On Trade and the Stability of (Armed) Peace

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Abstract: We consider a simple environment in which two sovereign states with overlapping ownership claims on an insecure resource/asset can choose to resolve their dispute either violently through war or peacefully through a negotiated division of the contested resource under the threat of war. Both approaches to conflict resolution depend on the states’ military capacities, but their outcomes are very different. War precludes international trade between the two states and may destroy productive resources; however, once a winner is declared, arming is unnecessary in future periods. By contrast, a peaceful resolution avoids destruction and supports mutually advantageous trade through negotiation; yet, settlements must be renegotiated and the states must continue to arm in future periods to settle their ongoing dispute. Paying special attention to the importance of trade in this context, we explore the conditions under which war and peace arise as perfectly coalition-proof equilibria over time. In addition to examining the prospects of “armed” peace, we also study the effects of trade on arming incentives and welfare. Our analysis reveals that, depending on time preferences, more liberal (open) trade regimes can have a pacifying effect on international relations. Furthermore, the international distribution of asset ownership plays a key role. Among other things, uneven ownership may be conducive to the stability of peace.

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If an economically self-sufficient man starts a feud against another autarkic man, no specific problems of "war-economy" arise. But if the tailor goes to war against the baker, he must henceforth produce his bread for himself.

Ludwig von Mises (1949, p. 824)

1 Introduction

To what extent does trade between nations induce more peaceful relations between them? There is, of course, a time-honored hypothesis advanced by some scholars of international relations that, because war significantly reduces or completely destroys opportunities for trade, the cost of war rises with increasing integration of national economies (e.g., Polachek, 1980). The so-called "liberal" view, then, suggests that with globalization and/or trade liberalization, we should expect extended peace.

Although the liberal hypothesis builds on a very basic principle in economics, that trade is mutually beneficial, very few economists have explored this hypothesis formally. Skaperdas and Syropoulos (2001) augment a Heckscher-Ohlin model of trade with a conflict over resources between two identical and small countries. In that setting, where arming and thus costs of conflict are endogenous and thus trade-regime dependent, trade’s effects on the degree of conflict and payoffs relative to autarky depend intuitively on exogenously given world price relative to that under autarky. The analysis, however, considers just the case of two small countries that trade with the rest of the world, and thus does not really get at the possible influence of interdependence between them.¹ Garfinkel and Syropoulos (2017a) explore the importance of economic interdependence between two large countries who compete for claims to a resource and subsequently trade with each other. While explicitly capturing the endogeneity of arming and the accompanying security costs like Skaperdas and Syropoulos (2001), they find that, with some exceptions having to do with extreme asymmetries between countries’ mix of initial resources, trade induces lower arming and greater payoffs. However, neither of these analyses consider the costs of conflict in terms of disrupting trade. By contrast, Martin et al. (2008) consider this disruptive effect of war, but do not explicitly capture the resource costs of conflict.

In this paper, we combine these two approaches, in an effort to gain further insight into how greater trade openness matters for international relations over time, whether it fuels conflict or pacifies it. Our analysis builds a model of trade where, along the lines of Armington (1969), each country has a unique and inalienable technology for producing a single intermediate input to produce a final good for consumption. Diversity of inputs

¹ Also, see Garfinkel et al. (2015) who extend that analysis, examining possible asymmetries in initial resources as well as more general functional forms for preferences and technologies.
enhances each countries’ ability to produce that final good, and herein lie the possible gains from trade. We augment that model with conflict over claims of ownership to a resource used to produce the intermediate good. A key feature of the analysis is the distinction it makes between mobilization of resources to arm and the deployment of those arms in open conflict, along the lines of Garfinkel and Skaperdas (2000) and McBride and Skaperdas (2007, 2014).2

The basic setup of the model is as follows: once the contending countries have made their arming decisions, they choose how to resolve their dispute. One option involves violent or open conflict, modeled as a winner-take-all contest, with some fraction of the nation’s remaining resources (after arms have been produced) being destroyed as a result. At the same time, open conflict destroys the possibility of trade between the two countries.3 The other option involves armed peace. More specifically, after having made their arming decisions, the two countries try to negotiate a peaceful division of the contested resource, leaving open the possibility of subsequent trade. Whatever arms they choose, the contending countries always have a short-run incentive to negotiate a peaceful settlement, for that option allows them to divide the contested resource without either having to deploy arms and suffer war’s destructive effects or to foreclose on trade.

