Appendix

**Proof of Lemma 1:** Define \( m_3^1 = c_3^1 - 1 \) (\( m_i^* = c_i^* - 1 \)) to be imports of good 3 (1) by the representative country in the CU (ROW). Totally differentiating the market-clearing condition (3) yields

\[
(\varepsilon + \varepsilon^* - 1) \left[ \frac{d q}{q} \right] = \left[ \frac{\partial m_i^1 / \partial \tau^*}{m_i^1 / (1 + \tau^*)} \right] \frac{d \tau^*}{1 + \tau^*} - \left[ \frac{\partial m_3^1 / \partial \Phi}{m_3^1 / \Phi} \right] \frac{d \Phi}{\Phi}
\]

(A.1)

where \( \varepsilon = \frac{\partial m_i^1 / \partial q}{m_i^1 / q} \), and \( \varepsilon^* = \frac{\partial m_i^1 / \partial q}{m_i^1 / q} \). Totally differentiating the import demand functions in (4) yields

\[
\varepsilon^* = \frac{q \beta + \sigma (1 - \beta) [q (1 + \tau^*)]^\alpha}{q \beta + (1 - \beta) [q (1 + \tau^*)]^\alpha} + \frac{\sigma (1 - \beta) [q (1 + \tau^*)]^\alpha}{\alpha + (1 - \beta) (1 - [q (1 + \tau^*)]^\alpha)}
\]

(A.2a)

\[
\varepsilon = \frac{q (\beta + \alpha) + (\sigma - 1) \beta q^{1 - \alpha \Phi}}{q (\beta + \alpha) - \beta q^{1 - \alpha \Phi}} + \frac{(\sigma - 1) \beta q^{1 - \alpha \Phi}}{(1 - \beta) + \beta q^{1 - \alpha \Phi}}
\]

(A.2b)

\[
\frac{\partial m_i^1 / \partial \tau^*}{m_i^1 / (1 + \tau^*)} = -\frac{\sigma (1 - \beta) [q (1 + \tau^*)]^\alpha}{[q (\beta + \alpha) - \beta q^{1 - \alpha \Phi}] + \frac{1}{\alpha + (1 - \beta) (1 - [q (1 + \tau^*)]^\alpha)}}
\]

(A.2c)

\[
\frac{\partial m_3^1 / \partial \Phi}{m_3^1 / \Phi} = -\beta q^{1 - \alpha \Phi} \left[ \frac{1}{q (\beta + \alpha) - \beta q^{1 - \alpha \Phi}} + \frac{1}{(1 - \beta) + \beta q^{1 - \alpha \Phi}} \right]
\]

(A.2d)

\[
\frac{d \Phi}{\Phi} = \alpha \left( \frac{d \tau}{1 + \tau} - \frac{1}{1 + (1 + t)^\alpha} \frac{d t}{1 + t} \right)
\]

(A.2e)

Market stability requires \( \varepsilon + \varepsilon^* - 1 > 0 \). For \( \alpha > 1 \), we have \( \varepsilon > 1 \) and \( \varepsilon^* > 1 \) for all finite \( \alpha \) and all \( \beta \in (0, 1) \). Substituting (A.2) into (A.1) yields the comparative statics results of Lemma 1. 

**Proof of Proposition 1:** Part (a). Note that the optimal tariff formula for ROW can be written as

\[
\frac{1}{\tau^*} = \varepsilon - 1, \text{ where } \varepsilon \text{ is given by (A.2b)}.
\]

This yields the condition
\[
\frac{1}{\tau^*} = \frac{\sigma \beta \Phi}{q^\alpha (\beta + \alpha) - \beta \Phi} + \frac{(\sigma - 1) \beta \Phi}{(1 - \beta) q^{\alpha - 1} + \beta \Phi}
\]  
(A.3)

The LHS of (A.3) is decreasing in \( \tau^* \) and approaches 0 as \( \tau^* \to \infty \). Since \( q \) is a decreasing function of \( \tau^* \) and the RHS of (A.3) is a decreasing function of \( q \), the RHS is positive and finite for all \( q \) at which trade exists, and is decreasing in \( \tau^* \). Therefore, there will exist a unique (finite) best-response tariff \( \bar{\tau}^*(\Phi) \) for \( \sigma \geq 1 \), which is decreasing in \( \Phi \). This follows from the fact that the RHS of (A.3) is increasing in \( \Phi \), and that \( q^\alpha \) rises less than proportionally to an increase in \( \Phi \) (Lemma 1).

