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On the Nonlinear Specifications of Short-Term Interest Rate Behavior: Evidence from Euro-Currency Markets

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Abstract. This paper presents a coherent nonlinear interest rate model that incorporates the dynamics of the error correction specification into the traditional term structure model. The joint tests based on six Euro-Currency rates indicate that the linear specification should be rejected. The estimated equation suggests that the linear components—the change of the long-term interest rate and the error correcting term are highly significant. The nonlinear components involving the higher order of the independent variables, the cross products, the lagged error squares, and/or the ARCH effect also present significant explanatory power for predicting short-term Euro-Currency rate changes, confirming the non-linear specifications.

Key words: Euro currency, nonlinear models, error correction model, term structure of interest rates

JEL Classification: E43 and F3

1. Introduction

Recent empirical studies on interest rate time series have arrived at several important findings. First, the statistical tests (Dickey-Fuller; Augmented Dickey-Fuller; and Phillips and Perron) fail to reject unit roots in the levels of interest rates, hence there is evidence, albeit inconclusive, that the levels of the interest rates are nonstationary (Choi and Wohar (1991), Mougoue (1992), Chiang and Chiang (1995)). Second, cointegration tests provide sound evidence attesting to the fact that the interest rates tend to follow a common stochastic trend (Arshanaplli and Doukas (1994), Engsted and Tanggaard (1994)), pointing out that short-term interest rates have a tendency to move together closely. Third, some evidence shows that the unbiased expectation hypothesis of the term structure of interest rates performs better in the recent data, Mishkin (1988)), indicating that the changes in short-term interest rates can be predicted by the slope of the yield curve.

The empirical evidence gathered to date carries two important implications. First, since the coefficient of the long-run equilibrium equation in the term structure of interest rate model, is close to unity, the error correction term will contain information very similar to that offered by the slope of the yield curve. Second, the error correction model (ECM) appears to be a more appropriate representation for describing the interest rate behavior since it contains both the short-run dynamics and the long-run equilibrium characteristics.

The research of Kugler (1990), Choi and Wohar (1991), Bradley and Lumpkin (1992) collectively present concrete evidence to support this model specification.

Even though the ECM furnishes a useful empirical framework and performs impressively, the functional relationship itself has essentially been specified in a linear form.[1] There are good reasons to believe that the short-term interest rate behavior should be modeled in a nonlinear fashion. First, the reactions of the short-rate changes to a long-short spread (specified in either expectations models or via an error correction representation) are likely to be in a nonlinear form. Second, the evidence obtained from both transfer function and (G)ARCH results indicates that the functional relationship and error structure of interest rate models favor nonlinear specifications (Chiang and Chiang (1995)).

In view of these considerations, this paper presents a coherent nonlinear interest rate model that incorporates the dynamics of the error correction specification into the traditional interest rate model. The organization of the paper is as follows: Section 2 provides a general discussion of the features of the error correction model; Section 3 covers the various forms of nonlinear models; Section 4 describes the statistics testing for nonlinearity while reporting the testing results; Section 5 gives the nonlinear estimates for the term structure models; and Section 6 contains the summary and conclusions.

2. Error correction representation and cointegration

To illustrate the key tenet of the error correction model (Engle and Granger (1987)), let us consider two time series: short-term interest rate, r_t, and long-term interest rate, R_t. Each series is first-difference stationary, that is, integration of order one, I(1). It is generally true that any linear combination of these two series is also I(1).

However, if there is a linear combination of these two series such that $u_t = r_t - b_0 - b_1 R_t$ is level stationary, i.e., u_t is I(0), then r_t and R_t are said to be cointegrated of order $(1, 1)$, where b_1 is a cointegrating parameter. The cointegrating, equilibrium, equation is given by:

$$r_t = b_0 + b_1 R_t + u_t, \tag{1}$$

where u_t is a random disturbance term. Equation (1) specifies the long-run relationship between r_t and R_t. This specification is consistent with the view that the short-term interest rate has been observed to fluctuate around the ''normal'' level of a given interest rate. This normal rate is usually referred to as the long-term interest rate. As stated by the expectations theory of the maturity structure of interest rates, the long-term interest rate is viewed as a weighted average of current and expected short-term interest rates. Apparently, the error correction term u_t reflects market information in a state of disequilibrium that is bound to be corrected so as to move toward the long-run level. Expressing this notion in an error correction model (ECM), we write:

$$\Delta r_t = G(L)\Delta R_t + P(L)\Delta r_t - \psi u_{t-1} + \varepsilon_t, \tag{2}$$

where Δ is the first-difference operator and $G(L)$ and $P(L)$ are finite order lag polynomials. In regression form, the model can be written as:

$$\Delta r_t = C + \gamma_0 \Delta R_t + \sum_{j=1}^{m} \gamma_j \Delta R_{t-j} + \sum_{j=1}^{n} \pi_j \Delta r_{t-j} - \psi u_{t-1} + \varepsilon_t, \tag{3}$$

where C, γ, π, and ψ are nonzero constant parameters; ε_t is an error term. Equation (3) offers a robust framework for modeling interest rate behavior. In its approach to model time series, ECM integrates the differences and levels of interest rates into a unified structure; thus, ECM combines the short-run dynamics (changes) with a desirable long-run relationship (level). Notice that the sign of the error correction term is negative, meaning that when the short rate is higher (lower) than the long rate, the short rate is expected to fall (rise) (this adjustment is determined by the parameter ψ). The estimated parameters in equation (1) have an important implication: if, in particular, the null $[b_0 b_1]' = [01]'$ in equation (1) cannot be rejected, the lagged error term u_{t-1} is equal to the lagged spread $(r_{t-1} - R_{t-1})$, which is the slope of the yield curve. Although the model form is consistent with the term structure equation model, the underlying notion is much closer to the regressive expectations model proposed by the Modigliani and Sutch (1966) than to the rational expectations model by Mankiw and Miron (1986).

3. Nonlinear specifications

In deviating from a linear model, there are an infinite number of ways to specify nonlinear models.[2] In this paper, we focus only on the models that are pertinent to empirical estimation. To illustrate, let us consider a general function such that the short rate is related to a set of exogenous variables denoted by $\mathbf{z}_t = \{z_{1t}, z_{2t}, \ldots, z_{kt}\}$. A general expression of this function can be described by a function f and a vector of parameters β, and written as:

$$\Delta r_t = f(\mathbf{z}_t; \beta). \tag{4}$$

The relationship between output and input variables can be expressed as several nonlinear functions as follows.

3.1. Power series model

Assuming that, in the error correction model, the Δr_t is related to a power series of u_{t-1}:

$$\Delta r_t = \beta_0 + \beta_1 u_{t-1} + \beta_2 u_{t-1}^2 + \cdots + \beta_m u_{t-1}^m + \varepsilon_t, \tag{5}$$

where $(u_{t-1}, u_{t-1}^2, \ldots, u_{t-1}^m)$ are error correcting terms in higher powers obtained from equation (1) above, ε_t is assumed to satisfy all of the assumptions of OLS estimations. For empirical estimation, if the number of observations is large enough, the OLS estimators of the regression coefficients will contain desirable properties. However, in practice, the right-hand-side variables $(u_{t-1}, u_{t-1}^2, \ldots, u_{t-1}^m)$ in equation (5) are often highly correlated, so that the variances of the estimated coefficients are likely to be very large, leading to the insignificance of the estimated coefficients.[3,4] Moreover, the highly nonlinear expression in (5) is usually not prepared to explain any particular market behavior. A more common view of a nonlinear model is given by:

$$\Delta r_t = \beta_0 + \beta_1 u_{t-1} + \beta_2 u_{t-1}^2 + \varepsilon_t. \tag{6}$$

Equation (6) is a typical nonlinear model with respect to the variable u_{t-1}. This model reflects the fact that, in addition to a linear relationship, u_{t-1}^2 is a nonlinear term.