However, when the countries take a longer-run perspective, settlement need not emerge as a subgame perfect, Nash equilibrium. The reason is that settlement in the current period concerns the division of resources only in that period; absent the possibility for the two countries to commit to a division of the contested resource in the future, settlement requires some diversion of resources away from the production of goods for consumption in the future as well as in the current period. Open conflict in the current period, by contrast, gives the victor a strategic advantage in future conflict, so that fighting today reduces future arming costs relative to those under settlement. In fact, despite its effect to rule out trade, open conflict is always a subgame perfect, Nash equilibrium. Moreover, depending on the possible gains from trade, the shadow of the future, and the degree of conflict’s destructiveness, open conflict could Pareto dominate peaceful settlement. In such cases, open conflict is a "strong perfect equilibrium" or, equivalently (in the two player setting we consider), a "perfectly coalition-proof" equilibrium (Bernheim et al., 1987). We view this equilibrium concept especially relevant in our setting since the two parties in conflict presumably communicate with one another in the process of their negotiations. Our analysis characterizes the set of conditions under which peaceful settlement is immune

2See Fearon (1995), Skaperdas and Syropoulos (1996), and Powell (2006) for similar approaches. Also see Garfinkel and Syropoulos (2015) who study the effects of trade on domestic conflict between identical groups within a small country that possibly trades with the rest of the world.

3While some have found that conflict has little to no significant effect on trade (e.g., Barbieri and Levy, 1999), Glick and Taylor (2010) present compelling evidence of a negative and significant effect that persists over time.
to both unilateral and coalitional deviations and thus emerges as the equilibrium outcome.

Like the other analyses mentioned above that have studied the distinction between mobilization of resources to produce guns and the decision to use those guns in open conflict, the analysis of this paper finds that a peaceful settlement is more likely to be immune to both unilateral and coalitional deviations, and thus more likely to emerge in equilibrium, when the shadow of the future is weak and the destructive effects of war are large.

The main contribution of this paper, however, is to characterize the importance of trade openness and to link that openness to differences in the initial distribution of resources across countries. The key to this link is summarized in the per-period gains from trade, which as in standard trade models, depends negatively on the elasticity of substitution between the two differentiated inputs in the production of final consumption goods, the level of trade costs, and the distribution of initial resources.

Our analysis shows that there exists a threshold level of the destructiveness of war such that for levels above that neither trade nor the initial distribution of asset ownership matter for the stability of peace. Otherwise, trade openness matters. If large enough, the gains from trade alone render armed peace stable, for all possible initial resource distributions. More generally, however, the initial distribution plays an important role—along with the destructiveness of war, the strength of the shadow of the future, as well as the other factors that influence the gains from trade—in the determination of the stability of armed peace. Interestingly, in this case, peace could be stable only for sufficiently uneven distributions.

2 A Basic Model of Trade and Resource Conflict

Consider a global economy consisting of two countries, identified with the superscript $i = 1, 2$ that interact over two periods. At the beginning of the first period, each country $i$ holds a claim over an asset (e.g., land, water, or oil well) that generates a stream $R^i$ of services per period of time, where $R^1 + R^2 = \bar{R}$. Once held securely, the resource can be used to (ultimately) produce consumables.

However, the initial claims are not entirely secure. Instead, whatever is held by country $i$ initially is available only for the production of "military capacity" or "guns" for short. Denoted by $G^i$ for $i = 1, 2$ and treated as a nontraded commodity, guns are employed only in an effort to influence the probability of winning under conflict or to take control of a share of the residual resources that remain after both countries arm that period. More precisely, we assume that $G^i$ is produced on a one-to-one basis from $R^i$ (with $G^i \leq R^i$), leaving $X^i = R^i - G^i$ of the resource to go into a final pool whose ownership will be contested by the two countries.

Let $\bar{G} \equiv G^1 + G^2$ denote total arming. Then, the total amount of resources contested
by the two countries is given by $\tilde{X} \equiv X^1 + X^2 = \tilde{R} - \tilde{G}$. Once ownership claims over
the asset are settled — either through warfare or a peaceful division — each country $i$ can produce, on a one-to-one basis, a distinct and potentially tradable commodity $Z^i$. We view this commodity as an intermediate input to the production of a consumption good. Importantly, the technology for producing $Z^i$ in each country $i$ is unique and inalienable.\(^4\)

In what follows, we present the details of our framework in two steps. First, we describe
the general equilibrium trade model, given the resources securely held by each country
after their ownership claims have been resolved. This model serves as the backbone of our
analysis. Second, we describe mechanisms of conflict resolution that are available to the
contending states.

### 2.1 General Equilibrium Trade Model

With the resources secured in the resolution of the dispute between the two countries, each
one $i$ produces $Z^i$ units of its distinct intermediate input. For ease of exposition, let us
refer to this quantity as country $i$’s "effective endowment." Furthermore, let us assume all
markets are perfectly competitive and that the final good for consumption in each country $i$
is produced according to the following constant elasticity of substitution (CES) technology:

$$F (D^i_1, D^i_2) = \left[ \sum_{j=1,2} (D^j_i)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}},$$

(1)

where $D^j_i$ denotes the quantity of intermediate good $j \in \{1, 2\}$ demanded and employed in
country $i \in \{1, 2\}$ and $\sigma > 1$ is the elasticity of substitution between intermediate inputs.\(^5\)

If the two countries do not trade with each other (perhaps because of conflict), then each
country $i$ produces $F(\cdot) = Z^i$ (since $D^j_i = 0$, $i \neq j$). Therefore, country $i$’s payoff function
under autarky is given by

$$w^i_A = Z^i,$$

(2)

where subscript "A" indexes "autarky."

