i,t}$, and $\alpha_{\tau,i}$ and $\beta_{\tau,t}$ are non-negative scalar parameters satisfying $(\alpha_{\tau,i} + \beta_{\tau,i}) < 1$. Since Q_t does not generally have ones on the diagonal, we scale it to obtain a proper correlation matrix $V_{\tau,t}$. Thus,

$$V_{\tau,t} = (diag(Q_{\tau,t}))^{-1/2}\, Q_{\tau,t}\, (diag(Q_{\tau,t}))^{-1/2}, \tag{6}$$

where $(diag(Q_{\tau,t}))^{-1/2} = diag\,(1/\sqrt{q_{\tau,11,t}},\ 1/\sqrt{q_{\tau,22,t}})$.

Now $V_{\tau,t}$ in equation (6) is a correlation matrix with ones on the diagonal and off-diagonal elements less than one in absolute value, as long as $Q_{\tau,t}$ is positive definite. A typical element of $V_{\tau,t}$ is in the form of:

$$\rho_{\tau,12,t} = q_{\tau,12,t} / \sqrt{q_{\tau,11,t} q_{\tau,22,t}} \ , \tag{7}$$

The dynamic correlation coefficient, $\rho_{\tau,12,t}$, can be obtained by using the element of $Q_{\tau,t}$ in equation (5), which is given by (5)′ below:

$$q_{\tau,ij,t} = (1 - a_{\tau,i} - b_{\tau,i})\overline{\rho}_{\tau,ij} + a_{\tau,i}\ \eta_{\tau,i,t-1}\eta'_{\tau,j,t-1} + b_{\tau,i} q_{\tau,ij,t-1}, \tag{5$'$}$$

The mean reversion requires that $(a_{\tau,i} + b_{\tau,i}) < 1$. The estimates of dynamic correlation coefficients (DCC)[11], $\rho_{\tau,12,t}$, between DJIA and NASDAQ index returns for one day, 10 minutes, and 30 minutes are shown in Figures 5a, 5b, and 5c, respectively. [12]

<Insert Figures 5a-5c about here>

Several observations are immediately apparent from these figures. First, although the correlation coefficients lie mainly in the range of 0.6 to 0.8 for most of the time, the estimated coefficients are time varying, reflecting some sort of portfolio shifting of the indices. Second, from a historical perspective, the variations of the correlations are seen to be declining, and the series appears to be more stable and displays less variance after the end of 2001. This suggests that both return series are more or less subjected to common factors in the post-2001 period, such as systematic risk, macroeconomic announcements, or Fed policy. This implies that the benefit of diversifying by holding a combination of DJIA and NASDAQ stocks has declined in recent years. Third, correlation variations occur more frequently during downturns than upturns. This may be attributable to sector rotation between the new economy and the old economy in early 2000 or to diverse beliefs and expectations triggered by outbreaks of news. Fourth, the correlation coefficients increase their variability with frequent scales. It becomes more apparent in highly volatile periods. For example, if we look at the data between April 4, 2000, and April 12, 2000, the correlation coefficients for the daily data even display some negative values. To gain more insight into the dynamic nature of these DCCs, the correlation coefficients have been fitted into a time series model, which allows the variances to evolve over time. Since plots of $\rho_{\tau,ij,t}s$ (from Figures 5a-5c) show non-stationarity, a first difference is required. Further, statistics (not reported) from autocorrelation and partial autocorrelation functions for the 10-minute and 30-minute series indicate that the MA(1) model appears to be a parsimonious representation. Thus, we write mean and variance equations as:

$$\Delta\rho_{\tau,ij,t} = \mu - \theta_1\, \upsilon_{\tau,t-1} + \upsilon_{\tau,t}\,, \tag{8}$$

$$h_{\tau,\rho,t} = \omega_{\tau,0} + \omega_1\upsilon^2_{\tau,t-1} + \omega_2 h_{\tau,\rho,t-1}\,. \tag{9}$$

where μ, θ_1, and ω are parameters, and $\upsilon_{\tau,t}$ is the shock term. The variances expressed in (9) are assumed to evolve with a GARCH(1, 1) process, as popularized by Bolleslev *et al.* (1992).

Since investment strategy, environment, and investor sentiment and psychology have displayed a distinct change since September 11, 2001, we use this date as a breakpoint to examine the DCC changes for 10-minute, 30-minute, and daily correlation series. The results of the MA(1)-GARCH(1, 1) model are reported in Table 5.[13] As shown in statistics of means and standard deviations, the mean values are consistently greater and have lower variances across all of the scales for the postcrisis period. This implies that both the DJIA and the NASDAQ indices have been commonly driven by certain market forces in a relatively stable fashion. The variations are still subject to macroeconomic news, announcements, and dynamic social/political factors. Interestingly, the mean equation of $\Delta\rho_{\tau,ij,t}$ for intraday daily data consistently reveals an MA(1) pattern; no particular pattern is shown on the coefficient of the daily series. It is generally recognized that an MA(1) process is equivalent to AR(∞), meaning that the $\Delta\rho_{\tau,ij,t}$ is highly correlated in the high-frequency data. The correlation coefficients exhibit even higher values in the post-2001 crisis period. Although the pattern is rather stable, the message derived from the GARCH coefficients indicates that the correlation coefficients are time varying.

<Insert Table 5 about here>

It is interesting to note that the analysis in this section is consistent with the results from phase correlations between the DJIA and the NASDAQ. Wu *et al*. (2006) found that the distributions of phase differences between the DJIA and the NASDAQ show an impressive change of phase correlation after the events of September 11, 2001, and the scenario persisted in later trading activities. The phenomenon has been attributable to speedy communications and a greater sensitivity to investors' psychology and to socio-political events after the September 11 shock to stock markets.

6. Scaling Analysis

To gain more insight into understanding the collective behaviors revealed by activities in stock markets, we perform scaling analysis on the DJIA and the NASDAQ indices at different scales. To elucidate, let us define the probability distribution P as a normalized distribution of a measure Z, which satisfies

$$\int_{-\infty}^{\infty} P(Z_{\tau,t}) dZ_{\tau,t} = 1 \qquad (10)$$

where $Z_{\tau,t}$ is the measure of stock return, and τ (=10 minutes, 20 minutes, …etc.) is a multiple of the primary time sampling unit Δt. Figures 6a and 6b depict the probability distributions $P\left(Z_{\tau,t}\right)$ of the intraday frequencies for both DJIA and NASDAQ return changes $Z_{\tau,t}$ observed at five different time intervals τ, ranging from 10 to 1,950 minutes. These distributions are scale-dependent, and the shorter the time interval, the narrower the width of the distribution. It has been reported that a properly normalized version of return can have its probability distribution behave as a rescaled-like distribution, such that probability distributions of normalized returns for different time scales can converge into a single curve (Voit 2003; Wu *et al*. 2006). The probability distribution of the normalized return can be described well by the double-exponential distribution at not-too-long τ (Silva, Prange, and Yakovenko, 2004), which can be modeled by the Heston model with stochastic volatility (Dragulescu and Yakovenko, 2002). These features for the DJIA and the NASDAQ have been demonstrated and discussed by Wu, *et al.* (2006).

<Insert Figures 6a-b about here>

As suggested by Mantegna and Stanley (1995, 2000), it is possible to map probability distributions with different time sampling scales into a single curve by performing scaling analysis. We plot $P\left(Z_{\tau,t}=0\right)$ of two indices against the time sampling intervals τ as shown in Figure 7a. Within a truncated time scale, the distributions of $P\left(Z_{\tau,t}=0\right)$ in relation to τ plotted in logarithmic scale are linear

(Mantegna and Stanley, 2000); the best fitting straight lines (also plotted in Figure 7a) obey the following (Wu *et al*., 2006):

$$\log_{10} P\left(Z_{\tau,t}=0\right)=c-\frac{1}{\alpha}\log_{10}\tau \,, \tag{11}$$

where c is a constant, and α is a quantity characterizing the class of distribution. It has been shown that the α value in equation (11) can be used to determine the stability of a distribution, which, in turn, enables us to determine the stability of the process under consideration (see Mantegna and Stanley, 2000). By measuring the slope of the fitting straight line, we obtain $\alpha \approx 1.84$ for the DJIA and $\alpha \approx 1.75$ for the NASDAQ; both of the α values are greater than 1.4 (Mantegna and Stanley, 1995), and less than or equal to 2 ($\alpha \le 2$), satisfying the condition for stable Lévy distributions (Voit, 2003).