3.2. Bilinear models

Let us consider alternative nonlinear models, usually called Bilinear Models. The bilinear specification takes the form:

$$\begin{aligned} \Delta r_t - \pi_1 \Delta r_{t-1} - \pi_2 \Delta r_{t-2} - \cdots - \pi_p \Delta r_{t-p} \\ = \varepsilon_t - \theta_1 \varepsilon_{t-1} - \cdots - \theta_q \varepsilon_{t-q} \\ + \sum_{i=1}^{m} \sum_{j=1}^{k} \eta_{ij} \Delta r_{t-i} \varepsilon_{t-j}, \end{aligned} \tag{7}$$

where ε_t is a white noise term; $(\pi_1, \pi_2, \ldots, \pi_p)$, $(\theta_1, \theta_2, \ldots, \theta_q)$ and η_{ij} are constant parameters; and $\varepsilon_{t-1} \ldots \varepsilon_{t-q}$ are the moving average terms. The last term on the right-hand-side is a nonlinear component. The nonlinearity involves testing the hypothesis of $\eta_{ij} = 0$ for equation (7). Rejection of $\eta_{ij} = 0$ implies a confirmation of the nonlinear specification. However, if the null cannot be rejected, the model reduces to a univariate ARIMA process.[5]

To estimate a nonlinear model like equation (7), it is necessary to determine the orders of the model, including the values of p, q, m, and k. After these values are decided, then the parameters α, π, θ, and η_{ij} can be estimated.[6]

In empirical estimation, Granger and Newbold (1976) and Granger and Andersen (1978) simplify the bilinear model that is still sufficiently accurate to capture the spirit of the original model. For instance, this simplification occurs by writing $p = 1$, $q = 0$, and $i = j$. The diagonal model thus becomes:

$$\Delta r_t = \alpha + \pi_1 \Delta r_{t-1} + \eta \Delta r_{t-1} \varepsilon_{t-1} + \varepsilon_t. \tag{8}$$

This model has been used widely by Tsay (1986) and Tong (1990). A special feature of this model is that the interaction term, $\Delta r_{t-1} \varepsilon_{t-1}$, reflects the information of nonlinearity.

3.3. Nonlinear time series models

Priestly (1991) demonstrates that the input, ε_t, and output, Δr_t, relationship can be represented by Volterra series expansions as:

$$\Delta r_t = \mu + \sum_{i=0}^{\infty} \varphi_i \varepsilon_{t-i} + \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \varphi_{ij} \varepsilon_{t-i}\varepsilon_{t-j} + \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty} \varphi_{ijk} \varepsilon_{t-i}\varepsilon_{t-j}\varepsilon_{t-k} + \cdots, \tag{9}$$

where the parameters φ_i, φ_{ij}, and φ_{ijk} are constant parameters (Priestley, 1991). The values of i, j, and k are non-negative (0 to ∞) so that Δr_t depends only on current and past values and not on future ones. Obviously, Δr_t is nonlinear if any of the higher order coefficients $\{\varphi_{ij}\}, \{\varphi_{ijk}\}, \ldots$ are nonzero. If not, a linear filter representation (also known as the Wold representation) for Δr_t is obtained.

Focusing on finite dimensional realization, one popular form of nonlinear models is:

$$\Delta r_t = \alpha + \varphi_1 \varepsilon_{t-1} + \varphi_2 \varepsilon_{t-1}\varepsilon_{t-2} + \varphi_{11}\varepsilon_{t-1}^2 + \varepsilon_t. \tag{10}$$

Equation (10) is a nonlinear MA(1) representation, the squared term captures a similar spirit to that of an ARCH(1) model, while nonlinearity in the product term $\varepsilon_{t-1}\varepsilon_{t-2}$ reflects the impact of the interaction between ε_{t-1} and ε_{t-2}. The nonlinearity involving higher orders and various cross-products can also be derived.[7]

4. Test for nonlinearity

The changes of short rates are assumed to be associated with a vector of independent variables $\{\mathbf{z_t}\}$, which are partitioned as $(\mathbf{z_{1t}}\mathbf{z_{2t}})$. The $\mathbf{z_{1t}}$ is in the dimension of $(1 \times p)$ representing p linear terms, and $\mathbf{z_{2t}}$ is in $(1 \times k)$ representing k nonlinear terms. The corresponding parameters are $(\boldsymbol{\varphi_1}\boldsymbol{\varphi_2})'$. Thus,

$$\Delta r_t = \mathbf{z_{1t}}\varphi_1' + \mathbf{z_{2t}}\varphi_2' + \varepsilon_t. \tag{11}$$

The nonlinear term denoted by $\mathbf{z_{2t}}$ will be the power series, cross product terms, or Box-Cox transformation. Specifically, the linear components, $\mathbf{z_{1t}}$, and nonlinear components, $\mathbf{z_{2t}}$, are respectively, given by:

$$\mathbf{z_{1t}} = \{\Delta R_t, \Delta R_{t-1}, \Delta r_{t-1}, u_{t-1}\}, \tag{12}$$

$$\mathbf{z_{2t}} = \{u_{t-1}^2, \Delta r_{t-1}\varepsilon_{t-1}, \varepsilon_{t-1}\varepsilon_{t-2}, \text{ and } \varepsilon_{t-1}^2\}. \tag{13}$$

Since the variables in the estimated models are likely to be highly correlated, the parsimonious principle proposes that only a small set of variables should be included in the test equations. The linear components in equation (12) are derived from the arguments found in equation (3) while the nonlinear components are based on the nonlinear model considered in Section 3. Specifically, u_{t-1}^2 is derived from the power series given by (6); the $\Delta r_{t-1}\varepsilon_{t-1}$ term is taken from the bilinear model in equation (7) or (8); the $\varepsilon_{t-1}\varepsilon_{t-2}$ term is based on the nonlinear time series model in (9); and the ε_{t-1}^2 term is obtained from (10) or based on the ARCH specification.

Given such a construction, testing the nonlinearity of the model is equivalent to testing the null, H_0: $\varphi_2' = \mathbf{0}$ (in k-dimensions). The rejection of the null is consistent with some forms of the nonlinear specification.

4.1. Test statistics for nonlinearity

Formal tests for nonlinearity are discussed by Tsay (1986), Luukkonen et al. (1987), Harvey (1990), Petruccelli (1990). A good summary is provided by Tong (1990). The tests employed in this study are the Lagrange Multiplier (LM) test (Tong (1990) and Charemza and Deadman (1992), the likelihood ratio (F-test) test (Harvey (1990), and the portmanteau (Q^2 test) statistics (McLeod and Li (1983). The procedures involving the nonlinear tests are described in the following sections.