\(^4\)In other words, the contest does not include the blueprints to produce the foreign intermediate. The
analysis could be extended to enable all countries to produce both tradable intermediate inputs (through
differential access to the relevant technologies, as in standard Ricardian type trade models) without altering
the key insights of our analysis in a substantive way. (For example, we could relax this assumption by
assuming that the winner can produce the rival’s input using a less efficient technology.) Our strategy
of focusing on nationally differentiated goods that could be traded internationally conforms to Armington
(1969) and is adopted primarily for technical reasons—essentially to bypass some technical complications
related to discontinuities in best-response functions—without changing our main findings in a significant way.

\(^5\)Our assumption that $\sigma > 1$ is needed to ensure that autarky is well-defined. It’s role is similar to the
one in models of monopolistic competition (i.e., it reflects the benefit of employing heterogeneous inputs in
production).
Now let $Y^i$, $p^j_i$ and $\gamma^i_j$ respectively denote country $i$'s income, domestic price and expenditure share for good $j$ in any given time period. One can verify that $\gamma^i_j \equiv (p^j_i/P^i)^{1-\sigma}$, where $P^i \equiv \left[ \sum_j (p^j_i)^{1-\sigma} \right]^{1/(1-\sigma)}$ is the relevant price index. In turn, our specification in (1) implies that the demand for good $j = 1, 2$ in country $i$ is given by $D^i_j = \gamma^i_j Y^i/p^j_i$. Thus, country $i$'s indirect payoff can be written as

$$w^i = Y^i/P^i, \quad i = 1, 2,$$

where $Y^i = p^i_i Z^i$.

Trade of intermediate goods takes place in the presence of "iceberg" type trade costs, reflecting geographic trade barriers. In particular, for each unit of the intermediate input $j$ that country $i$ imports, $\tau \geq 1$ must be shipped by its trading partner $j$ ($\neq i = 1, 2$). (Thus, $\tau - 1$ units "melt" in transit.) Let $\pi_j$ be the "world" price of good $j = 1, 2$. Then $\pi^i \equiv \pi_j/\pi_i$ and $p^i \equiv p^j_i/p^i_i$ are the world and domestic relative price of country $i$'s importable, respectively. Competitive pricing and arbitrage imply these prices satisfy $p^i = \tau \pi^i$ and, naturally, are endogenously determined through a world market-clearing condition. This condition requires the value of country $i$'s imports to be equal to the value of country $j$'s exports (appropriately adjusted to take into account the "shrinkage" in transit); that is, $\tau \pi_j D^i_j = \tau \pi_i D^j_i$ ($i \neq j = 1, 2$). Applying the forms of the demand functions derived earlier and the fact that $Y^i = p^i_i Z^i$, we rewrite this condition as

$$\pi^i = \frac{\gamma^i_j Z^i}{\gamma^i_i Z^i},$$

where $\gamma^i_j = (\tau \pi^i)^{1-\sigma} / \left[ 1 + (\tau \pi^i)^{1-\sigma} \right]$ for $i \neq j = 1, 2$. The (implicit) solution to (4), captured by $\pi^i_T$, where "T" refers to "trade," is the relative price of country $i$'s importable that clears the world market.

Next, define $\mu^i (\cdot) \equiv \left[ 1 + (\tau \pi^i)^{1-\sigma} \right]^{-\frac{1}{1-\sigma}} (= p^i_i/P^i)$. Our assumption that $\sigma > 1$ implies $\mu^i (\cdot) > 1$. Using this definition (together with $Y^i = p^i_i Z^i$) back in (3) enables us to obtain the following expression for country $i$'s payoff under trade:

$$w^i_T = \mu^i (\cdot) Z^i.$$

This expression shows that, under trade, a country's payoff depends on its capacity to produce intermediate good $Z^i$, as is the case under autarky. However, because $\mu^i (\cdot)$ depends

\begin{itemize}
  \item[6] The analysis could also be extended to consider the possible use of import tariffs. We abstract from this possibility here for simplicity.
  \item[7] Inspection of (4) reveals that $\pi^i_T$ depends on $(Z^i/Z^j)$, $\sigma$, and $\tau$. It is easy to check that this solution exists and is unique. Moreover, in the special case of unimpeded trade (where $\tau = 1$), the world market clearing price is given by $\pi^i_T = (Z^i/Z^j)^{1/\sigma}$.
\end{itemize}
on the world market clearing price $\pi_T^i$ and this price, in turn, depends on $(Z^i, Z^j)$, payoff $w_T^i$ now is a function of both countries’ output levels $(Z^i, Z^j)$.