Note that the stable non-Gaussian type of the probability distributions is a stochastic process with infinite variance characterized by distributions with power-law tails. Power-law distributions also imply a lack of a characteristic scale. We then rescale the probability distribution function $P\left(Z_{\tau,t}\right)$ and return changes $Z_{\tau,t}$ as suggested by Mantegna and Stanley (1995). It follows

$$Z_{s,\tau,t}=\frac{Z_{\tau,t}}{\tau^{1/\alpha}}\,,\;\; P_s\left(Z_{s,\tau,t}\right)=\frac{P\left(Z_{\tau,t}\right)}{\tau^{-1/\alpha}} \tag{12}$$

where the subscript s is used to denote scaled quantities. Figures 7b and 7c show the scaled plots of the probability distributions with $\alpha = 1.84$ for the DJIA and $\alpha = 1.75$ for the NASDAQ, respectively. Apparently, probability distributions of time scales can coincide with each other very well.

Evidently, if a stock market can be treated as a physical system, the data suggest that either the DJIA (with $\alpha = 1.84$) or the NASDAQ (with $\alpha = 1.75$) can converge to a stable state by itself after a shock, without relying on external regulation or intervention. This also implies that stock markets such as the DJIA and the NASDAQ are considered to have mature, self-governing mechanisms for maintaining their own stability. Meanwhile, the scaling analysis indicates that the DJIA, with a slightly higher α value than that of the NASDAQ, is equipped with a relatively better self-stabilizing capacity and presents a more uniform time series pattern.

<Insert Figures 7a-7c about here>

7. Conclusions

In this paper, we investigate the statistical properties of high-frequency data on stock returns. The analysis applies to both the Dow Jones Industrial Average (DJIA) and the NASDAQ indices. The evidence indicates that the empirical regularities are highly influenced by opening returns that contain overnight and other information of irregular length. The statistics show that both the NASDAQ and the DJIA have excessively high returns during opening time, although the NASDAQ, on average, has a higher return than the DJIA. The higher returns in the NASDAQ are matched by higher volatilities over all of the trading intervals over the business day. The evidence also shows that the high-frequency-return variances for a given scale produce a smile curve and the curvature is seen to be increasing with the time scale.

By examining the AR(1) pattern across time and frequencies, we often find that the coefficients are somehow affected by data that include the opening interval. However, when we fit the intraday stock returns into an AR(1)-GARCH(1, 1) model, we find consistent results for both 10-minute and 30-minure return horizons. The evidence shows that the DJIA returns are negatively autocorrelated, while the NASDAQ returns are positively autocorrelated, meaning that investors in the DJIA behave as positive feedback traders, whereas investors in the NASDAQ are a negative feedback group.

By examining the dynamic correlation coefficients between the DJIA and the NASDAQ returns over time, the return correlations are positive and fluctuate mainly in the range of 0.6 to 0.8. The statistics show that the correlation coefficients are time varying, reflecting some sort of dynamic portfolio allocations among different financial assets. By inspecting the time series path of conditional correlation coefficients, we find that the variations of the coefficients are declining and appear to be more stable for the post-2001 period. This suggests that both markets are sensitive to or driven by some common factors, such as systematic risk, macroeconomic announcements, Fed policy, or investor psychology. This also implies that the benefit of diversifying by holding a combination of DJIA and NASDAQ stocks has declined in recent years.

By checking for conditions of a stable Lévy distribution, we find that both the DJIA and the NASDAQ can converge to a stable equilibrium after shocks without relying on an external regulation or intervention. This implies that both markets appear to be stable and mature and are governed by a self-stabilizing mechanism, especially the DJIA market.

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Table 1 Summary statistics across different frequencies of return and volatility series

This table summarizes the return ($R_{\tau,t}$) statistics across different frequencies of intraday, interday, and interweek for both the DJIA and the NASDAQ. The sample period covers Aug. 1, 1997, to Dec. 31, 2003, for a total of 60,884 10-minute observations.

	DJIA									NASDAQ									
τ	Mean	Med.	Max.	Min.	Std. Dev.	Skew.	Kurt.	J.-B.	Prob.	Mean	Med.	Max.	Min.	Std. Dev.	Skew.	Kurt.	J.-B.	Prob.	Observ.
10-min	0.000006	0	0.034	-0.0326	0.001974	0.167669	23.36597	1052495	0	0.0000125	0.000015	0.07377	-0.0683	0.003978	0.24868	29.14976	1735339	0	60884
20-min	0.000012	0	0.03689	-0.04292	0.002786	0.059889	18.74973	314653.8	0	0.0000248	0.000050	0.09912	-0.06611	0.005607	0.515363	19.97172	366701.2	0	30442
30-min	0.000018	0	0.04339	-0.0447	0.003411	0.264554	15.26449	127427.4	0	0.0000382	0.000050	0.14961	-0.0788	0.00699	0.762882	23.8069	368044.5	0	20294
40-min	0.000024	0	0.03685	-0.04647	0.003934	-0.126545	12.73828	60185.15	0	0.0000496	0.000080	0.13195	-0.06955	0.007932	0.471845	16.59085	117710.2	0	15221
50-min	0.000030	0.000075	0.04731	-0.04894	0.00443	-0.032442	13.01785	50916.79	0	0.0000626	0.000050	0.1129	-0.07885	0.008902	0.47037	13.41195	55448.36	0	12176
60-min	0.000036	0.000020	0.04568	-0.0434	0.004839	0.11569	10.18474	21847.34	0	0.0000767	0.000100	0.11919	-0.07476	0.009908	0.45797	12.1145	35477.76	0	10147
70-min	0.000041	0.000030	0.04035	-0.0586	0.0052	-0.099977	10.91561	22719.79	0	0.0000883	0.000130	0.13467	-0.07106	0.010627	0.463904	12.00188	29676.59	0	8697
80-min	0.000047	0	0.04772	-0.06083	0.005556	0.00203	10.54224	18037.4	0	0.0001010	0.000140	0.14252	-0.07561	0.011323	0.572785	12.1004	26676.1	0	7610
90-min	0.000053	0	0.05044	-0.05621	0.005922	0.094176	9.375432	11465.43	0	0.0001130	0.000160	0.12283	-0.07159	0.012055	0.34779	9.290563	11288.84	0	6764
100-min	0.000059	0	0.0482	-0.05177	0.006263	-0.011898	9.286831	10026.13	0	0.0001270	0.000130	0.16023	-0.09494	0.012754	0.541165	12.86781	24997.6	0	6088
110-min	0.000064	0	0.0413	-0.04717	0.006532	0.085298	8.365525	6644.942	0	0.0001380	0.000190	0.12811	-0.07976	0.013252	0.397428	8.811231	7932.576	0	5534
120-min	0.000070	0.000030	0.04717	-0.04681	0.006844	0.004339	8.271554	5873.976	0	0.0001540	0.000160	0.17218	-0.07951	0.014148	0.609759	12.45802	19222.74	0	5073
130-min	0.000076	0.000060	0.04089	-0.05899	0.00702	0.005924	7.941249	4764.188	0	0.0001680	0.000160	0.11997	-0.13896	0.014726	0.176103	8.862001	6729.295	0	4683
390-min	0.000233	0.000480	0.05857	-0.06803	0.012658	-0.016781	5.541697	420.2569	0	0.0005380	0.001390	0.17424	-0.11252	0.02686	0.168805	5.40815	384.6023	0	1561
1-day	0.000234	0.000300	0.06348	-0.07184	0.01285	-0.09432	5.87337	557.2809	0	0.0003760	0.001560	0.14173	-0.09669	0.021685	0.197128	5.56106	451.2681	0	1613
2-day	0.000472	0.001480	0.07741	-0.08749	0.01858	-0.293446	4.972641	142.2508	0	0.0007680	0.002850	0.1422	-0.11893	0.030997	-0.037244	4.621624	88.49917	0	806
3-day	0.000682	0.001780	0.0958	-0.10872	0.02137	-0.376276	5.52345	155.1512	0	0.0011020	0.003930	0.14274	-0.13514	0.036568	-0.350185	4.151999	40.66928	0	537
4-day	0.000961	0.002010	0.13305	-0.12363	0.026925	-0.295379	6.277294	186.2137	0	0.0015360	0.003550	0.15109	-0.16624	0.043674	-0.272676	3.919767	19.19925	0.000068	403
5-day	0.001099	0.002525	0.09837	-0.1426	0.027533	-0.357011	5.447198	87.18964	0	0.0017600	0.006675	0.18969	-0.17358	0.046693	-0.24838	4.26787	24.87807	0.000004	322
1-week	0.001096	0.002950	0.08426	-0.14263	0.026995	-0.508843	5.282978	86.94677	0	0.0017240	0.002895	0.18978	-0.25305	0.045226	-0.492549	6.38399	172.8701	0	334
2-week	0.002115	0.005920	0.11924	-0.14502	0.036082	-0.528108	4.70597	28.01374	0	0.0033530	0.005140	0.25753	-0.1805	0.063106	0.017988	4.063278	7.875821	0.019489	167
3-week	0.003071	0.007070	0.10407	-0.13242	0.044471	-0.274524	3.074423	1.419836	0.491685	0.0055250	0.017370	0.20654	-0.33079	0.083506	-0.577294	4.536858	17.0894	0.000195	111
4-week	0.004148	0.006060	0.11911	-0.15116	0.05314	-0.394225	3.262172	2.387595	0.303068	0.0062160	0.009640	0.27388	-0.21807	0.087565	0.109033	3.383367	0.672725	0.714364	83
5-week	0.004884	0.010855	0.15898	-0.20933	0.064307	-0.358524	3.81427	3.237287	0.198167	0.0086670	0.009990	0.26439	-0.27257	0.107436	0.035241	3.116665	0.05109	0.974778	66

