MEM633 Robust Control Systems I HW#2

Problem 1: Is the following vectors linearly independent in the field of real numbers?

$$\begin{bmatrix} 2-3j \\ 1+4j \end{bmatrix}, \quad \begin{bmatrix} 8+2j \\ 1+j \end{bmatrix}, \quad \begin{bmatrix} j \\ 5 \end{bmatrix}$$

Is the set linearly independent in the field of complex numbers? Explain.

Problem 2: Consider the example in page 2-11 of the notes. Suppose the representations of x, $\hat{e}^1$, $\hat{e}^2$, e^1, and e^2 with respect to the basis $\{e^1, e^2\}$ are known, use the equation $\hat{a} = Pa$ in page 2-12 to derive the representations of x, $\hat{e}^1$, $\hat{e}^2$, e^1, and e^2 with respect to the basis $\{\hat{e}^1, \hat{e}^2\}$.

Problem 3: Consider the operator that rotates a vector in $\mathbb{R}^2$ counterclockwise by 45 degrees with respect to the origin. Write a matrix representation for this operator.

Problem 4: Is there a solution for the following linear equation? Find the set of all solutions for the equation and explain.

$$\begin{bmatrix} 5 & 3 & 2 \\ 1 & 2 & -1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 3 \\ 1 \end{bmatrix}$$

Problem 5: Consider the matrix

$$A = \begin{bmatrix} 5 & 3 & 2 & 1 \\ 1 & 2 & -1 & 3 \\ 0 & 1 & -1 & 2 \end{bmatrix}$$

which maps $\mathbb{R}^4$ into $\mathbb{R}^3$. Find the null space of A.

Problem 6: Show that if λ_i is an eigenvalue of A, then $f(\lambda_i)$ is an eigenvalue of the matrix function $f(A)$.

Problem 7: Consider the linear space $\mathbb{R}^2$.

(a) Sketch all the points with $\|x\|_1 = 1$.

(b) Sketch all the points with $\|x\|_2 = 1$.

(c) Sketch all the points with $\|x\|_\infty = 1$.