What can one say about the gains from trade in this context? A comparison of (2) and (5) reveals that country $i$’s gains from trade are captured by $\mu^i (\cdot) > 1$.

As discussed in the next section and shown in the Appendix, these gains from trade are increasing in the degree of dissimilarity between traded goods ($\sigma \downarrow$) and increasing with trade cost reductions ($\tau \downarrow$). As we will see later on, in the context of this model, the reason countries may prefer to settle on their disputes peacefully is twofold: (i) to avert the possibly destructive consequences of war, and (ii) to generate and internalize possible gains from trade in intermediate inputs.

To shed light on these issues we must examine how countries’ effective endowments $(Z^i, Z^j)$ and, through them, their payoff functions, depend on arming decisions under conflict and settlement.

### 2.2 Resolving Resource Disputes

We now turn to the determination of the countries’ effective endowments. Recall that, if country $i$ uses some portion of its initial resource endowment $R^i$ to produce $G^i$ guns (i.e., $G^i \leq R^i$), $X^i = R^i - G^i \geq 0$ residual units of the resource go into a common resource pool that is contested peacefully or through warfare. Using the notation introduced earlier, the size of the contested pool is given by $\bar{X} = \sum_i X^i = \bar{R} - \bar{G}$. Therefore, an increase in the quantity of guns produced by either country $i$ reduces the aggregate size of the contested pool. For this reason, arming is costly. The benefits of arming for each country depend on the manner in which they jointly resolve their ownership claims over $\bar{X}$, either through (i) overt, and possibly destructive, conflict/war or through (ii) a peaceful division of $\bar{X}$ in the shadow of conflict ("armed peace").

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8We describe the dependence of $\pi_T^i$ and $w_T^i$ on all variables of interest in the Appendix. Among other things, there we show that $w_T^i$ is increasing both in $Z^i$ and in $Z^j$. However, in contrast to autarky (where $w_A^i$ is linear in $Z^i$), $w_T^i$ is concave in $Z^i$ due to an adverse terms-of-trade effect.

9Owing to the endogeneity of $\pi_T^i$, the size of these gains is also endogenous. Moreover, if either $p' = \pi_T^i \rightarrow 0$ or $p' \rightarrow \infty$ then $\mu^i \rightarrow 1$ and $w_T^i \rightarrow w_A^i$. Further, $\lim_{\sigma \rightarrow \infty} w_T^i = \lim_{\tau \rightarrow \infty} w_T^i = w_A^i$. As $\sigma \rightarrow \infty$, the intermediate inputs become indistinguishable, and thus trade is pointless. By the same token, trade is eliminated when trade costs become sufficiently large.

10The structure of this trade model with $\sigma > 1$ ensures that gains from trade arise due to the "diversity" of inputs in the production of the final good. As noted earlier, the model could be modified to capture trade based on Ricardian-type comparative advantage. It could also be extended to allow for the possibility of trading a fixed number of differentiated goods or an endogenously determined number of varieties, as in Krugman (1980). Neither of these extensions changes the fundamental insights of our analysis.

11The analysis could be extended to consider the possibility that a fraction $\kappa \in [0, 1]$ of $X^i$ is secure and the remaining $(1 - \kappa)X^i$ units are subject to appropriation. This modification would allow us to study the implications of various degrees of insecurity including the extreme cases of perfect security ("Nirvana"), which arises when $\kappa = 1$ and is the norm in standard neoclassical theory, and other intermediate cases, where $\kappa \in (0, 1)$. We abstract from this generalization here because it does not affect the key insights regarding the comparison of conflict and settlement.
Under conflict, arming is the means through which a country improves its probability of winning. Under settlement, arming enhances a country’s fallback (or "threat point") payoff, thereby enabling it to command a larger share of the contested pool in the negotiations. Naturally, the extent to which countries arm depends on their perceived marginal benefits and opportunity costs of arming, as well as their expectations of actions in future periods. Two features of our analysis stand out. First, countries’ arming decisions will be endogenously determined as functions of their expectations regarding their manner in which they will resolve conflict. Second, countries will actually decide between open conflict and peaceful settlement. We now describe these two forms of conflict resolution in more detail.