Table 2 Time series estimates of AR(1) for the DJIA and the NASDAQ across different time frequencies

τ		DJIA		NASDAQ	
		Coefficient	P-value	Coefficient	P-value
10-min	δ_τ	0.00000604	0.4498	0.0000126	0.4399
	ϕ_τ	-0.001695	0.6758	-0.000938	0.8169
	R^2	0.00028		0.000002	
30-min	δ_τ	0.0000183	0.4716	0.0000385	0.4383
	ϕ_τ	0.003285	0.64	0.005783	0.4102
	R^2	0.000841		0.000242	
60-min	δ_τ	0.0000361	0.465	0.0000768	0.4643
	ϕ_τ	-0.00124	0.9007	0.007409	0.4556
	R^2	0.001899		0.001201	
120-min	δ_τ	0.0000709	0.4423	0.000152	0.4431
	ϕ_τ	0.027851	0.0476	0.014206	0.3118
	R^2	0.006283		0.000112	
390-min	δ_τ	0.000261	0.3782	0.000592	0.3819
	ϕ_τ	-0.022378	0.3797	-0.098186	0.0001
	R^2	0.006283		0.00964	

Notes: Total sample included 60,879 observations for the 10-minute series after adjustment. The estimated equation is:

$$R_{\tau,t} = \delta_\tau + \phi_\tau R_{\tau,t-1} + \varepsilon_{\tau,t} ,$$

where $R_{\tau,t}$ is stock return applied to the *DJIA* and the *NASDAQ* series; δ_τ is a constant term; ϕ_τ is a constant coefficient; the subscript τ is a scale index; $\varepsilon_{\tau,t}$ is a vector of random error terms.

Table 3 Time series estimates of AR(1)-GARCH(1, 1) of DJIA and NASDAQ high-frequency returns

Panel A: DJIA including opening returns								
τ	Obs.	ϕ_τ	P-value	α_τ	P-value	β_τ	P-value	$\alpha_\tau + \beta_\tau$
10-min	60528	-0.0196	0.0000	0.2061	0.0000	0.7482	0.0000	0.9543
30-min	20176	-0.0123	0.0742	0.2959	0.0000	0.4744	0.0000	0.7703
60-min	10088	-0.0020	0.0749	0.0302	0.0000	0.9648	0.0000	0.9950
120-min	5045	0.0096	0.4578	0.0415	0.0000	0.9497	0.0000	0.9912
390-min	1613	-0.0348	0.1808	0.0921	0.0000	0.8836	0.0000	0.9757
1-day	1613	-0.0305	0.2581	0.0882	0.0000	0.8838	0.0000	0.9721
Panel B: NASDAQ including opening returns								
τ	Obs.	ϕ_τ	P-value	α_τ	P-value	β_τ	P-value	$\alpha_\tau + \beta_\tau$
10-min	60528	0.0271	0.0000	0.3371	0.0000	0.6952	0.0000	1.0323
30-min	20176	0.0134	0.0389	0.3586	0.0000	0.5233	0.0000	0.8819
60-min	10088	0.0025	0.3042	0.0284	0.0000	0.9695	0.0000	0.9979
120-min	5045	0.0283	0.0244	0.0358	0.0000	0.9612	0.0000	0.9970
390-min	1613	-0.1273	0.0000	0.0936	0.0000	0.8949	0.0000	0.9885
1-day	1613	0.0014	0.9600	0.1007	0.0000	0.8888	0.0000	0.9895
Panel C: DJIA excluding opening returns								
τ	Obs.	ϕ_τ	P-value	α_τ	P-value	β_τ	P-value	$\alpha_\tau + \beta_\tau$
10-min	58976	-0.0080	0.0583	0.1388	0.0000	0.8361	0.0000	0.9749
30-min	18624	-0.0139	0.0384	0.0470	0.0000	0.9460	0.0000	0.9929
60-min	9312	-0.0051	0.5705	0.0307	0.0000	0.9649	0.0000	0.9955
120-min	4656	0.0074	0.5791	0.0400	0.0000	0.9523	0.0000	0.9923
Panel D: NASDAQ excluding opening returns								
τ	Obs.	ϕ_τ	P-value	α_τ	P-value	β_τ	P-value	$\alpha_\tau + \beta_\tau$
10-min	58976	0.0528	0.0000	0.2024	0.0000	0.7970	0.0000	0.9995
30-min	18624	0.0298	0.0000	0.0505	0.0000	0.9474	0.0000	0.9979
60-min	9312	0.0064	0.4712	0.0320	0.0000	0.9660	0.0000	0.9981
120-min	4656	0.0312	0.0185	0.0355	0.0000	0.9619	0.0000	0.9974

Notes: There is only one observation in 390-min interval per day; hence, there is no observation after excluding the opening interval in the 390-min and daily frequency series. The AR(1)-GARCH(1, 1) model is

$$R_{\tau,t} = \delta_\tau + \phi_\tau R_{\tau,t-1} + \varepsilon_{\tau,t}$$
$$\sigma^2_{\tau,t} = \omega_\tau + \alpha_\tau \varepsilon^2_{\tau,t-1} + \beta_\tau \sigma^2_{\tau,t-1}$$

where τ represents different frequencies.