A. Lagrange Multiplier test: LM

The basic principle of the Lagrange Multiplier statistic is to obtain the R^2 by taking the residuals from the regression on the restricted model $\{\mathbf{1}, \mathbf{z_{1t}}\}$. and regressing them on the complete set of independent variables $\{\mathbf{1}, \mathbf{z_{1t}}, \mathbf{z_{2t}}\}$. Specifically,

(1) Regress Δr_t on $\{\mathbf{1}, \mathbf{z_{1t}}\}$, where $\mathbf{z_{1t}} = \{z_{11t}, z_{12t}, \ldots, z_{1pt}\}$, then derive the residuals $\{\hat{\varepsilon}_t\}$ from the regression;
(2) Regress $\{\hat{\varepsilon}_t\}$ on $\{\mathbf{1}, \mathbf{z_{1t}}, \mathbf{z_{2t}}\}$ and obtain the coefficient of determination, R^2;
(3) Form the Lagrange Multiplier statistic by the number of observations T times the R^2.

That is:

$$LM = TR^2. \tag{14}$$

The LM has a χ_k^2 distribution under the null (Harvey, 1990, pp. 170–175).

B. Likelihood ratio test: F-test

This statistic is equivalent to LM test if the sample is asymptotically large. It aims to test the joint significance of nonlinear terms. Thus, the residual sum of the squares from the linear model RSS_1 is compared with results from the model that includes the nonlinear terms RSS_0. The steps are:

(1) Regress Δr_t on $\{1, \mathbf{z_{1t}}\}$, where $\mathbf{z_{1t}} = \{z_{11t}, z_{12t}, \ldots, z_{1pt}\}$, form the residuals $\{\hat{\varepsilon}_t\}$ and the residual sum of squares RSS_1 (the restricted residual sum of the squares);
(2) Regress Δr_t on $\{1, \mathbf{z_{1t}}, \mathbf{z_{2t}}\}$, where $\mathbf{z_{2t}} = \{z_{21t}, z_{22t}, \ldots, z_{2kt}\}$ (these elements may involve certain powers and cross-products, such as, $z_{21t} = z_{11t}^2$), obtain the residual sum of squares RSS_0 (the unrestricted RSS);
(3) Form the F-statistic as:

$$F = [(RSS_1 - RSS_0)/k]/[RSS_0/(T - p - k)]. \tag{15}$$

Under the null hypothesis that the nonlinear restrictions imposed are true, the statistic has an F distribution with (k, T-p-k) degrees of freedom.

C. Portmanteau test: Q^2-statistic for the squared residuals

This test is designed to examine the autocorrelations of the squares of the residuals from a time series model or a regression model. The procedures are:

(1) Regress Δr_t on $\{1, \mathbf{z_{1t}}\}$, where $\mathbf{z_{1t}} = \{z_{11t}, z_{12t}, \cdots, z_{1pt}\}$, form the residuals $\{\hat{\varepsilon}_1\}$ and the squares of the residuals;
(2) Let $\hat{r}_k$ denote the sample autocorrelation of the squared residuals, calculate as:

$$\hat{r}_k = \sum_{j=1}^{T-k} (\hat{\varepsilon}_j^2 - \hat{\sigma}^2)(\hat{\varepsilon}_{j+k}^2 - \hat{\sigma}^2) \Big/ \sum_{j=1}^{T-k} (\hat{\varepsilon}_j^2 - \hat{\sigma}^2)^2. \tag{16}$$

where $\hat{\sigma}^2 = \sum_{j=1}^{T} \hat{\varepsilon}_j^2 / T$; and

(3) Construct the portmanteau statistic:

$$Q^2 = T(T + 2) \sum_{k=1}^{m} \hat{r}_k^2 / (T - k). \tag{17}$$

Since the test involves the use of squared residuals, the statistic may detect nonlinearity. The asymptotic null distribution is χ_m^2 if the true squared innovations are independent. Alternatively, Lawrence and Lewis (1985) have suggested examining the cross-covariances between $\hat{\varepsilon}_t$ and $\hat{\varepsilon}_t^2$. These cross-covariances should be zero if the model is linear.

It is generally recognized that the Q^2 statistic is more relevant for testing linearity against ARCH-type models, while other types of non-linear specification may be detected by the LM test or the likelihood ratio test.

Table 1. The statistical results for testing linearity

Currency	LM	F	Q_4^2
Panel A. 1-month rate			
US	4.94	1.21	5.23
CD	14.99**	3.89**	6.69
BP	3.83	0.91	0.06
GM	6.45	1.58	10.65**
JY	19.50**	5.08**	49.07**
SF	7.91**	1.99**	2.01
Panel B. 3-month rate			
US	26.95**	7.55**	4.05
CD	8.69*	2.16*	7.21
BP	2.79	0.67	33.03**
GM	19.98**	5.33**	5.43
JY	19.96**	5.34**	12.14**
SF	21.14**	5.70**	1.96
Panel C. 6-month rate			
US	16.44**	4.30**	39.57**
CD	6.70	1.14	19.57**
BP	6.51	1.61	22.58**
GM	7.10	1.75	11.89**
JY	9.70**	2.39*	23.46**
SF	11.54**	2.91**	0.40

[a] **Indicates statistically significant difference from zero at the 5% level or better; *indicates significant at the 10% level.
[b] The critical values of χ_4^2 for 1%, 5%, and 10% levels are 13.3, 9.49, 7.78, respectively; the critical values of $F_{(4,167)}$ for 1%, 5%, and 10% levels are 3.48, 2.45, 1.99, respectively.

4.2. Results for testing nonlinearity

Results for testing linearity are reported in Table 1.[8] The Euro-Currency rate data are examined in terms of changes for 1-month, 3-month, and 6-month rates, which are, respectively, shown in Panels A through C. Clearly, the *LM* test produces similar qualitative results as those of the *F* test. The evidence indicates that, with the exceptions of the 1-month rate for the US and BP, the null hypothesis of ‘‘without’’ nonlinearity is rejected by the data. The rejections occur in at least one situation. The test results thus indicate that the nonlinear model should be considered when specifying the test equations.

5. Nonlinear estimations of the term structure models

5.1. A nonlinear form in the error correction term

To start with, we simply add the squared error correction term u_{t-1}^2 to the basic equation. This formula is based on the rationale that the adjustment process is nonlinear. So the estimated equation takes the following form:

Table 2. Empirical results based on the nonlinear specification of the error correction term: White-Newey-West procedure