Open conflict takes the form of a "winner-take-all" contest. Let $\phi^i$ denote the probability that state $i$ wins the contest. This probability is specified by the following contest success function (CSF):

$$\phi^i = \phi^i(G^i, G^j) = \begin{cases} 
\frac{G^i}{G} & \text{if } G > 0 \\
\frac{1}{2} & \text{if } G = 0
\end{cases}, \quad i \neq j = 1, 2. \tag{6}$$

Since $\phi^i_{G^i} > 0$ and $\phi^j_{G^j} < 0$, the winning probability to country $i$ is increasing in its own guns and decreasing in the guns of its opponent. Furthermore, the CSF is symmetric, such that $G^i = G^j = G \geq 0$ implies $\phi^i = \phi^j = \frac{1}{2}$.\(^\text{13}\)

Conflict in the first period has three other substantive features. First, it can be destructive, leaving a fraction $\beta \in (0, 1]$ of the common resource pool intact in the first and subsequent periods.\(^\text{14}\) Such destruction tends to detract from the relative appeal of conflict. Second, any departure from settlement destroys opportunities for trade in the first and second periods. This assumption, which is clearly extreme, also tends to detract from the relative appeal of conflict. Our rationale for imposing it here is to capture a salient feature of the "liberal" argument, that the larger are the potential gains from trade, the

\(^\text{12}\)We require that the choices, studied below in detail, are immune to both unilateral deviations and coalitional deviations. The latter requirement is appropriate, specifically because countries can communicate freely during the negotiation process.

\(^\text{13}\)This functional form, first introduced by Tullock (1980), belongs to a more general class of contest success functions (CSFs), $\phi^i(G^i, G^j) = f(G^i)/\sum_j f(G^j)$ where $f(\cdot)$ is a non-negative and increasing function, axiomatized by Skaperdas (1996). See Hirshleifer (1989), who explores the properties of two important functional forms of this class, including the "ratio success function" where $f(G) = G^b$ with $b \in (0, 1]$. Though the results to follow remain qualitatively unchanged under this more general specification, we use the specification in (6), assuming $b = 1$ for simplicity (and, for our analysis of unilateral deviations from settlement, for tractability).

\(^\text{14}\)This destructive effect might alternatively be due to permanent damage to each country’s technological apparatus/infrastructure, reducing their effectiveness of transforming the contested resource into the intermediate input. The analysis can be easily extended to entertain other types of destruction that could influence arming decisions (e.g., conflict could destroy resources at different rates depending on the time period considered) without altering the key insights.
higher is the opportunity cost of conflict and thus the more likely it is that countries will opt for peaceful settlement.\footnote{Later on we discuss how the analysis and insights may change if the victor (e.g., an imperial power) may also appropriate the foreign technology (from, say, its colony) and put it to use in period $t = 2$. This type of appropriately raises the relative appeal of conflict.} Finally, open conflict in the first period confers a strategic advantage on the victor in the future. In particular, the winner of the war in the first period takes control not only of the contested pool after destruction $\beta \tilde{X}$, but also of the resource that survives destruction $\beta \tilde{R}$ in the next period, without having to arm at that time.\footnote{Here we are effectively assuming that the winning side captures the resource (net of destruction) in both periods and the losing side perishes. What we have in mind is that defeat in conflict undermines the losing side’s capacity, organization, and possibly even its will to enter conflict in the future. Put differently, one could view conflict here as crippling the losing side’s ability to engage in future conflict or, more starkly, eliminating the loser. Importantly, the contest is not over the intermediate inputs or the consumption good.} This last feature of open conflict is clearly extreme as well. However, it provides a useful benchmark that highlights the potential benefits of conflict and is consistent with the idea that resource ownership is insecure.

Settling the dispute peacefully amounts to agreeing on a division of $\tilde{X}$ and to engaging in trade. A rationale for "cooperation" arises here because, for any given quantities of guns within a time period, negotiation and settlement (i) support bilateral trade and the payoff gains that normally accompany it, and (ii) preempt the destructive consequences of conflict. Nonetheless, peace is costly. In particular, under the reasonable assumption that binding contracts on future settlements are not feasible, future divisions of the resource are subject to renegotiation. What is more, peaceful settlements do not eliminate countries’ incentives to arm because arming is not contractible and arming affects the division of the contested pool ("armed peace").

Indeed, since conflict involves arming only in the first period while settlement does not necessarily eliminate the need to arm in any period, conflict could dominate settlement in terms of payoffs. Furthermore, even if settlement turns out to be jointly optimal (i.e., settlement delivers higher payoffs to both sides than conflict), states could find it individually optimal to violate an "agreement" by deviating from it unilaterally. We wish to identify the possible conditions under which settlement is stable (i.e., immune to coalitional and unilateral deviations that produce conflict). The focus on sequential "coalition proof" equilibria seems appropriate in this context because policymakers can communicate.

Let $\lambda^i$ be the fraction of the common pool $\tilde{X}$ that state $i$ would obtain under settlement in a given period. The timing of actions in the first period is as follows:

**Stage 1.** Each state $i$ chooses its arming $G^i (\leq R^i)$, treating its rival’s decision $G^j (j \neq i)$ as given. This up-front decision is irreversible and non-contractible.

**Stage 2.** The two countries enter into negotiations about how to divide the contested pool,
\[ X = \bar{Z} - \bar{G}, \text{ in the current period.} \]

2a. If both states agree on a division, \( X \) is distributed accordingly. Country \( i \)'s effective endowment becomes \( Z^i = \lambda^i X \).