Table 4 Time series analysis of dynamic conditional correlation coefficients at various time scales

Coefficient	$\Delta\rho_{\tau,t}$ (1-day)		$\Delta\rho_{\tau,t}$ (30-min)		$\Delta\rho_{\tau,t}$ (10-min)	
	before crisis	after crisis	before crisis	after crisis	before crisis	after crisis
Panel A: Mean and standard deviation of $\rho_{\tau,t}$						
μ_ρ	0.664	0.852	0.726	0.865	0.725	0.857
σ_ρ	0.158	0.094	0.129	0.060	0.127	0.061
Panel B: Mean equation						
C	-0.0005	-8.72E-06	-0.0002	0.0003	-3.23E-05	-2.17E-05
	(0.628)	(0.018)	(0.229)	(1.869)**	(0.068)	(0.099)
θ_1	-	-	0.894	0.975	0.887	0.929
			(31.99)***	(35.90)***	(60.00)***	(54.28)***
Panel C: Variance equation						
ω_0	1.115E-05	2.4E-06	0.001	0.003	0.002	0.0002
	(6.55)***	(3.77)***	(0.90)	(3.039)***	(2.28)***	(1.64)
ω_1	0.054	0.115	0.027	0.363	0.048	0.024
	(9.97)***	(4.03)***	(0.98)	(3.17)***	(2.22)***	(2.87) ***
ω_2	0.936	0.738	0.896	0.076	0.771	0.909
	(174.84)***	(12.35)***	(8.40) ***	(0.28)	(8.78)***	(20.23)***
LB(10)	7.436	3.41	14.809**	4.094	21.86***	10.203

Notes:

a. The estimated equations are:

$$\Delta\rho_{\tau,\mathrm{t}} = \mu - \theta_1 \upsilon_{\tau,t-1} + \upsilon_{\tau,t} \text{ and } h_{\tau,\rho,t} = \omega_{\tau,0} + \omega_1 \upsilon_{\tau,t-1}^2 + \omega_2 h_{\tau,\rho,t-1}.$$

b. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively. The numbers in parentheses are standard errors.

c. $\Delta\rho_{\tau,t}$ (10-min) denotes change in conditional correlation coefficient for the (10-minute) series, etc.

d. LB (10) is the Ljung-Box statistics test for autocorrelation up to the 10^{th} lag.

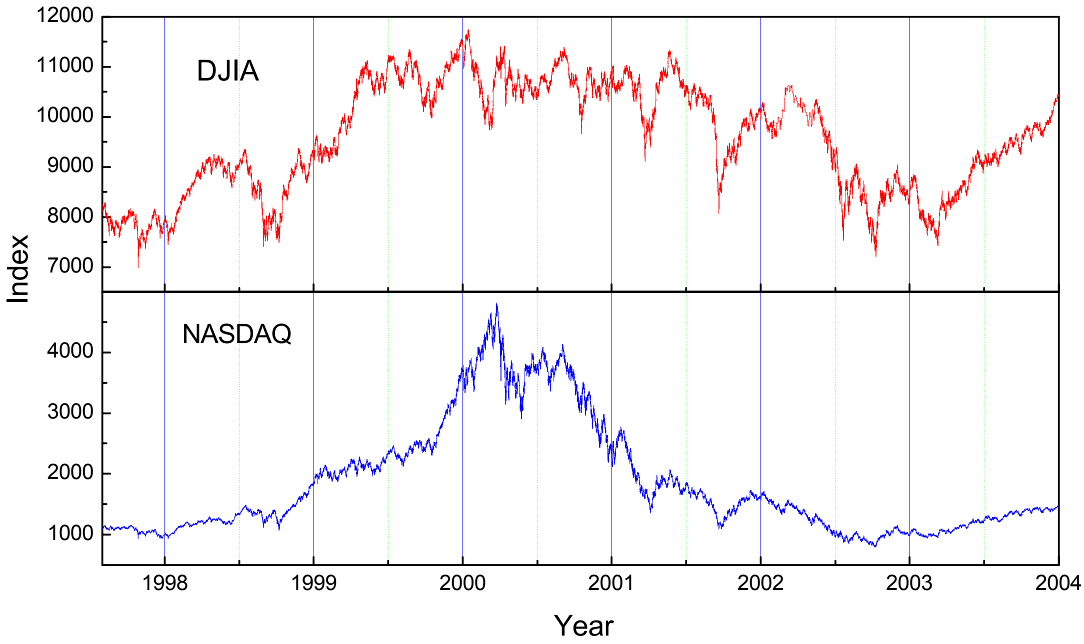


Figure 1 Time series plots of 10-minute frequencies of DJIA and NASDAQ indices (Aug. 1, 1997 - Dec. 31, 2003).

