

1. Suppose

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

- (a) Is the realization controllable? Observable?
- (b) Try to write  $C(sI - A)^{-1}B$  as  $D_L^{-1}(s)N_L(s)$  and  $N_R(s)D_R^{-1}(s)$  with  $\deg D_L(s) = 6$ , and  $\deg D_R(s) = 3$ .
- (c) Repeat part (b) with  $\deg D_L(s) = 4$  and  $\deg D_R(s) = 2$ .

2. Check in several different ways whether the following pairs of matrices are right coprime.

(a)  $\begin{bmatrix} s & 0 \\ -s & s^2 \end{bmatrix}$  and  $\begin{bmatrix} 0 & -(s+1)^2(s+2) \\ (s+2)^2 & s+2 \end{bmatrix}$ .

(b)  $\begin{bmatrix} s & s \\ -s(s+1)^2 & -s \end{bmatrix}$  and  $\begin{bmatrix} (s+1)^2(s+2)^2 & 0 \\ 0 & (s+2)^2 \end{bmatrix}$ .

(c)  $\begin{bmatrix} 2s+1 & s^2+1 \\ (s+1)^2 & s^2+2s \end{bmatrix}$  and  $\begin{bmatrix} 2s^2+3s+5 & s^2+4s+1 \\ s^2+5s-1 & s^2+s-1 \end{bmatrix}$ .

Try also to find a gcd if they are not coprime.