**Part (b):** The fact that there is a unique value at which \( \partial U^*/\partial \tau^* = 0 \) ensures that ROW’s welfare is increasing in \( \tau^* \) for all tariffs less than the best-response tariff rate, which establishes (i). Differentiate (6) with respect to \( q \) to obtain

\[
\frac{\partial V^*/\partial q}{V^*/q} = \beta q \left[ \frac{1}{1 - \beta + \alpha + \beta q} - \frac{1}{(1 - \beta)[q(1 + \tau^*)]^\alpha + \beta q} \right]
\]

\[
+ (1 - \beta) \sigma \left[ \frac{1}{1 - \beta + \beta q^{\alpha - 1} (1 + \tau^*)^{1 - \alpha}} + \frac{1}{(1 - \beta) + \beta q^{1 - \alpha} (1 + \tau^*)^{-\alpha}} \right]
\]

(A.4)

The first term in brackets will be negative if \( \alpha + (1 - \beta)(1 - [q(1 + \tau^*)]^\alpha) > 0 \), which from (4a) will be satisfied if ROW imports goods 1 and 2. Since the second term in (A.4) is negative \( V^*(q, \tau^*) \) must be decreasing in \( q \). Application of Lemma 1 then yields parts (ii) and (iii) of part (b) of the proposition.

**Part (c):** Differentiation of (6) establishes that \( \partial U^*/\partial \tau < 0 \) for \( \tau > 0 \), so a reduction in \( \tau \) with \( q \) constant will raise ROW welfare.  

**Proof of Proposition 2:** **Part (a):** To prove this part of the proposition, we first derive the optimal (external) tariff formula for the representative union member 1, given the internal tariff \( t \). With \( t \) kept fixed, the ratio of consumption of good 1 to 2 in union member is, by equation (1), equal to

A-2
\[
\frac{c_2}{c_1} = (1 + t)^{-\alpha}. \tag{A.5}
\]

Substituting this in the utility function of union member 1 yields

\[
U = \left[ \beta \mu (c_1^{\lambda})(\alpha-1)\lambda + (1-\beta)(c_3^{\lambda})(\alpha-1)\lambda \right]^{\alpha/\lambda - 1}
\]

where

\[
\mu = \frac{1}{2} \left[ 1 + (1 + t)^{-\alpha} \right].
\]

Differentiating the above \( U \) totally and rearranging terms yields a condition that must be true for the policy that maximizes the representative union member’s welfare, i.e.,

\[
dU = 0 \quad \Rightarrow \quad -\left[ \frac{\mu \beta}{1 - \beta} \left( \frac{dc_1}{dc_3} \right) \right] = \left( \frac{c_3}{c_1} \right)^{\frac{\alpha}{\lambda - 1}} = \frac{1 + \tau}{q}. \tag{A.6}
\]

The second equality in the RHS of (A.6) follows from (1) and the fact that \( p_3/p_1 = (1 + \tau)/q \). From the trade balance condition (3), we can write \( c_3 \) as a function of the world price \( q \), i.e., \( c_3 = q(c_1(q) - 1) + 1 \) where \( c_1(q) \) is ROW’s demand function in (4a). Differentiating this condition with respect to \( q \) gives

\[
\frac{dc_3}{dq} = -(c_1 - 1)(\tau - 1). \tag{A.7}
\]

Utilizing the trade balance condition (3) and (A.6) in CU member 1’s budget constraint (2b), we have

\[
\frac{\beta \lambda c_1}{q} = \frac{(1 - \beta)(1 - c_3)}{q} + \alpha + \beta = -(1 - \beta)(c_1(q) - 1) + \alpha + \beta
\]

where

\[
\lambda = \frac{1}{2} \left[ 1 + (1 + t)^{-\alpha} \right].
\]
Differentiating this expression with respect to $q$, utilizing (A.7), and collecting terms yields

$$\frac{dc_i^1}{dq} = \frac{(1-\beta)(c_i^* - 1)}{\beta\lambda q} \varepsilon^*.$$  