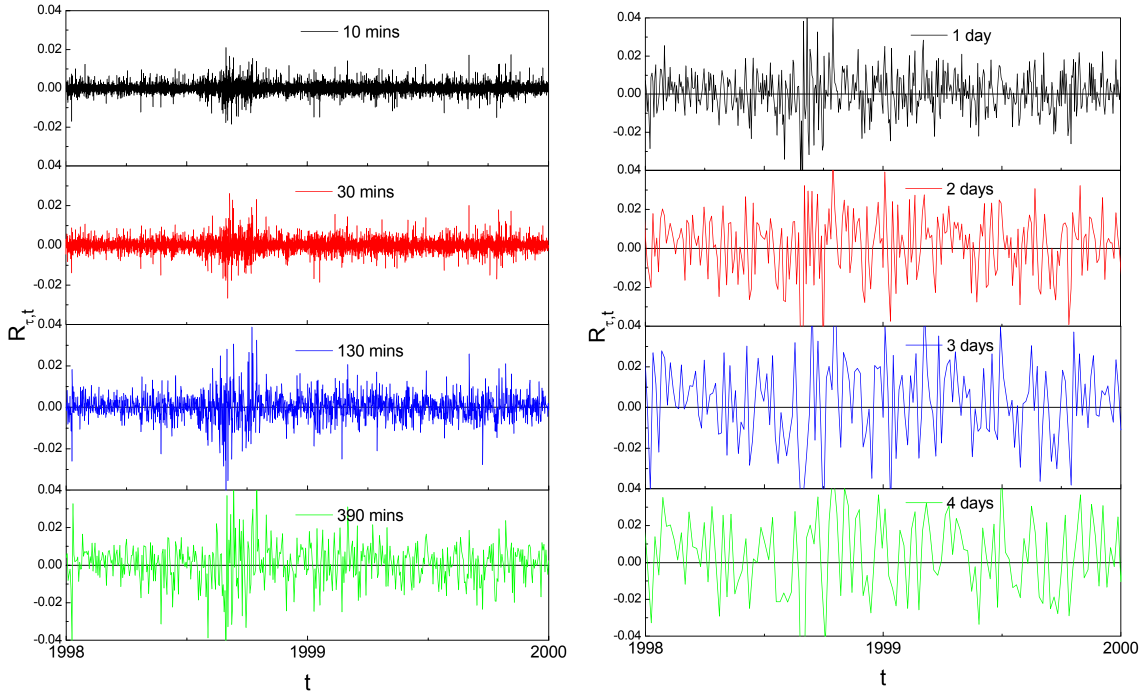


Figure 2 Time series plots of stock returns for intraday and interday DJIA data

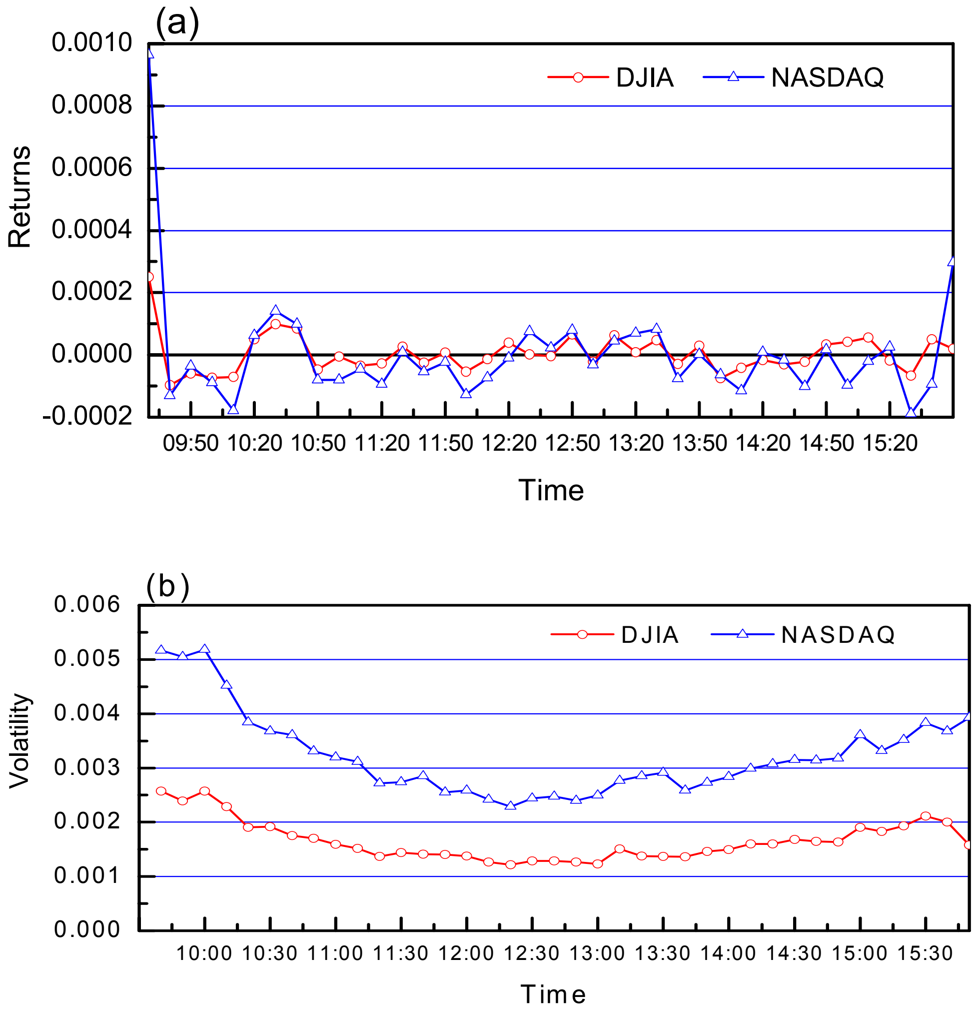
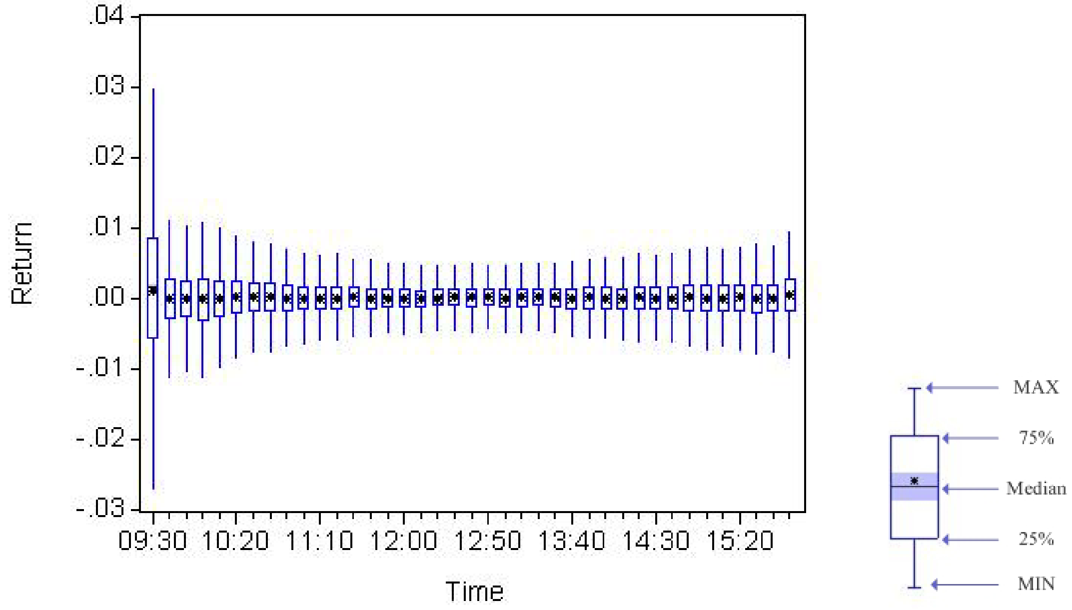
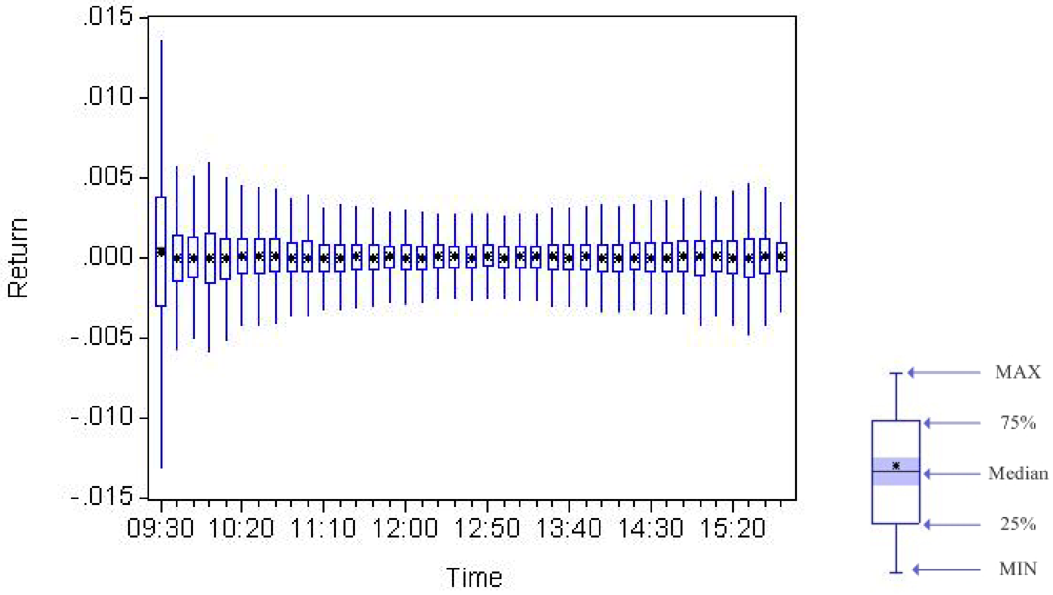


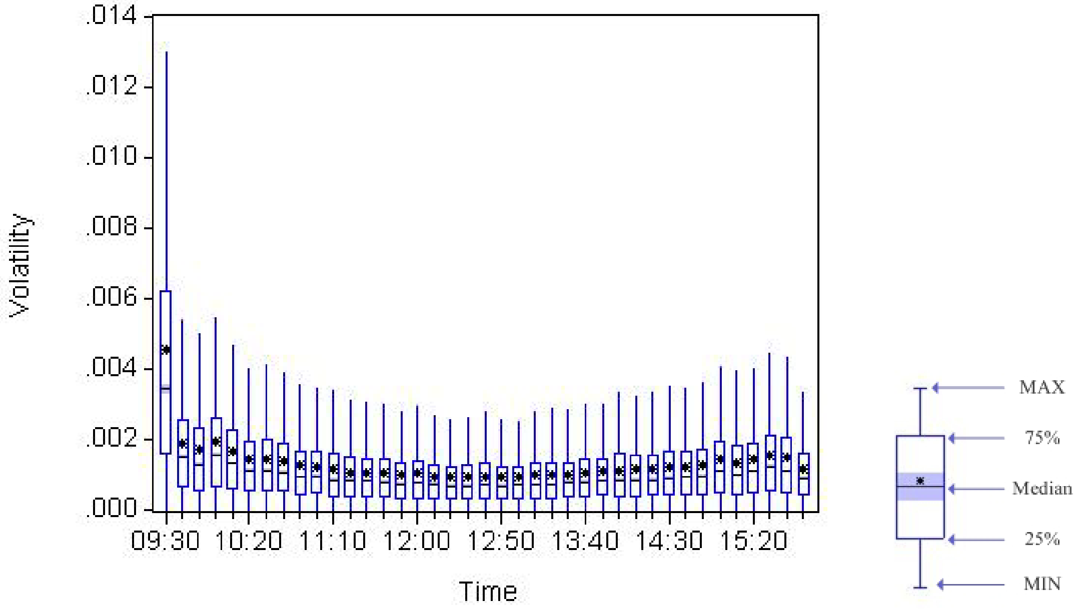
Figure 3.1 (a) Panel data of 10-minute intraday returns of DJIA and NASDAQ with opening interval, and (b) Panel data of 10-minute intraday volatility of DJIA and NASDAQ without opening interval.



