MEM 633 Robust Control I HW #3

1. $A = T\Lambda T^{-1}$ and $\Lambda = diag\left[\lambda_1, \lambda_2, ...\lambda_n\right]$ where $\lambda_1, \lambda_2, ...\lambda_n$ are distinct.

   Show that

   (a) $e^{At} = Te^{\Lambda t}T^{-1}$

   (b) $e^{\Lambda t} = diag\left[e^{\lambda_1 t}, e^{\lambda_2 t}, ...e^{\lambda_n t}\right]$

2. Consider the time-invariant system $\dot{x}(t) = Ax(t)$ where the nxn matrix A has distinct eigenvalues $\lambda_1, \lambda_2, ...\lambda_n$. The corresponding eigenvectors are $e_1, e_2, ...e_n$.

   Let $T = \left[e_1\ e_2\ .....\ e_n\right]$ and $v_1^{'}, v_2^{'}, ...v_n^{'}$ be row vectors of T^{-1}. Show that the solution of $\dot{x}(t) = Ax(t)$ can be written as

$$x(t) = \sum_{i=1}^{n} v_i^{'} \cdot x(0) \cdot e^{\lambda_i t} \cdot e_i$$

3. The A matrix in Problem 2 is given as

$$A = \begin{bmatrix} 0 & 1 \\ 6 & -5 \end{bmatrix}$$

   Write down the solution of $\dot{x}(t) = Ax(t)$ by using the result of Problem 2. You will see the system is unstable. However, $x(t)$ will be bounded if the initial state vector is in the stable subspace. Describe the stable subspace of the system.