Currency	Constant	ΔR_t	ΔR_{t-1}	Δr_{t-1}	u_{t-1}	u_{t-1}^2	SEE	R^2
Panel A. ($r_t =$ 1-month rate & R_t = 3-month rate)								
US	0.0009	0.808**	0.057	0.117	− 0.763**	− 3.028**	0.0048	0.74
	(0.20)	(10.39)	(0.55)	(0.82)	(7.71)	(0.54)		
CD	0.0006*	0.707**	0.001	0.030	− 0.499**	− 12.40**	0.0047	0.63
	(1.72)	(10.34)	(0.01)	(0.39)	(5.43)	(3.10)		
BP	− 0.0003	0.698**	− 0.077	− 0.040	− 0.857**	− 0.010	0.0060	0.69
	(0.49)	(9.05)	(0.84)	(0.58)	(11.65)	(0.01)		
GM	0.0002	0.903**	0.234**	− 0.211**	− 0.440**	− 18.29*	0.0024	0.82
	(0.85)	(13.01)	(2.34)	(3.20)	(5.04)	(1.74)		
JY	− 0.0003	0.889**	0.234**	− 0.197**	− 0.529**	12.98**	0.0044	0.74
	(0.65)	(9.64)	(2.42)	(2.41)	(4.04)	(2.27)		
SF	0.0002	0.994**	0.135**	− 0.045	− 0.702**	− 10.72**	0.0045	0.73
	(0.54)	(10.57)	(2.03)	(0.82)	(9.80)	(4.90)		
Panel B. ($r_t =$ 3-month rate & R_t = 6-month rate)								
US	0.0004	0.973**	0.169**	− 0.093	− 0.264**	− 30.67**	0.0029	0.90
	(0.73)	(43.49)	(3.04)	(1.42)	(3.48)	(8.84)		
CD	− 0.00002	0.954**	0.077	− 0.016	− 0.424**	2.361	0.0027	0.86
	(0.11)	(25.41)	(1.28)	(0.24)	(5.86)	(0.22)		
BP	0.0002	0.666**	0.050	− 0.084	− 0.256**	− 5.921*	0.0054	0.62
	(0.49)	(4.44)	(0.63)	(0.79)	(3.82)	(1.68)		
GM	− 0.0001	1.002**	− 0.006	− 0.034	− 0.308**	16.338	0.0015	0.92
	(0.36)	(14.26)	(0.08)	(0.48)	(3.84)	(0.69)		
JY	0.00012	1.059**	0.031	− 0.039	− 0.541**	− 7.991	0.0029	0.84
	(0.17)	(14.80)	(0.31)	(0.58)	(4.50)	(0.59)		
SF	0.0002	0.874**	0.056	− 0.001	− 0.354**	− 11.869**	0.0028	0.81
	(0.72)	(8.92)	(0.90)	(0.01)	(4.55)	(5.01)		
Panel C. ($r_t =$ 6-month rate & R_t = 12-month rate)								
US	0.0003	1.069**	0.305**	− 0.297**	− 0.189**	− 30.704**	0.0023	0.92
	(1.12)	(24.71)	(2.92)	(2.86)	(4.39)	(1.99)		
CD	− 0.00012	0.968**	0.096	− 0.095	− 0.309**	6.729	0.0029	0.83
	(0.50)	(27.16)	(1.00)	(0.98)	(6.49)	(1.60)		
BP	− 0.0002	1.048**	0.077	− 0.171**	− 0.475**	1.707	0.0067	0.63
	(0.19)	(14.19)	(0.66)	(2.09)	(3.30)	(0.35)		
GM	0.0001	0.964**	0.084	− 0.078	− 0.468**	− 26.829	0.0015	0.89
	(0.69)	(20.89)	(0.63)	(0.65)	(7.13)	(1.09)		
JY	− 0.0001	1.085**	0.104	− 0.143	− 0.359**	16.564**	0.0023	0.86
	(0.52)	(21.29)	(0.88)	(1.62)	(5.70)	(2.71)		
SF	− 0.0001	1.121**	0.076	− 0.074	− 0.524**	− 15.030**	0.0028	0.81
	(0.62)	(24.93)	(0.71)	(1.43)	(7.76)	(4.53)		

[a] The estimated equation is:

$$\Delta r_t = \alpha + \gamma_0 \Delta R_t + \sum_{j=1}^{m} \gamma_j \Delta R_{t-j} + \sum_{j=1}^{n} \pi_j \Delta r_{t-j} - \psi u_{t-1} + \varphi u_{t-1}^2 + \varepsilon_t,$$

where $m = n = 1, u_{t-1} = r_{t-1} - b_0 - b_1 R_{t-1}$.

[b] The absolute values of the *t*-statistics are presented in parentheses.

[c] **Indicates statistically significant difference from zero at the 5% level or better; *indicates significant at the 10% level. The critical level at 1% = 2.62, at 5% = 1.98, and at 10% = 1.65.

$$\Delta r_t = \alpha + \gamma_0 \Delta R_t + \gamma_1 \Delta R_{t-1} + \pi \Delta r_{t-1} - \psi u_{t-1} + \varphi u_{t-1}^2 + \varepsilon_t. \tag{18}$$

Since evidence suggests that the series $\{\varepsilon_t\}$ is not independent through higher moments, equation (18) will be estimated by using White-Newey-West consistent estimators and GARCH(1,1) procedures. These results are presented in Tables 2 and 3, respectively.

The estimated results indicate that all of the estimated coefficients for the current change in long rate and the one-period lagged error correcting term have the anticipated signs and are highly significant. With respect to the squared error correcting term, at least 50% of the coefficients are significant, especially for the cases of JY and SF. When the equations were re-estimated by assuming that the error structure follows the GARCH(1,1) process, no substantial change is found. As may be seen in Table 3, the only change is the statistical significance of the coefficients for the squared error correction term. The estimated results show that the nonlinearity appears to be dominated by the GARCH effect.

5.2. A general nonlinear estimation

The next step of our modeling strategy is to look for a larger information set to account for a broader nonlinear specification. Equations (19) and (20) reflect this idea:

$$\Delta r_t = \alpha + \gamma_0 \Delta R_t + \gamma_1 \Delta R_{t-1} + \pi \Delta r_{t-1} - \psi u_{t-1} + \varphi_1 u_{t-1}^2 + \varphi_2 \Delta r_{t-1} \varepsilon_{t-1} + \varphi_3 \varepsilon_{t-1} \varepsilon_{t-2} + \varepsilon_t, \tag{19}$$

$$h_t^2 = \alpha_0 + \varphi_4 \varepsilon_{t-1}^2 + \varphi_5 h_{t-1}^2. \tag{20}$$

Equation (19) is obtained by substituting (12) and (13) into (11) and reparameterizing. The first four terms of equation (19) are the linear components; the last three terms in (19) and the past innovation squares and the lagged variance in (20) are the nonlinear components. The estimated results for equations (19) and (20) are presented in Table 4. The estimated coefficients for the linear terms provide results similar to those reported by Chiang and Chiang (1995). The differences that are found are due to results associated with the nonlinear components. The coefficients for the nonlinear terms, including the squares of the error correcting term and the cross products, show some statistical significance. However, the inclusion of these nonlinear components render some of the coefficients on h_{t-1}^2 insignificant. Nevertheless, the coefficients on the ARCH component are highly significant[9]. A general conclusion derived from Table 4 is that nonlinear specification appears to be relevant, although the significance of the nonlinear terms varies across currencies and maturities.

5.3. Structural changes

Although the above estimated results are in favor of non-linear specification, it is important to know whether these functions persist over time. A number of techniques

Table 3. Empirical results based on the nonlinear specification of the error correction term: GARCH(1,1) procedure