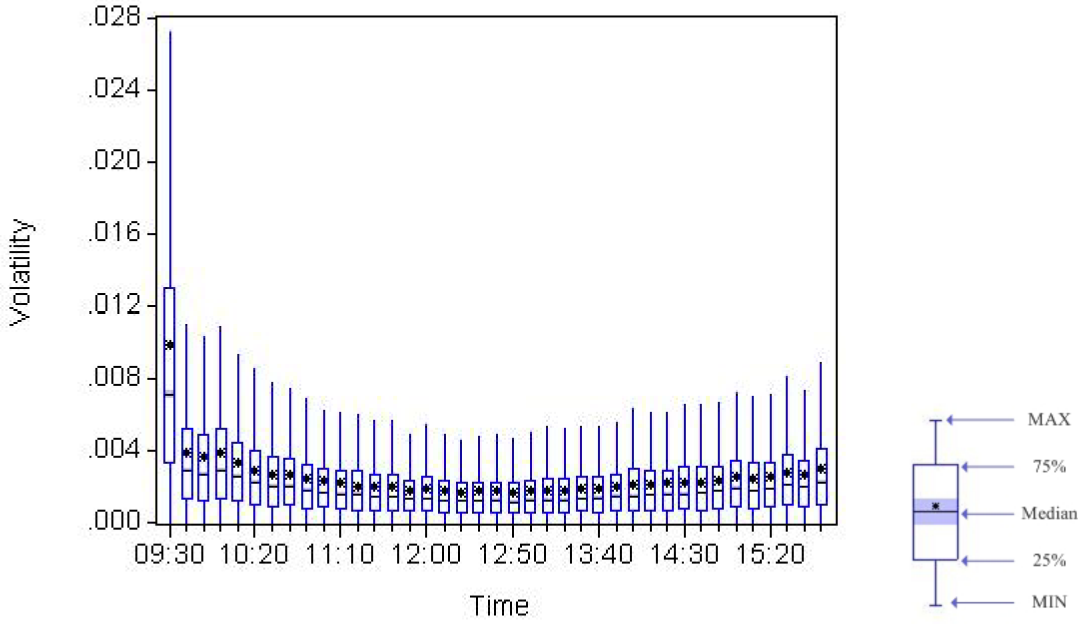
(a1) Boxplots of panel DJIA intraday returns for 10-mimute frequency



(a2) Boxplots of panel NASDAQ intraday returns for 10-mimute frequency



(b1) Boxplots of panel DJIA intraday volatility for 10-mimute frequency



(b2) Boxplots of panel NASDAQ intraday volatility for 10-mimute frequency

Figure 3.2 (a1 and a2) Boxplots of panel DJIA and NASDAQ intraday returns for 10-mimute frequency. (b1 and b2) Boxplots of panel DJIA and NASDAQ intraday volatility for 10-mimute frequency

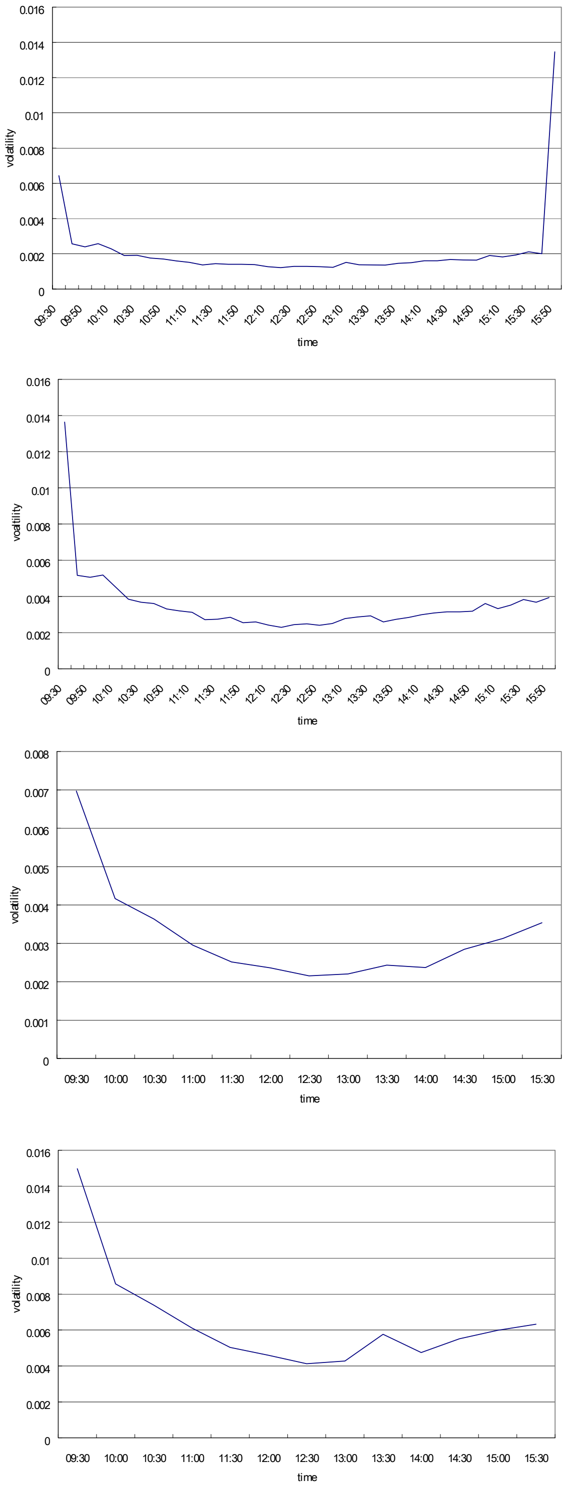


Figure 4 Panel intraday volatility at 10-minute and 30-minute scales with opening interval (a) Panel data of 10-minute intraday volatility of DJIA, (b) Panel data of 10-minute intraday volatility of NASDAQ, (c) Panel data of 30-minute intraday volatility of DJIA, and (d) Panel data of 30-minute intraday volatility of NASDAQ.