2b. If negotiations fail, the two sides enter into open conflict over \( X \). The effective endowments are \( Z_{\text{win}} = \beta \bar{X} \) for the winner and \( Z_{\text{loss}} = 0 \) for the loser.

**Stage 3.** If the two sides agree to settle their claims and no deviation from the agreement is recorded, the contenders engage in competitive trade. Conflict and deviations from settlement foreclose on current and future trades.

What happens in the second period (\( t = 2 \)) depends on the outcome of the two countries interactions in period \( t = 1 \). If war breaks out in period \( t = 1 \), there is no arming in period \( t = 2 \) and the winner, say country \( i \), enjoys the stream of benefits associated with controlling \( R_{\text{win}} = \beta \bar{R} \) units of the services of the primary resource at that time. In contrast, the loser \( j (\neq i) \) perishes. If peaceful settlement arises in period \( t = 1 \), the three stages specified above are repeated in period \( t = 2 \). For reasons that will become apparent shortly, conflict can be part of the equilibrium only in period \( t = 1 \); that is, settlement in period \( t = 1 \) always leads to settlement in period \( t = 2 \).

### 3 Equilibria under Conflict and Settlement

Having described the sequence of actions by the two sides, we now go on to describe their expected payoff functions when interactions occur over a two-period time horizon. Within this environment we then describe countries’ arming incentives and decisions both under war and peace with a view toward preparing the ground for our subsequent analysis of equilibria of the extended game.

#### 3.1 Conflict

Let \( u^i (G^i, G^j) \) be country \( i \)'s conflictual payoff function in the current period. Since trade is foreclosed under conflict, by (2) country \( i \)'s contingent payoff in the current period is linear in its effective contingent resource endowments or intermediate good output levels. On the basis of our earlier discussion, this payoff function can be defined as

\[
\begin{align*}
u^i \equiv u^i (G^i, G^j) &= \phi^i Z_{\text{win}}^i + (1 - \phi^i) Z_{\text{loss}}^i = \phi^i \beta \bar{X}.
\end{align*}
\]

It is worth noting that, in period \( t = 2 \), it is the above payoff that the contenders will compare to the payoff they would obtain if they chose to settle in the same period.
Now denote with $\delta \in [0, 1]$ countries’ (common) discount factor and with $U^i$ country $i$’s expected discounted lifetime payoff under overt conflict divided by $1 + \delta$, which we refer to as its "average" payoff under conflict. Since the winning side will control $R^i_{\text{win}} = \beta \tilde{R}$ and $R^i_{\text{loss}} = 0$ with probabilities $\phi^i$ and $1 - \phi^i$, respectively, in period $t = 2$, its average payoff will be

$$U^i = U^i(G^i, G^j) = \frac{1}{1 + \delta} [u^i(G^i, G^j) + \phi^i \beta \delta \tilde{R}],$$

for $i \neq j = 1, 2$. Applying (7) in the above equation and rearranging terms gives

$$U^i = \frac{\beta}{1 + \delta} \phi^i (\tilde{X} + \delta R), \; i \neq j = 1, 2. \tag{8}$$

The extent to which each country $i$ may arm will depend on the solution to $\max_{G^i} U^i$, s.t., $X^i \geq 0$ for $i = 1, 2$. Differentiation of country $i$’s expected payoff $U^i$ in (8) with respect to $G^i$ gives:

$$U^i_{G^i} = \frac{\beta}{1 + \delta} \left[ \phi^i G^i (\tilde{X} + \delta \tilde{R}) - \phi^i \right], \; i = 1, 2. \tag{9}$$

The first term inside the square brackets of (9) (multiplied by $\beta/(1 + \delta)$) captures country $i$’s average discounted marginal benefit to arming. This benefit arises because a marginal increase in $G^i$ improves state $i$’s probability of winning the war and controlling the output streams of $\tilde{X}$ and $\tilde{R}$. However, there is an opportunity cost to the increase in country $i$’s arming due to the reduction in the size of the pool $\tilde{X}$. This cost is captured by the second term in (9) (multiplied by $\beta/(1 + \delta)$). Inspection of the expression inside the square brackets of (9) also reveals that a longer shadow of the future ($\delta \uparrow$) raises the marginal benefit to arming relative to the corresponding marginal cost.