Currency	Constant	ΔR_t	ΔR_{t-1}	Δr_{t-1}	u_{t-1}	u_{t-1}^2	α_0	ε_{t-1}^2	h_{t-1}^2
Panel A. (r_t = 1-month rate & R_t = 3-month rate)									
US	0.0004	0.741**	0.205**	− 0.017	− 0.575**	− 0.729	0.057**	0.569**	0.397**
	(1.17)	(25.22)	(2.08)	(0.16)	(4.74)	(0.08)	(4.74)	(4.74)	(4.74)
CD	0.0006	0.662**	0.023	0.028	− 0.495**	− 10.759**	0.020**	0.08**	0.830**
	(0.32)	(18.37)	(0.27)	(0.32)	(6.95)	(2.80)	(2.05)	(2.11)	(13.69)
BP	− 0.00001	0.795**	− 0.004	0.014	− 0.505**	− 6.120	0.066**	0.743**	0.010
	(0.04)	(33.99)	(0.06)	(0.02)	(5.46)	(0.89)	(8.40)	(6.15)	(0.06)
GM	0.00007	0.860**	0.203*	− 0.202**	− 0.429**	− 12.743	0.004	0.044	0.883**
	(0.28)	(32.11)	(1.71)	(2.04)	(4.23)	(0.89)	(1.61)	(1.42)	(11.63)
JY	0.0001	0.906**	0.215**	− 0.202**	− 0.560**	15.812	0.018**	0.440**	0.492**
	(0.58)	(36.96)	(2.63)	(3.49)	(6.44)	(1.29)	(4.39)	(3.82)	(6.35)
SF	0.0004	0.967**	0.056	− 0.039	− 0.657**	− 10.271	0.009**	0.130**	0.813**
	(1.59)	(22.79)	(0.49)	(0.47)	(6.04)	(0.77)	(6.04)	(3.28)	(15.64)
Panel B. (r_t = 3-month rate & R_t = 6-month rate)									
US	0.0004	0.963**	0.169	− 0.093	− 0.264*	− 30.670	0.079**	0.050	0.060
	(0.83)	(28.67)	(1.32)	(0.82)	(1.77)	(1.52)	(2.93)	(0.55)	(0.05)
CD	0.00003	0.890**	0.176**	− 0.175**	− 0.225**	6.415	0.007**	0.655**	0.425**
	(0.20)	(58.46)	(3.28)	(2.92)	(3.60)	(0.34)	(3.24)	(4.79)	(13.69)
BP	− 0.0003	0.809**	0.118**	− 0.187	− 0.323**	− 2.303**	0.055**	0.330**	0.311**
	(1.18)	(31.34)	(4.92)	(0.63)	(4.18)	(2.15)	(8.40)	(6.15)	(1.99)
GM	0.00005	0.988**	− 0.081	− 0.064	− 0.259**	20.505	0.0001**	0.387**	0.657**
	(0.51)	(62.26)	(1.08)	(0.92)	(3.61)	(0.99)	(2.38)	(4.47)	(11.27)
JY	0.0001	0.998**	0.339**	− 0.315**	− 0.129**	7.095	0.003**	0.630**	0.341**
	(0.85)	(57.31)	(5.76)	(7.61)	(2.11)	(0.38)	(4.49)	(7.44)	(10.55)
SF	0.0002	0.911**	0.122	− 0.105	− 0.379**	− 6.454	0.004	0.398**	0.696**
	(1.15)	(54.30)	(0.99)	(0.88)	(3.69)	(0.25)	(1.02)	(4.83)	(13.05)
Panel C. (r_t = 6-month rate & R_t = 12-month rate)									
US	0.0002*	0.978**	0.207*	− 0.199*	− 0.145**	− 10.96	0.004**	0.430**	0.540**
	(1.88)	(52.55)	(1.69)	(1.70)	(2.45)	(1.06)	(5.14)	(4.75)	(8.30)
CD	− 0.0001	0.923**	0.148*	− 0.195**	− 0.300**	− 8.489	0.005**	0.443**	0.540**
	(0.59)	(53.34)	(1.92)	(2.54)	(6.66)	(0.59)	(3.57)	(3.31)	(8.37)
BP	0.0001	1.080**	0.131	− 0.168*	− 0.319**	2.887	0.053**	0.550**	0.294**
	(0.33)	(20.00)	(1.08)	(1.78)	(3.27)	(0.66)	(5.25)	(2.77)	(2.29)
GM	0.0003*	0.985**	− 0.079	0.068	− 0.507**	− 43.264*	0.005**	0.258**	0.545**
	(1.93)	(78.48)	(0.58)	(0.55)	(5.11)	(1.82)	(3.73)	(2.14)	(5.62)
JY	− 0.0001	1.013**	− 0.191*	0.205**	− 0.223**	7.041	0.003**	0.400**	0.518**
	(0.68)	(47.75)	(1.70)	(2.21)	(2.33)	(0.38)	(2.33)	(2.58)	(12.72)
SF	− 0.0001	1.126**	0.080	− 0.087*	− 0.482**	13.837*	0.011*	− 0.009	0.872**
	(0.50)	(31.86)	(1.34)	(1.93)	(4.13)	(1.80)	(1.82)	(1.19)	(12.31)

[a] The estimated equation is:

$$\Delta r_t = \alpha + \gamma_0 \Delta R_t + \sum_{j=1}^{m} \gamma_j \Delta R_{t-j} + \sum_{j=1}^{n} \pi_j \Delta r_{t-j} - \psi u_{t-1} + \varphi u_{t-1}^2 + \varepsilon_t,$$

where $m = n = 1, u_{t-1} = r_{t-1} - b_0 - b_1 R_{t-1}$.

[b] The absolute values of the t-statistics are presented in parentheses.

[c] **Indicates statistically significant difference from zero at the 5% level or better; *indicates significant at the 10% level. The critical level at 1% = 2.62, at 5% = 1.98, and at 10% = 1.65.

d. The values of the coefficient α_0 are multiplied by 10^{-4}.

Table 4. The estimates of the error correction model with nonlinear specification

Currency	C	ΔR_t	ΔR_{t-1}	Δr_{t-1}	u_{t-1}	u_{t-1}^2	$\Delta r_{t-1}\varepsilon_{t-1}$	$\varepsilon_{t-1}\varepsilon_{t-2}$	α_0	ε_{t-1}^2	h_{t-1}^2	$Q^2(4)$
Panel A. ($R = 3$-month rate, $r = 1$-month rate)												
US	0.0003	0.572**	0.180**	−0.052	−0.282**	4.177	2.301	0.074	0.088**	0.465**	0.001	0.22
	(0.87)	(26.42)	(2.58)	(0.74)	(3.60)	(0.75)	(0.63)	(0.01)	(3.36)	(8.80)	(0.01)	
CD	0.0005	0.656**	−0.062	0.067	−0.55**	−14.75**	9.243	19.163	0.028*	0.068	0.836**	1.29
	(1.09)	(16.22)	(0.66)	(0.76)	(7.11)	(3.53)	(1.26)	(1.05)	(1.72)	(1.64)	(10.46)	
BP	−0.0005	0.705**	−0.013	−0.018	−0.39**	17.537**	13.41**	13.53**	0.064**	0.407**	0.001	3.62
	(0.42)	(34.50)	(0.23)	(0.29)	(5.52)	(3.08)	(2.22)	(2.06)	(6.67)	(7.18)	(0.04)	
GM	0.00001	0.846**	0.170	−0.176**	−0.46**	−6.914**	−1.596	−16.914	0.005*	0.040	0.856**	3.89
	(0.02)	(28.76)	(1.32)	(2.30)	(3.44)	(2.07)	(0.12)	(0.28)	(1.68)	(1.34)	(9.80)	
JY	0.0002	0.741**	0.033	−0.098**	−0.40**	−2.718	−8.849	−20.59	0.035**	0.295**	0.28**	2.97
	(0.75)	(19.39)	(0.47)	(2.31)	(5.89)	(0.40)	(1.17)	(1.08)	(3.64)	(5.12)	(2.56)	
SF	0.0005	0.947**	0.138	−0.045	−0.574**	2.066	−23.72**	−15.608	0.013**	0.216**	0.74**	1.23
	(1.62)	(20.56)	(1.18)	(0.46)	(5.42)	(0.11)	(1.96)	(0.54)	(2.06)	(3.67)	(2.56)	
Panel B. ($R = 6$-month rate, $r = 3$-month rate)												
US	0.00002	0.940**	0.163**	−0.077	−0.188**	34.52**	5.671	−14.023	0.090**	0.496	−0.139	0.17
	(0.83)	(65.31)	(2.02)	(0.96)	(3.17)	(4.77)	(0.88)	(0.86)	(4.98)	(1.42)	(0.98)	
CD	0.00001	0.858**	0.006	0.040	−0.313**	−19.76*	4.460	−13.956	0.049**	0.24**	0.097	7.81
	(0.04)	(49.57)	(0.12)	(0.72)	(5.74)	(1.77)	(0.72)	(1.06)	(7.60)	(5.60)	(1.41)	
BP	−0.0004	0.779**	0.161*	−0.22**	−0.240**	1.777	0.588	−5.408	0.030*	0.194*	0.62**	4.84
	(1.27)	(27.77)	(1.75)	(1.97)	(4.04)	(0.59)	(0.96)	(0.66)	(1.67)	(1.67)	(3.06)	
GM	0.00001	0.988**	−0.020	−0.003	−0.254**	33.76	−18.954	−28.870	0.001**	0.23**	0.72**	3.89
	(0.03)	(62.25)	(0.80)	(0.03)	(3.44)	(1.34)	(0.95)	(0.31)	(2.56)	(3.16)	(11.49)	
JY	0.0002	0.847**	0.102	−0.22**	−0.381**	−12.66	−7.903	−37.58**	0.019**	0.22**	0.16*	4.66
	(1.16)	(57.34)	(1.20)	(3.06)	(6.65)	(0.92)	(1.13)	(2.18)	(7.04)	(5.94)	(1.72)	
SF	−0.0004	0.811**	−0.043	0.093	−0.248**	−24.21**	10.553	24.708	0.040**	0.87**	0.003	1.23
	(0.22)	(38.66)	(0.51)	(1.06)	(5.19)	(4.48)	(1.20)	(1.50)	(9.24)	(4.17)	(1.54)	
Panel C. ($R = 12$-month rate, $r = 6$-month rate)												
US	0.0001	0.901**	0.033	−0.018	−0.101**	−9.633*	18.921**	68.708**	0.009**	0.121**	0.451**	6.41
	(0.45)	(80.20)	(0.67)	(0.41)	(3.62)	(1.80)	(7.76)	(4.65)	(4.12)	(6.96)	(4.61)	
CD	0.00001	0.886**	0.197**	−0.196**	−0.237**	7.790	−15.817**	−2.078	0.002**	0.195**	0.737**	1.07
	(0.48)	(63.88)	(2.90)	(2.76)	(5.19)	(0.92)	(1.97)	(0.06)	(3.04)	(4.82)	(17.29)	