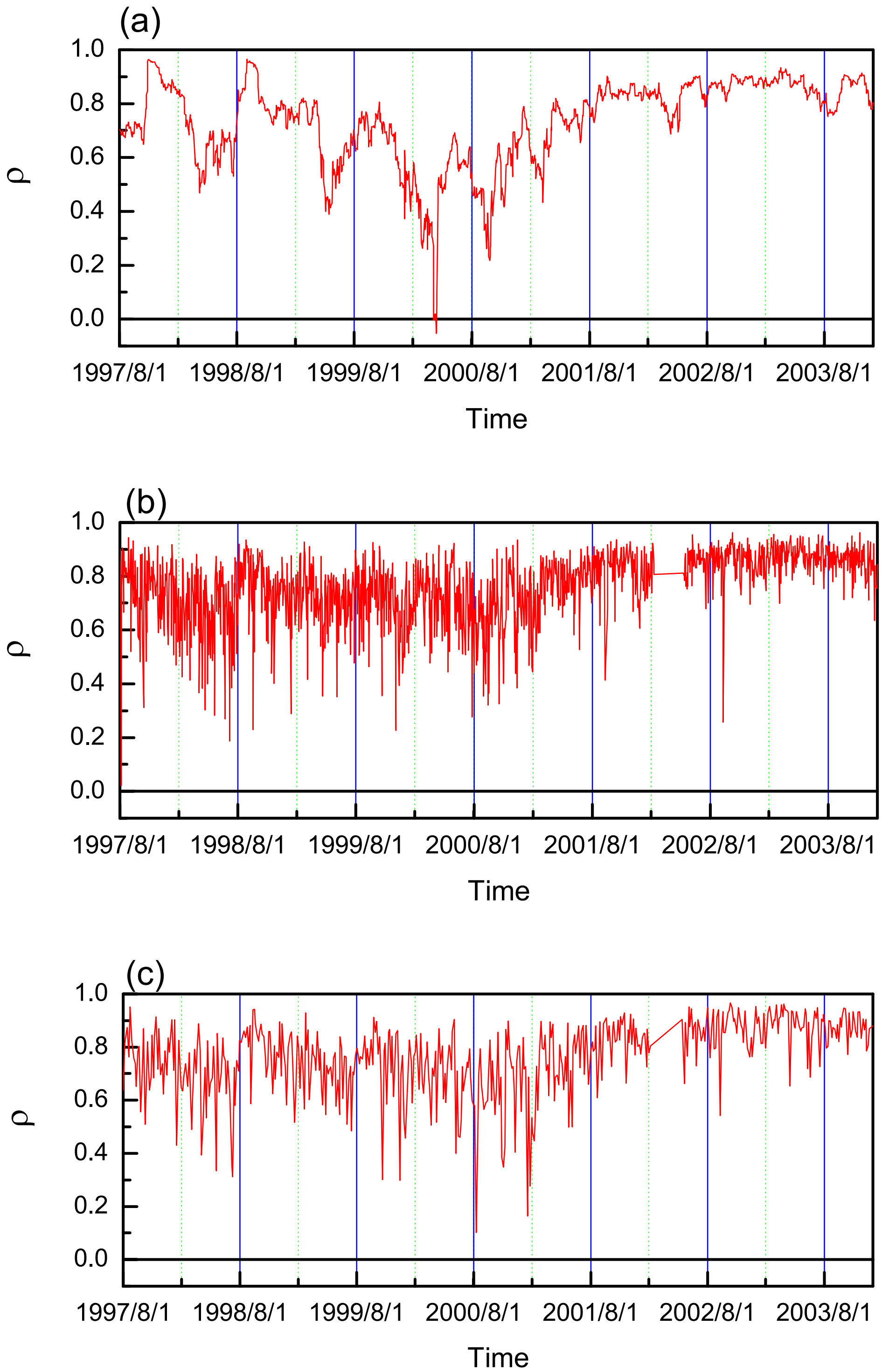


Figure 5 Dynamic conditional correlation between the DJIA and the NASDAQ:
(a) Daily dynamic conditional correlation between the DJIA and NASDAQ,
(b) 10-minute dynamic conditional correlation between the DJIA and the NASDAQ,
(c) 30-minute dynamic conditional correlation between the DJIA and the NASDAQ.

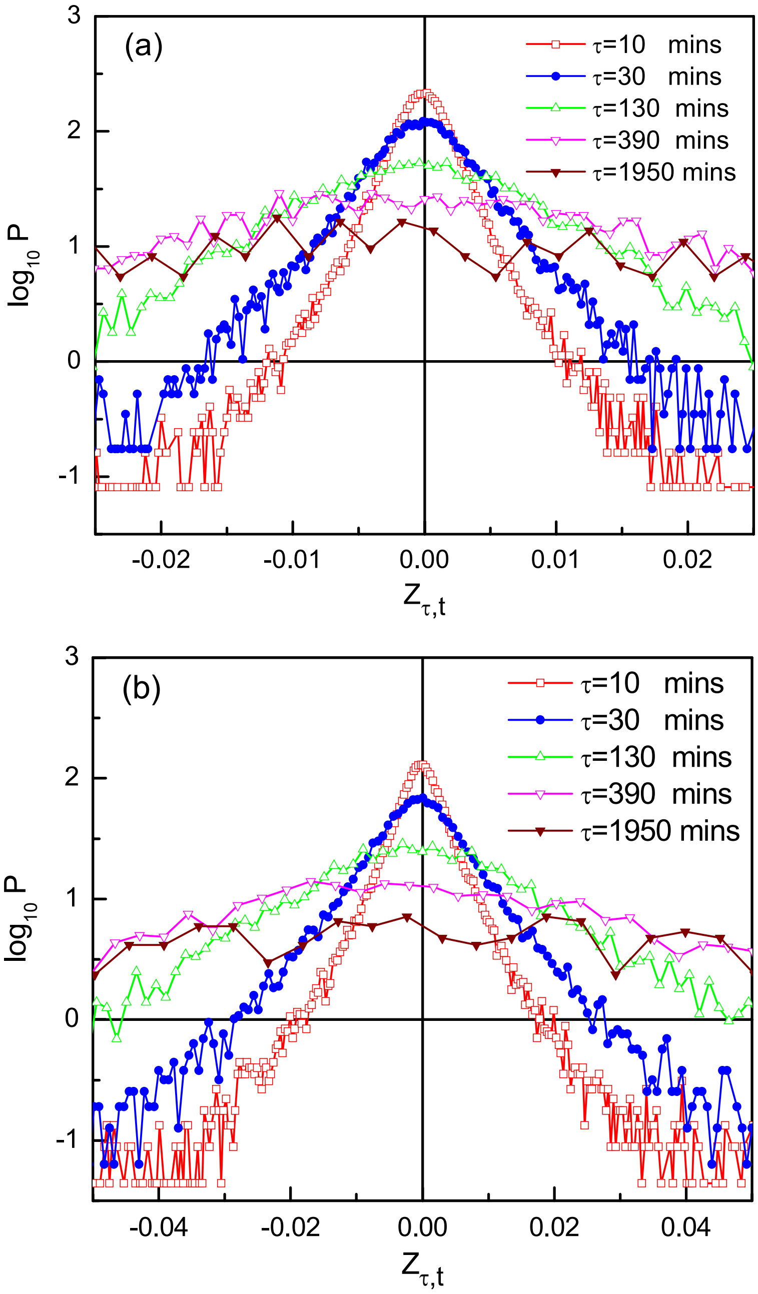


Figure 6 Probability distributions of return changes of (a) the DJIA, and (b) the NASDAQ for intraday data with time sampling intervals of multiples of 10 minutes.