(A.8)

Substituting (A.8) and (A.7) into (A.6) gives the following best-response formula for the union:

$$1 + \tau = \frac{\mu}{\lambda} \frac{\varepsilon^*}{\varepsilon^* - 1}.$$  

(A.9)

The existence of an optimal tariff satisfying (A.9) is illustrated in Fig. A.1. The LHS is represented by the ray $OL$. From the definitions of $\mu$ and $\lambda$, the ratio $\mu/\lambda$ is increasing in the internal tariff $\tau$ and is equal to 1 for $\tau = 0$. If ROW’s tariff $\tau^*$ is below the prohibitive level, the RHS of (A.9) will exceed 1 at $\tau = 0$. Moreover, the RHS is decreasing in $\varepsilon^*$, and $\varepsilon^*$ is increasing in $q$ as can be ascertained from (A.2). Since $q$ is increasing in $\tau$ (Lemma 1), the RHS of (A.9) is decreasing in $\tau$, as shown by the locus $R_0$ in Figure A.1. The intersection between loci $OL$ and $R_0$ determines the unique optimal tariff schedule $\tilde{\tau}(\cdot)$ at which (A.9) is satisfied. It follows from (A.2a) and Lemma 1 that $\varepsilon^*$ is an increasing function of $\tau^*$ (where the price adjustment is taken into account). An increase in $\tau^*$ thus leads to a leftward shift in the $R_0$ locus in Fig. A.1, so $\partial \tilde{\tau}(\cdot)/\partial \tau^* < 0$.

Now consider the effect of an increase in the internal tariff $\tau$ on the best-response (external) tariff of the union. At given $\tau$, the increase in $\tau$ will raise $\mu/\lambda$ (= $[1+(1+t)^{1-c}]/[1+(1+t)^{-c}]$) and by Lemma 1(b) will reduce $\varepsilon^*(q(\tau^*,\Phi),\tau^*)$. Both of these effects cause the $R$ schedule to shift upwards from $R_0$ to $R_1$ in Fig. A.1. We thus have $\partial \tilde{\tau}(\cdot)/\partial \tau > 0$.

Part (b): To prove this part, we differentiate (7) with respect to $q$ to obtain
\[
\frac{\partial V}{\partial q} = \frac{(\beta + \alpha)q}{(1-\beta) + (\beta + \alpha)q} - \frac{\beta q^{1-\sigma}\Phi}{(1-\beta) + \beta q^{1-\sigma}\Phi} 
\]

\[
\sigma \beta q^{1-\sigma} \left[ \frac{\Phi}{(1-\beta) + \beta q^{1-\sigma}\Phi} - \frac{\Psi}{(1-\beta) + \beta q^{1-\sigma}\Psi} \right]
\]

The term in the first brackets will be positive if and only if \((\beta + \alpha)q > \beta q^{1-\sigma}\Phi\), which from (4b) will be satisfied if the union imports good 3. The second bracketed term will be positive if an only if \(\Phi > \Psi\).

From the definitions of \(\Phi\) and \(\Psi\) in (5) and (8), respectively, this is tantamount to requiring that
\[
1 + \tau > \frac{[1 + (1 + t)^{1-\sigma}]/[1 + (1 + t)^{-\sigma}]}{(1 + (1 + t)^{1-\sigma})} \text{ (or, equivalently, that } 1 + \tau > \mu/\lambda).\]

A sufficient condition for this inequality to be true is \(0 \leq t \leq \tau\). This establishes the first part of Proposition 2(b). The remaining parts follow from the above and Lemma 1.

Part (c): Differentiation of (7) at fixed \(q\) yields
\[
\frac{\partial U}{\partial \Phi} = \frac{-\sigma(1-\beta)q^{1-\sigma}\Phi}{\beta + (1-\beta)q^{1-\sigma}\Phi}, \quad \frac{\partial U}{\partial \Psi} = \frac{\sigma}{\sigma - 1} \frac{(1-\beta)q^{1-\sigma}\Psi}{\beta + (1-\beta)q^{1-\sigma}\Psi}
\]

From (5) and (8) we have \(d\Phi/\Phi = \sigma(d\tau)/(1+\tau)\) and \(d\Psi/\Psi = (\sigma - 1)(d\tau)/(1+\tau)\) at fixed \(t\). Substituting these results in (A.11), it can be shown that \(U\) will be decreasing in \(\tau\) (with \(\tau^*\) adjusted to maintain \(q\) constant) if \(\Phi > \Psi\). As established in the proof of part (b), \(0 \leq t \leq \tau\) is a sufficient condition for this to hold.