Table 4. (*Continued*)

Currency	C	ΔR_t	ΔR_{t-1}	Δr_{t-1}	u_{t-1}	u^2_{t-1}	$\Delta r_{t-1}\varepsilon_{t-1}$	$\varepsilon_{t-1}\varepsilon_{t-2}$	α_0	ε^2_{t-1}	h^2_{t-1}	$Q^2(4)$
BP	0.0004	1.051**	0.008	− 0.013	− 0.272**	0.740	− 3.451	− 9.683	0.064**	0.194**	0.369**	6.25
	(1.18)	(17.64)	(0.06)	(0.13)	(5.52)	(0.25)	(0.89)	(0.97)	(5.81)	(4.03)	(3.58)	
GM	0.00001	0.989**	0.092	− 0.108	− 0.344**	− 42.34**	5.758	97.80**	0.001	0.415**	0.702**	5.28
	(1.36)	(82.58)	(0.88)	(1.02)	(4.09)	(2.14)	(0.32)	(3.77)	(1.13)	(6.74)	(20.25)	
JY	0.0001	0.846**	0.299**	− 0.276**	− 0.205**	7.645	− 2.822	49.44**	0.005**	0.092**	0.549**	3.63
	(0.73)	(53.13)	(5.96)	(6.11)	(6.29)	(1.05)	(0.88)	(3.35)	(5.83)	(7.51)	(18.99)	
SF	− 0.0002*	0.902**	0.162**	− 0.15**	− 0.286**	− 23.32	14.26*	− 1.410	0.031**	0.622**	− 0.056	0.06
	(1.72)	(50.49)	(2.41)	(2.40)	(3.11)	(1.55)	(1.78)	(0.87)	(8.12)	(4.03)	(1.23)	

[a] The estimated equations are:

$$\Delta r_t = \alpha + \gamma_0 \Delta R_t + \gamma_1 \Delta R_{t-1} + \pi \Delta r_{t-1} - \psi u_{t-1} + \varphi_1 u^2_{t-1} + \varphi_2 \Delta r_{t-1}\varepsilon_{t-1} + \varphi_3 \varepsilon_{t-1}\varepsilon_{t-2} + \varepsilon_t,$$

$$h^2_t = \alpha_0 + \varphi_4 \varepsilon^2_{t-1} + \varphi_5 h^2_{t-1}.$$

[b] The absolute values of the *t*-statistics are presented in parentheses.

[c] **Indicates statistically significant difference from zero at the 5% level or better; *indicates significant at the 10% level. The critical level at 1% = 2.62, at 5% = 1.98, and at 10% = 1.65.

d. The values of the coefficient α_0 are multiplied by 10^{-4}.

(Brown et al. (1975); Tsurumi (1980); and Harvey (1990)) have been used to test the stability of the parameter. However, in this paper we shall employ the popular likelihood ratio test as follows:

$$\gamma = [RSS_T - (RSS_1 + RSS_2)/k]/[(RSS_1 + RSS_2)/(T - 2K)], \tag{21}$$

where RSS_1, RSS_2, and RSS_T, are the residual sums of squares from the estimated equation on the first t observations, the remaining T-t observations, and the entire observations, respectively. Under the null hypothesis of no structural change, γ has an F-distribution with $(k, T-2k)$ degrees of freedom.

Using October, 1982 as a break up point to divide the entire sample (a procedure which is also known as date switching from a non-borrowed reserve regime to a borrowed reserve regime), we perform the F-test. The results are presented in Table 5.[10] The evidence shows that, with the exception of the 1-month and 6-month rate equation[5] for the U.S. dollar, the null hypothesis (that the coefficients for two separate periods are stable) cannot be rejected at the 1% level. There are a few incidences for which the null is rejected at the 5% level; however, the majority of the cases indicate that no significant structural change takes place in each subperiod and that the functional relationship appears to persist.

5.4. Evidence from a preferred equation

The choice of a good model is usually based on establishing well-supported significance, both in theory and in empirical findings. Financial economic theory provides a behavioral rationale for the model guidance, while the data justify the facts. Thus, in estimating a final equation, we propose a preferred equation that includes only the relevant variables, while eliminating the unnecessary variables (that is, the ones that are statistically insignificant). This principle is applied to the system equations in (19) and (20), equations which allow inclusion of longer lags in the error correction term. The estimated statistics are reported in Table 6.

The empirical results from Table 6 may be summarized as follows. First, the long-rates are highly significant. The estimated values of the coefficients are very close to unity, indicating that both the changes of the short rate and long rate are highly cointegrated, both in the short- and long-run relationships. Although the lagged long rate and the lagged short-rate are significant in some of the estimated equations, no consistent pattern can be traced. Some of the effect may have been absorbed in the lagged error correction terms.

Second, the error correction terms for various countries have maintained stable and consistent values. The coefficients have negative signs and are statistically significant. In some estimated equations, longer lags are found to be significant. This information, together with the significance of the squared term, indicates that the error correction processes found in most of the countries are nonlinear. This finding suggests that Engle and Granger's specification can only be viewed as a good approximation.