Denote with $B^i_c (G^j; \cdot)$ country $i$’s best reply to $G^j > 0$ ($j \neq i$) under conflict. Keep in mind that, depending on the actions of a country’s rival, it’s possible that its resource endowment $R^i$ constrains its arming decision. One can easily verify now that the first-order condition (FOC) implied by (9) together with the resource constraint deliver the following best-response functions under conflict:

$$B^i_c (G^j; \delta, R^i, \tilde{R}) = \min \left( R^i, \tilde{B}^i_c \right), \; i \neq j = 1, 2, \tag{10}$$

where $\tilde{B}^i_c \equiv -G^j + \sqrt{(1 + \delta) RG^j}$ is country $i$’s unconstrained best-response function. Clearly, the values of $\delta$, $\tilde{R}$ and $G^j$ jointly determine the shape of $B^i_c (\cdot)$. In particular, inspection of (10) reveals that, when the resource constraint is inactive for country $i$, its security policy is a strategic complement (resp., substitute) for its adversary’s guns when
\( \tilde{B}_c^i (G^j; \cdot) > G^j \) (resp., when \( \tilde{B}_c^i (G^j; \cdot) < G^j \)).\(^{17}\) Moreover, \( \partial \tilde{B}_c^i / \partial \delta > 0 \), so a longer shadow of the future induces a less unconstrained country to adopt a more aggressive stance in its security policy.\(^{18}\) These features of \( B_c^i (G^j; \cdot) \), which are illustrated in Fig. 1, play prominent roles both in the determination of a conflictual equilibrium and, as we will see, the contenders’ incentives to deviate from negotiated settlements.\(^{19}\)

Let us now study in some detail the non-cooperative equilibrium associated with "conflict". Denote with \( G_c^i \) the quantity of guns country \( i \) would produce in this equilibrium and define

\[
R_c^i_L \equiv \left( 1 - \frac{1 - \delta}{2} \right) \tilde{R} / 2 \quad \text{and} \quad R_c^i_H \equiv \left( 1 + \frac{1 - \delta}{2} \right) \tilde{R} / 2,
\]

where subscript "\( L \)" ("\( H \)") identifies a certain "low" ("high") endowment threshold and superscript "\( c \)" refers to "conflict". Clearly, \( R_c^i_H - R_c^i_L = (1 - \delta) \tilde{R} / 2 \geq 0 \) for \( \delta \leq 1 \). Utilizing the properties of \( B_c^i (G^j; \cdot) \) implied by (10) together with the fact that \( R^i + R^j = \tilde{R} \) lead to

\[
G_c^i = \begin{cases} 
R^i & \text{if } R^i \in (0, R^i_L] \\
R^i_H & \text{if } R^i \in [R^i_L, R^i_H] \\
-R^j + \sqrt{(1 + \delta) \tilde{R} R^j} & \text{if } R^i \in [R^i_H, \tilde{R}]
\end{cases}
\]  

(11)

for \( i \neq j = 1, 2 \).

A distinguishing feature of the equilibrium is that, when the initial pattern of asset ownership is sufficiently even (specifically, when \( R^i \in (R^i_L, R^i_H) \)) arming is invariant to changes in the international distribution of asset ownership and \( G_c^i = G_c^j \) for \( i \neq j = 1, 2 \). In contrast, if the initial pattern of ownership is sufficiently asymmetric, only one country, the smaller one, will be constrained by its resource endowment regardless of the value of \( \delta \).

Fig. 1 depicts the above possibilities in some detail. The initial pattern of resource ownership is captured by points along the thick straight line connecting points \( \tilde{R} \) and \( \tilde{R} \) on the two axes (by construction, \( R^1 + R^2 = \tilde{R} \) along this line). For points on \( \tilde{R} \) to the left of \( A \), such as point \( E_1 \), country 1’s arming decision is constrained by its endowment because \( R^1 \in (0, R^1_L) \).\(^{20}\) In contrast, country 2’s best-response function coincides with the unconstrained curve \( O E''_2 E \). Thus, the equilibrium is at point \( E'_1 \). Using similar logic we find that,

---

\(^{17}\)One can verify that the slope of \( \tilde{B}_c^i \) along the 45° line equals 0 since \( d\tilde{B}_c^i / dG^j = -U_{GJ}^i / U_{GJ}^i \) and \( U_{GJ}^i | \bar{G} = G \) = 0.

\(^{18}\)This is so because a larger discount factor is associated with a more valuable prize in the contest (see (9)).

\(^{19}\)It should be noted that Fig. 1 does not depict the non-negativity constraint on guns to maintain proper scale. As will become clear later, this constraint is never active. (But if it were active for, say, country 1, then \( B_c^i \) would coincide with the relevant range of the horizontal axis.)

\(^{20}\)In this case, country 1’s best-response function has a kink at point \( E''_1 \) and is captured by curve \( O E''_1 E'_1 E_1 \).
for $R^1 \in [R^c_L, \bar{R}]$ (or, equivalently, for $R^2 \in (0, R^c_H]$), country 2 is the resource-constrained
country and point $E'_2$ is the relevant equilibrium. If $\delta < 1$, at most one country’s arming
decision can be constrained by its initial resource endowment. Moreover, an unconstrained
equilibrium (point $E$) arises for all resource allocation in the $AB$ segment of $\bar{R}\bar{R}$ (because
$R^1 \in [R^c_L, R^c_H]$).

