MEM 633 Robust Control I HW #4

1. Find realizations in controller and observability forms of the transfer function,

$$H(s) = \frac{2s^3 + 13s^2 + 31s + 32}{s^3 + 6s^2 + 11s + 6}$$

   Give both block diagrams and state-space equations.

2. (a) Show, when all inverses exist, that

$$(sI - A)^{-1} - (sI - B)^{-1} = (sI - A)^{-1}(A - B)(sI - B)^{-1}$$

   and

$$(sI - A)^{-1} - (vI - A)^{-1} = (sI - A)^{-1}(v - s)(vI - A)^{-1}$$

   (b) Use the above results to show that for a realization $\{A, b, c\}$ with

   $u(t) = e^{vt} \cdot u_s(t)$, $u_s(t)$ is the unit step function,

   the output can be written as

$$L[y(t)] = c(sI - A)^{-1}[x_0 - (vI - A)^{-1}b] + c(vI - A)^{-1}(s - v)^{-1}b$$

3. (a) If $\{A, b, c, d\}$, $d \neq 0$, is a realization with $H(s) = c(sI - A)^{-1}b + d$, show that

$$\left\{A - \frac{bc}{d},\ \ \frac{b}{d},\ \ \frac{-c}{d},\ \ \frac{1}{d}\right\}$$

   is a realization for a system with transfer function 1/H(s).

   (b) If we are given $\{A, b, c, d\}$, $d \neq 0$, show that the zeros of $c(sI - A)^{-1}b + d$ are the eigenvalues of the matrix $A - \frac{bc}{d}$.