Proof of Lemma 2(b): For the Cobb-Douglas case the demand functions are given by (4) with \(\sigma - 1\). Using (1) and these demand functions in the utility function of union member 1, \(U = (c_1^{1-\beta})(c_2^{1-\beta})(c_3^{1-\beta})\) yields
\[
U = \left[ \frac{(1-\beta) + q(\beta + \alpha)}{(1-\beta) + \beta \Phi} \right]^{1+\tau} q^{\beta/\tau} (1 + t)^{-\beta/\tau}
\]

where \(\Phi = \frac{1+\tau}{2} \frac{2+\tau}{1+\tau}\).
A Kemp-Wan tariff reduction in the external tariff $\tau$ should keep $\Phi$ constant, therefore

$$\frac{d\tau}{1 + \tau} = \frac{1}{2 + t} \frac{dt}{1 + t}. \quad (A.13)$$

If the Kemp-Wan tariff reduction is undertaken, the world price under the agreement will be unaffected. Recall that a hat (^) over variables indicates percentage change (e.g., $\hat{x} = dx/x$). Differentiating (A.12) and making use of (A.13), we obtain the effect of a Kemp-Wan adjustment in external tariffs on the utility of the union under the agreement to be

$$\hat{U}_A = -\frac{\beta}{2} \left[ \frac{t}{2 + t} \frac{dt}{1 + t} \right]. \quad (A.14)$$

Since the RHS of the above expression (excluding $dt$) is negative for $t > 0$, a reduction in the internal tariff $t$ will raise the welfare of the union under the agreement. This effect is independent of the magnitude of $\tau$, thus establishing the result. \[\|\]

**Derivation of Equations (13) and (14):** Under no trade with ROW, the representative union member country 1 will consume its endowment of good 3. From (1), its consumption of goods 1 and 2 will be linked by the condition $c_2^1 = (1 + t)^{-\sigma} c_1^1$. The autarky price of good 1 within the union, denoted $q_N$, will be determined by the requirement that the demand for a representative bloc 1 good, $\frac{\beta}{2} (c_1^1 + c_2^1)$, be equal to the local supply of good 1, $\beta + \alpha$. Using (1) and the fact that $c_1^2 = c_2^1$, the market-clearing condition can be written as shown in (14) in the text. It is then direct to verify that the associated utility level is given by (13).

**Proof of Proposition 4:** Part (a): We show that if $\sigma > 1$ and the union makes a Kemp-Wan adjustment in its external tariff $\tau$, then (i) $\hat{U}_D > \hat{U}_A > 0$ and (ii) $\hat{U}_N > \hat{U}_A > 0$. These inequalities establish that the
LHS of (11b) in the text increases by less than the RHS, so that the Kemp-Wan adjustment in the external tariff is not incentive compatible. Since the LHS of (11b) is increasing in \( \tau \) at the initial point, the value of \( \tau \) that maintains incentive compatibility must exceed the Kemp-Wan tariff. To establish inequality (i), note that \( \hat{U}_A \) is obtained by evaluating (15) at \( \Theta(\tau_A, t) \). To calculate the effect of the deepening of integration on \( U_D \), recall that \( \partial U / \partial \tau = 0 \) when the union chooses to deviate optimally from the tariff agreement.

Since the local effect of the external tariff is zero, there is no loss of generality in assuming that the union makes a Kemp-Wan adjustment in its optimal tariff as a result of the change in \( t \). Therefore, \( \hat{U}_D \) is obtained from (15) evaluated at \( \Theta(\tau_A, t, t) \). Since \( \tilde{\tau}(\tau_A, t) > \tau_A \), Lemma 2 yields \( \hat{U}_D > \hat{U}_A > 0 \).