Third, there is a strong GARCH(1,1) effect. This evidence reveals that the volatile changes in the short rates are not independent of previous interest rate volatility. Specifically, the volatile changes in short rates are positively influenced by the most recent

Table 5. F-test of the parameter stability

Currency	US	CD	BP	GM	JY	SF
1-month Rate	2.31	1.13	1.48	0.94	2.01	0.65
	(0.03)	(0.35)	(0.14)	(0.50)	(0.03)	(0.79)
3-month Rate	1.47	1.28	1.65	2.48	1.47	2.13
	(0.15)	(0.24)	(0.09)	(0.01)	(0.15)	(0.02)
6-month Rate	2.97	0.71	0.68	2.11	0.90	0.89
	(0.01)	(0.73)	(0.75)	(0.06)	(0.54)	(0.55)

[a] The values in the parentheses are the significance levels for testing the null hypothesis of constant coefficient.
[b] The critical value for $F(11{,}131)$ at the 1% and 5% levels of significance are approximately 2.50 and 1.90, respectively.

interest rate fluctuations. These results are consistent with much of the conventional market behavior analyzed for financial asset prices.

Fourth, in addition to the linear component of the independent variables, changes of the short-rate are significantly explained by nonlinear terms. Depending on the currencies and maturities, the significance of the nonlinear variables is shown in: a squared, cross product, and/or (G)ARCH terms. Thus, the traditional linear regression specification of the interest rate model (Mankiw (1986), Mankiw and Miron (1986)), one that does not consider nonlinear specifications, is likely to have a specification error. Moreover, the significance of the nonlinear terms of the non-GARCH components indicates that the GARCH-type specification may not sufficiently capture all the relevant information for nonlinearity.[11]

6. Conclusions

This paper presents empirical estimations in order to examine the likely existence of nonlinearity. The joint tests for linearity suggest that a linear specification should be rejected. Since it is possible to have infinite forms of nonlinearity, the selection of appropriate nonlinear variables and, in turn, the model form is confined to the conventional nonlinear models (bilinear, nonlinear time series, and power series). The nonlinearity in these models usually takes the form of the higher order of the independent variables, the cross products, the lagged error squares, or the GARCH effect. In the empirical estimations, the choice of a nonlinear variable is based on the parsimonious principle and levels of statistical significance.

In sum, the empirical evidence in this paper indicates that in addition to the changes of the long rate, ΔR_t, and the error correction term, u_{t-1}, which are always highly significant in this study, at least one of the nonlinear variables is significant. These nonlinear variables are the squares of the lagged error correcting term (u_{t-1}^2), the cross products of the lagged innovations and the change of short rate $(\Delta r_{t-1}\varepsilon_{t-1})$, the cross products of the lagged innovations $(\varepsilon_{t-1}\varepsilon_{t-2})$, and/or the GARCH effect $(\varepsilon_{t-1}^2$, and $h_{t-1}^2)$.

Table 6. The estimates of the nonlinear model with preferred equation

Currency	C	ΔR_t	ΔR_{t-1}	Δr_{t-1}	u_{t-1}	u_{t-i}	u^2_{t-1}	$\Delta r_{t-1}\varepsilon_{t-1}$	$\varepsilon_{t-1}\varepsilon_{t-2}$	α_0	ε^2_{t-1}	h^2_{t-1}	$Q^2(4)$
Panel A. ($R=3$-month rate, $r=1$-month rate)													
US	0.0001	0.806**	0.136**		− 0.268**	− 0.343**	− 8.481**			0.29**	0.107**	0.390**	5.33
	(0.26)	(40.87)	(4.94)		(5.17)	(8.95)	(2.53)			(5.80)	(4.28)	(2.14)	
CD	0.0006	0.678**			− 0.523**		− 13.448**			0.004	0.059	0.784**	1.22
	(1.43)	(18.58)			(8.51)		(4.22)			(0.57)	(0.95)	(2.47)	
BP	− 0.0005**	0.748**			− 0.542**		− 20.221**			0.047**	0.552**	0.005	1.01
	(2.01)	(46.53)			(9.76)		(4.49)			(6.85)	(9.26)	(0.55)	
GM	− 0.00001	0.865**			− 0.665**	− 0.220**				0.004**	0.009**	0.851**	0.90
	(0.33)	(28.58)			(6.98)	(2.19)				(2.54)	(2.48)	(15.62)	
JY	0.0003	0.741**			− 0.517**	− 0.081**		− 12.583**		0.029**	0.409**	0.271**	3.99
	(1.51)	(28.19)			(10.82)	(2.37)		(1.97)		(4.05)	(6.80)	(3.89)	
SF	0.0005*	0.942**			− 0.639**			− 18.907**		0.014**	0.187**	0.745**	1.69
	(1.85)	(22.63)			(7.72)					(2.13)	(4.43)	(10.60)	
Panel B. (R = 6-month rate, $r=3$-month rate)													
US	− 0.0001	0.933**			− 0.267**		− 26.599**			0.060**	0.890**	− 0.050	0.29
	(0.25)	(52.80)			(2.97)		(5.32)			(5.69)	(4.51)	(1.55)	
CD	0.0001	0.852**			− 0.277**		− 29.478*			0.061**	0.205**	0.021	6.16
	(0.55)	(48.04)			(7.03)		(7.56)			(9.32)	(7.35)	(0.32)	
BP	− 0.0003	0.778**	0.142*	− 0.206**	− 0.242**					0.037**	0.210**	0.544**	4.82
	(1.33)	(29.28)	(1.76)	(2.21)	(7.08)					(1.97)	(1.99)	(2.65)	
GM	0.00004	0.988**			− 0.301**					0.001**	0.291**	0.690**	0.51
	(0.40)	(75.59)			(4.54)					(2.36)	(4.52)(12.28)		
JY	0.0003**	0.97**		− 0.05**	− 0.285**					0.003**	0.286**	0.286**	3.02
	(2.59)	(76.56)		(2.87)	(6.06)					(5.36)	(11.03)	(27.21)	
SF	− 0.0005	0.801**			− 0.203**		− 12.603**			0.039**	0.777***	− 0.049	0.84
	(0.36)	(36.61)			(6.04)		(3.21)			(7.91)	(4.81)	(0.68)	
Panel C. ($R=12$-month rate, $r=6$-month rate)													
US	0.0002**	1.004**			− 0.422**	0.229**				0.002**	0.332**	0.670**	0.83
	(3.16)	(63.01)			(4.45)	(2.17)				(2.74)	(5.71)	(13.29)	
CD	0.00004	0.877**	0.24**	− 0.22**	− 0.226**			− 17.687**		0.003**	0.193**	0.699**	0.76
	(0.35)	(68.70)	(3.96)	(3.39)	(2.44)			(2.44)		(3.49)	(5.47)	(17.96)	

Table 6. (*Continued*)