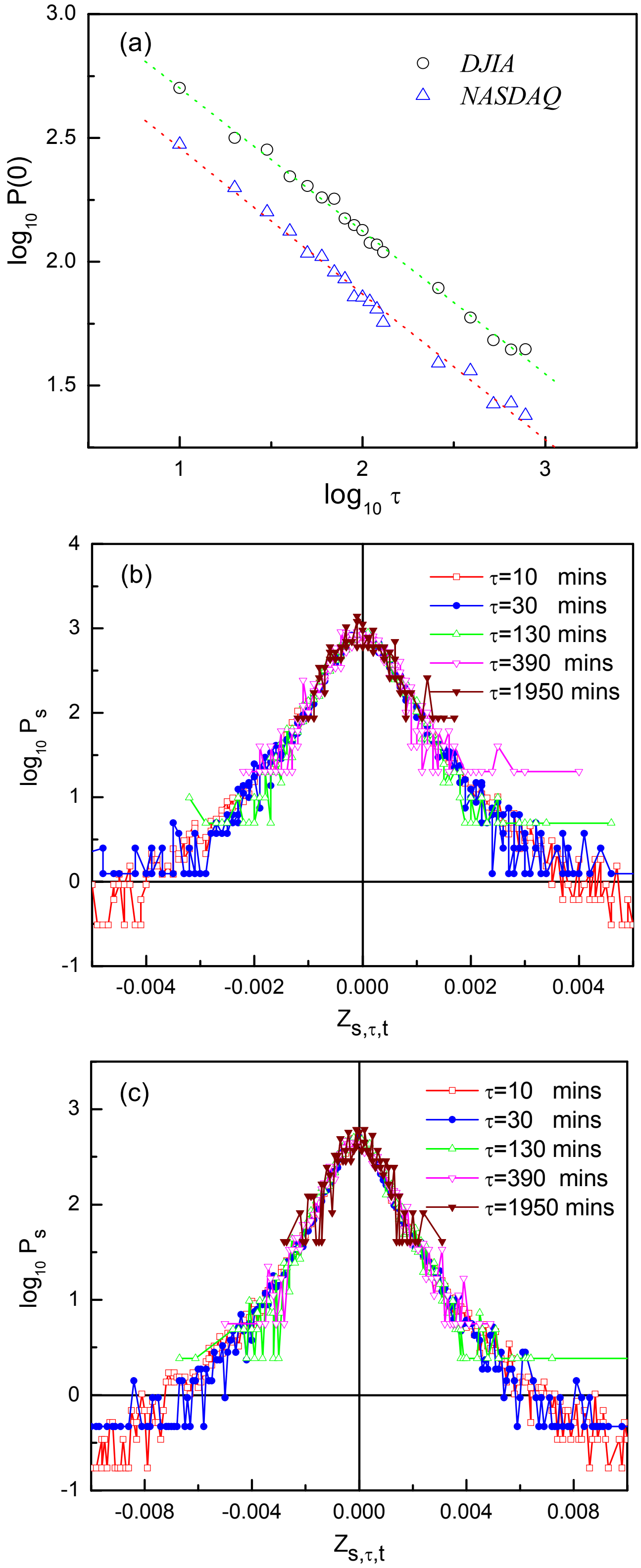


Figure 7 (a) Probability of return variation $P\left(Z_{\tau,t}=0\right)$ as a function of the time sampling intervals τ. The slope of the best-fit straight line is -0.54 ± 0.01 for DJIA, and -0.57 ± 0.01 for NASDAQ. Scaled plot of the probability distributions with (b) $\alpha=1.84$ for the DJIA, and (c) $\alpha=1.75$ for the NASDAQ.

Endnotes

[1] Alternatively, Engle and Russell (1998) have developed the autoregressive conditional duration (ACD) model to investigate high-frequency stock market data. In the ACD model the expected duration between trades depends on past durations. Here we follow Andersen and Bollerslev's approach to investigate the time series properties of two high-frequency stock returns. However, unlike Andersen and Bollerslev (1997) and Mian and Adam (2001), we do not omit the closing-to-opening returns. Rather, we keep them in the data to conduct sensitivity analyses. After eliminating the omitted days for which all of the 10-minute values of the index were not available, a total of 1,543 trading days with 60,177 observations of 10-minute index values was obtained

.

[2] Examination of their dynamic correlations between two series can be found in section 5.

[3] It may be seen that most of the interday intervals show left skewness, especially the DJIA. As noted by Andersen and Bollerslev (1997), the negative skewness may be interpreted as evidence of the "leverage" and/or volatility feedback effects discussed by Black (1976), Campbell and Hentschel (1992) and Bekaert and Wu (2000).

[4] The statistics show that daily returns have a skewness of -0.09 and kurtosis of 5.87 for the DJIA and skewness of 0.197 and kurtosis of 5.56 for the NASDAQ.

[5] We do not plot the NASDAQ to save space.

[6] Evidence of the U-shape pattern of intraday volatility can be found in Wood *et al.* (1985), Andersen and Bollerslev (1997), and Ito *et al.* (1998), among others. The theoretical models on the U-shaped pattern appear in Foster and Viswanathan (1990) and Slezak (1994).

[7] The smile curve of volatility reflects the fact that the highest point of return volatility occurs around the opening time 9:30 interval (0.006216804), followed by the 10:00,

9:40, and 9:50 intervals, respectively, and hits the lowest point around the 12:20 interval (0.001240272), followed by the second lowest at the 13:00 interval and third lowest at the 12:10 interval, respectively

.

[8] The parameters will be estimated by the log-maximum likelihood method. The density function in Nelson (1991) is given by

$$f(\mu_{\tau,t},\sigma_{\tau,t},\nu)=\frac{\nu[\Gamma(3/\nu)]^{1/2}}{2[\Gamma(1/\nu)]^{-3/2}\sigma_{\tau,t}}\exp\left[-\left|\frac{\varepsilon_{\tau,t}}{\sigma_{\tau,t}}\right|^{\nu}\left[\frac{\Gamma(3/\nu)}{\Gamma(1/\nu)}\right]^{\nu/2}\right]$$

where $\Gamma(\cdot)$ is the gamma function and ν is a scale parameter or degree of freedom to be estimated. For ν =2, the GED yields the normal distribution, while for ν =1 it yields the Laplace or double-exponential distribution. Given initial values of $\varepsilon_{\tau,t}$ and $\sigma^2_{\tau,t}$, the parameter vector $\Theta\equiv(\delta_\tau,\phi_\tau,\omega_\tau,\alpha_\tau,\beta_\tau,\nu)$ can be estimated by log-maximum likelihood method (log-MLE) over the sample period. The log-maximum likelihood function can be expressed as

$$L(\Theta)=\sum_{t=1}^{T}\log f(\mu_{\tau,t},\sigma_{\tau,t},\nu)$$

where $\mu_{\tau,t}$ is the conditional mean and $\sigma_{\tau,t}$ is the conditional standard deviation. Since the log-likelihood function is non-linear, the numerical procedure is used to derive estimates of the parameter vector.

[9] As argued by Sentana and Wadhwani (1992) and expounded by Antoniou *et al.* (2005), positive feedback traders buy stocks after prices rise and sell stocks after prices fall. Shiller (1989) found that a main reason that prompted investors to sell their stocks in October 1987 was that stock prices had fallen, thus inducing a fear of contagion in other investors. In contrast, the negative feedback traders sell stocks after prices increase and buy stocks after prices decline. Shiller argued that feedback models suggest that price is determined in part by its own lagged values, increases in price tending at times to foster further increases. However, as argued by Shiller, there is little, even a negative, serial correlation of price changes (Shiller, 1989, p. 375).

[10] A variety of papers have documented the fact that correlations across major stock markets change over time. King, Sentana, and Wadhwani (1994) find the covariances of stock returns change over time. Kaplanis (1988) compares the matrices of returns across 10 markets and finally rejects the constant correlations hypothesis. Koch and Koch (1991) use Chow tests to examine stock returns in 1972, 1980, and 1987 and find higher correlations in more recent years. Some evidence shows that correlations

tend to increase during unstable periods (Forbes and Rigobon, 2002). Longin and Solnik (1995) find that correlations between the major stock markets rise in periods of high volatility. Karolyi and Stulz (1996) report that covariances are high while returns on the national indices are high and when "markets move a lot." All these papers are based mainly on daily data. Very few attempts have been devoted to analyzing dynamic conditional correlations in high-frequency data. Moreover, we are interested in exploring the results from varying different scales of data in the context.

[11] An alternative definition of the correlation coefficient (more precisely, cross-correlation coefficient, see Laloux *et al.*, 1999; Plerou *et al.*, 1999) denoted by $C_{\tau,ij,t}$ is defined as the statistical overlap of the fluctuations $\delta R_{\tau,i,t} = R_{\tau,i,t} - E\left(R_{\tau,i,t}\right)$ between the two stocks i and j, that is, $C_{\tau,ij,t} = \frac{E\left(\delta R_{\tau,i,t}\delta R_{\tau,j,t}\right)}{\sigma_{\tau,i,t}\sigma_{\tau,j,t}}$, where $R_{\tau,t}$ is the logarithmic return, and $\sigma^2_{\tau,i,t} = E\left(\left[\delta R_{\tau,i,t}\right]^2\right)$. The average $E(\cdot)$ is over a time period T. We are interested in exploring whether different scales of data types would cause different results in dynamic cross correlations. Based on this equation, we can perform two analyses: one with T fixed to one day, and the other with T fixed to a certain number of events. Using two ways (with and without deleting opening intervals) to investigate the DCC, we found that there is no difference between the two, and we do not report it here. Additional methods for measuring correlation can be found in Tsay (2002); Tse and Tsui (2002).

[12] In our case, we fixed $T = 38$ after removing the 09:30 data point. The discontinuation of the correlation coefficients in the figures is due to missing data for the sample period from February 9, 2002, to May 9, 2002.

[13] In their analysis of intraday foreign exchange rate and S&P 500 futures, Andersen and Bollerslev also find that the intraday returns display an MA(1)-GARCH pattern.