To show (ii), we differentiate (13) to obtain

\[
\hat{U}_N = \left[ \frac{\left( c_1^i \right)^{\frac{\alpha-1}{\alpha}} \left( 1 + (1+t)^{1-\alpha} \right)}{(1-\beta) + \left( c_1^i \right)^{\frac{\alpha-1}{\alpha}} (1 + (1+t)^{1-\alpha})} \right] \left( \frac{dc_1^i}{c_1^i} - \frac{\sigma}{\left[ 1 + (1+t)^{\alpha-1} \right]} \frac{dt}{1+t} \right).
\]  

(A.15)

From (14),

\[
\frac{dc_1^i}{c_1^i} = \left[ \frac{\sigma}{1 + (1+t)^\alpha} \right] \frac{dt}{1+t}.
\]  

(A.16)

Since \( c_3^i = 1 \) under autarky, (1) implies that \( \left( c_1^i \right)^{(\alpha-1)/\alpha} = (p_N)^{1-\alpha} \) under this state. Using this fact and substituting (A.16) into (A.15), we have

\[
\hat{U}_N = \left[ \frac{(\beta/2)[1 + (1+t)^{1-\alpha}]}{(1-\beta)(p_N)^{\alpha-1} + (\beta/2)[1 + (1+t)^{1-\alpha}]} \right] \frac{d\Psi}{\Psi}.
\]  

(A.17)

The autarky price \( q_N \) for the union will exceed \( q/(1+\tau_A) \), so a comparison of (A.17) with (15) establishes that \( \hat{U}_D > \hat{U}_A > 0 \). This proves part (a) for the case \( \sigma > 1 \). The same argument can be applied to show that for the case of \( \sigma < 1 \) we will have \( 0 < \hat{U}_N < \hat{U}_D < \hat{U}_A \). This means that the Kemp-Wan tariff
reduction raises the LHS of (11b) by more than the RHS, so that the external tariff must be reduced by more than the Kemp-Wan reduction to maintain strict equality in (11b).

**Part (b):** We establish the result by showing that under a Kemp-Wan adjustment of the external tariff we will have \( \hat{U}_N = \hat{U}_D = \hat{U}_A \), so that the incentive constraint of the union is unaffected. The effect of the Kemp-Wan adjustment of the agreement is given by (A.13). To derive the payoff under cheating, we choose \( q \) to maximize welfare as in (A.6) which yields the necessary condition

\[
dU = 0 \quad \Rightarrow \quad -\left[ \frac{\beta}{1-\beta} \right] \left( \frac{dc^1_i}{dc^3_i} \right) = \frac{c^1_i}{c^3_i} = \frac{1 + \tau}{q}.
\]

Substituting (A.7) and (A.8) into (A.18) yields the optimal tariff formula

\[
\Phi = \frac{\epsilon^*}{\epsilon^* - 1}.
\]

When the union deviates from the agreement, the value of \( \Phi \) is the same for all values of \( t \), which indicates that the terms of trade in the optimal deviation is independent of the internal tariff \( t \). Therefore, in the Cobb-Douglas case there is a Kemp-Wan adjustment in the external tariff that the union imposes in the event of cheating, so the change in \( \tau \) under cheating is given by (A.13), which yields \( \hat{U}_D = \hat{U}_A \).

To complete the proof, it remains to show that \( \hat{U}_N = \hat{U}_A \). Substituting into (14) for the Cobb-Douglas case yields

\[
c^1_i = 2 \left[ \frac{\beta + \alpha}{\beta} \right] \left( \frac{1 + t}{2 + t} \right).
\]

Substituting this result into the Cobb-Douglas utility function and using (1) we obtain the autarky utility level

\[
U_N = 2 \left[ \frac{\beta + \alpha}{\beta} \right] \left( 1 + t \right)^\frac{\alpha}{2} (2 + t)^{-\beta}.
\]

Differentiating (A.20) yields

\[
\hat{U}_A = \frac{\beta}{2} \left[ \frac{t}{2 + t} \right] \frac{dt}{1 + t}.
\]
This is identical to the result obtained for the agreement and cheating effects. Since each of the payoff terms in (11b) increases by the same proportion under a Kemp-Wan tariff adjustment in the agreement, the Kemp-Wan tariff adjustment will be incentive compatible.
Figure A.1:

Existence of an Optimal Tariff for the Customs Union in the Presence of Tariffs on Internal Trade