Currency	C	ΔR_t	ΔR_{t-1}	Δr_{t-1}	u_{t-1}	u_{t-i}	u^2_{t-1}	$\Delta r_{t-1}\varepsilon_{t-1}$	$\varepsilon_{t-1}\varepsilon_{t-2}$	α_0	ε^2_{t-1}	h^2_{t-1}	$Q^2(4)$
BP	0.0003**	1.028**			− 0.271**	0.106**				0.023**	0.034**	0.820	4.91
	(2.01)	(46.53)			(9.76)	(4.49)				(6.85)	(9.26)	(0.55)	
GM	− 0.00001**	0.998**			− 0.306**		− 42.306**			0.001**	0.329**	0.722**	3.41
	(3.89)	(111.42)			(4.86)		(2.63)			(2.54)	(7.91)	(36.44)	
JY	0.0001	0.851**	0.311**	− 0.286**	− 0.181**			− 64.62**		0.005**	0.125**	0.499**	2.91
	(1.11)	(52.16)	(6.12)	(6.17)	(5.85)			(4.89)		(6.09)	(7.76)	(19.09)	
SF	− 0.0003**	0.924**	0.136**	− 0.12**	− 0.399**					0.028**	0.885**	0.030	0.07
	(2.79)	(63.06)	(2.36)	(2.44)	(4.68)					(6.37)	(3.79)	(1.45)	

[a] The estimated equations are:

$$\Delta r_t = \alpha + \gamma_0 \Delta R_t + \gamma_1 \Delta R_{t-1} + \pi \Delta r_{t-1} + \beta_1 u_{t-1} + \beta_i u_{t-i} + \varphi_1 u^2_{t-1} + \varphi_2 \Delta r_{t-1}\varepsilon_{t-1} + \varphi_3 \varepsilon_{t-1}\varepsilon_{t-2} + \varepsilon_t,$$

$$h^2_t = \alpha_0 + \varphi_4 \varepsilon^2_{t-1} + \varphi_5 h^2_{t-1}.$$

[b] The absolute values of the t-statistics are presented in the parentheses.

[c] **Indicates statistically significant difference from zero at the 5% level or better; *indicates significant at the 10% level. The critical level at 1% = 2.62, at 5% = 1.98, and at 10% = 1.65.

[d] The values of the coefficient $alpha_0$ are multiplied by 10^{-4}.

[e] All of the lags for u_{t-i} are in second-orders (u_{t-2}). The only exception is the 1-month rate for the US dollar, which is u_{t-4}.

Notes

1. The linear models are popular for two reasons. First, they may not appear restrictive and may serve to describe the functional relationship. Second, some types of linear models are compatible with nonlinear specifications, especially the variables that have been transformed in a log-linear equation.
2. Two types of nonlinear models are dependent on whether they are linear with respect to the parameters. The first type of model is nonlinear with respect to the variables, while linear with respect to the parameters to be estimated. The second is nonlinear with respect both to the variables and to the parameters (Kmenta (1986)).
3. The functional form can be alternatively and more flexibly expressed by the BoxCox transformation. Specifically, we write:

$$\Delta r_t = b_0 + b_1 g^{(\lambda)}(u_{t-1}) + \varepsilon_t, \text{ where } g^{(\lambda)}(u_{t-1}) = \left(u\frac{\lambda}{t-1} - 1\right)/\lambda.$$

If $\lambda = 1$, the linear model results, while if $\lambda = -1$, the equation will be a reciprocal of the variable. In fact, the values of λ in between 1 and -1 will produce different functional forms. For example, if $\lambda = 0$, then the model will become a log or a semilog form. In shaping a model, it is difficult to impose a particular value of λ a priori. Usually, the choice of λ depends on its statistical properties.
4. Nonlinearity in the parameters can also be specified, as it usually arises from a model transformation that leads estimated parameters to be nonlinear. For instance, equation, such as $\Delta r_t = \beta_0 + \beta_1 u_{t-1} + \varepsilon_t$, with the first-order autoregressive process, $\varepsilon_t = \rho\varepsilon_{t-1} + w_t$, can be transformed into a nonlinear model as:

$$\Delta r_t = (1-\rho)\beta_0 + \rho\Delta r_{t-1} + \beta_1 u_{t-1} - \rho\beta_1 u_{t-2} + w_t.$$

This model is a linear function of the variables Δr_{t-1}, u_{t-1}, and u_{t-2}, but it is a nonlinear function of the mean function parameters β_0, β_1, and ρ. Note that this model form is very similar to the one presented by Chiang and Chiang (1995).
5. Sometimes, the error series ε_t in (7) is assumed to be well behaved, so that the moving average (MA) terms can be dropped. When a constant, α, is added and rearranged, the equation becomes:

$$\Delta r_t = \alpha + \sum_{i=1}^{p} \pi_i \Delta r_{t-i} + \sum_{i=1}^{m}\sum_{j=1}^{k} \eta_{ij}\Delta r_{t-i}\varepsilon_{t-j} + \varepsilon_t.$$

6. Depending on the order of i and j in η_{ij}, the model can be specified as the diagonal. If $i > j$, the model is called superdiagonal; if $i < j$, it is called subdiagonal. The estimations of these models can be very tedious and time consuming, especially the determination of the orders in the equation. The selection procedure is usually based on Akaike's AIC criterion which attains its minimum value.
7. Threshold models constitute another form of nonlinear models; these models have been introduced by Tong (1983, 1990) in a sequence of research papers. The basic idea is to start with a linear model for a series such as Δr_t, and then allow the parameters to vary according to the values of a finite order of past Δr_t, or a finite number of past values of an associated series $\{z_t\}$. For example, a p-order threshold autoregressive model (TAR(p)) takes the form of:

$$\Delta r_t = \pi_0^{(i)} + \pi_1^{(i)}\Delta r_{t-1} + \cdots + \pi_p^{(i)}\Delta r_{t-p} + \varepsilon_t^{(i)},$$

where $\varepsilon_t^{(i)}$ are strict white noise processes, $\pi_0^{(i)}$, $\pi_1^{(i)}$, and $\pi_p^{(i)}$ are constant parameters, $i = 1, 2, \ldots, (\Delta r_t, \ldots, \Delta r_{t-p}) \in \Re(i)$, where $\Re(i)$ is a given region of the p-dimensional Euclidean space $\Re(i)^p$. A simplified version of this model can be achieved by setting $i = 1$ and 2. The model is nonlinear since it involves two regimes denoted by the superscripts 1 and 2. The detailed procedures of the model selection and estimations are provided by Tong (1983, 1990).
8. In the estimations, we employ monthly data from January 1977 to December 1992. The Eurocurrency interest rates, including the data for U.S. dollars (US), German marks (GM), British pounds (BP), Canadian dollars (CD), Swiss francs (SF), and Japanese yens (JY), are measured by 1-month, 3-month, 6-month, and 12-month Euro-deposit rates. All the data are measured at the end of the month and are obtained from various issues of *Harris Bank: Foreign Exchange Weekly Review.*

9. Since empirical evidence shows that the short rate and the adjacent long rate are cointegrated (Arshanaplli and Doukas (1994); Chiang and Chiang (1995)), it follows that the error correction term u_{t-1} corresponds essentially to the short-long spread $r_{t-1} - R_{t-1}$ in the term structure model. Thus, we also estimate the system involving equations (19) and (20) by replacing $r_{t-1} - R_{t-1}$ for u_{t-1}. The results are very similar to those that we report in Table 4. These results are available upon request.
10. The sample period for conducting the F-test runs from October 1979 through October 1992. We use October 1979 as a beginning period because the U.S. monetary authority (the Fed) changed its operating procedures at this time. However, we obtained a similar result when the sample period February, 1978 through October 1992 was used in the test.
11. It is of interest to point out that the estimated results in Table 6 are quite consistent with the nonlinearity tests arrived in Section 4.2. Importantly, the lowest values of Q_4^2 in Table 1 (BP 1-month rate, SF 3- and 6-month rates), correspondingly, give rise to the most insignificant coefficients of h_{t-1}^2 in Table 6.

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