For additional insight, suppose that initially $R^1$ is close to 0. Successive reallocations
of resources that cause $R^1$ to rise are associated with movements along $\bar{R}\bar{R}$ in the
direction of the arrows depicted in Fig. 1. These reallocations shift the conflictual equilibrium along
country 2’s unconstrained best-response function (e.g., from $E''_2$ to $E'_1$) until point $E$ is
reached. Provided $\delta < 1$, the equilibrium remains at $E$ for all possible reallocations in
segment $AB$. However, when the reallocations go beyond point $B$, country 2’s resource
constraint becomes active and the equilibrium moves along country 1’s unconstrained best-
response function in the direction of origin $O$. The solid-line (resp., dashed-line) pink curve
describes country 1’s (resp., 2’s) guns.

We may summarize the above ideas as follows:

**Proposition 1 (Arming)** Under conflict, there exists a unique equilibrium in arming. For
$i \neq j = 1, 2$ and any given $\bar{R}$ such that $R^i + R^j = \bar{R}$, equilibrium arming decisions are as
follows:

(a) (i) If $R^i \in (0, R^c_L]$, then $G^i_c = R^i$ and $G^j_c = \hat{B}^j(R^i, \delta)$.
(ii) If $R^i \in [R^c_L, R^c_H]$, then $G^i_c = G^j_c = R^c_L(\delta)$.
(iii) If $R^i \in (R^c_H, \bar{R})$, then $G^i_c = \hat{B}^j(R^i, \delta)$ and $G^j_c = R^i$.
(b) $d(R^c_H - R^c_L)/d\delta < 0$ and $\lim_{\delta \to 1} R^c_L = \lim_{\delta \to 1} R^c_H = \bar{R}/2$.
(c) Larger discount factor values $\delta$ imply more arming by unconstrained countries.

If the international structure of asset ownership is sufficiently even, the resource con-
straint on arming in period $t = 1$ is inactive for both countries and they produce equal
quantities of guns. On the other hand, if asset ownership is sufficiently uneven, the less
affluent economy specializes completely in the production of arms in period $t = 1$ whereas
its more affluent adversary diversifies its production and also produces more guns. A longer
shadow of the future (\(\delta \uparrow\)), implies higher arming for any country whose production is di-
versified. At the same time, increases in $\delta$ reduce the range of resource allocations under
which arming decisions are equalized (i.e., $d(R^c_H - R^c_L)/d\delta < 0$).

Part (a.i) underscores the allocation of resource endowments that constrain country $i$’s
but not country $j$’s arming decision and what that means for equilibrium arming. Part
(a.ii) unveils the range of endowments under which both countries’ resource constraint on
arming is inactive. Part (a.iii) is essentially the reverse of part (a.i). Part (b) emphasizes

\(^{21}\)If $\delta = 1$, the interval $AB$ collapses to a single point as $R^c_L = R^c_H = \bar{R}/2$. 

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that the range of endowment allocations under which arming is equalized internationally falls with increases in $\delta$. Fig. 2(a) describes the equilibrium in guns as a function of the distribution of factor ownership for alternative values in the discount factor $\delta$.

Recall that $\bar{G}_c = G^i_c + G^j_c$. A consequence of Proposition 1 is that $\bar{G}_c$ is increasing in $\delta$ under all factor ownership allocations because the increased value of the "prize" induces all unconstrained countries to arm more heavily. In contrast, $d\bar{G}_c/dR^i > 0$ for $R^i \in (0, R^L_c)$, $d\bar{G}_c/dR^i = 0$ for $R^i \in [R^L_c, R^H_c]$ and $d\bar{G}_c/dR^i < 0$ for $R^i \in [R^H_c, \bar{R}]$. In words, when the resource constraint is binding for one country, a relaxation of this constraint through resource reallocations expands global arms because the constrained country produces more guns and, owing to strategic complementarity, the unconstrained country reciprocates by producing even more guns. Exactly the opposite is true if a country’s constraint becomes tighter. Resource reallocations do not affect $\bar{G}_c$ if neither country’s resource constraint is active.

Having examined countries’ equilibrium arming decisions under conflict, we now examine the dependence of equilibrium payoffs under conflict $U^i_c$ ($i = 1, 2$) on the distribution of factor ownership and the shadow of the future ($\delta$). In general, changes in $R^i$ and/or in $\delta$ affect country $i$’s average payoff $U^i_c$ directly and indirectly (through their possible impact on $G^i_c$ and $G^j_c$). When country $i$’s resource constraint is inactive, the payoff effect of $G^i_c$ on $U^i_c$ vanishes (envelope theorem). In contrast, when country $i$’s resource constraint on its arming is binding, changes in parameters that enable country $i$ to move closer to its unconstrained optimum improve its payoff.

How do changes in $G^j_c$ affect $U^i_c$? One can verify from (8) and the properties of the CSF in (6) that $U^i_{G^j_c} < 0$; so under conflict an increase in a country’s arming affects its rival’s payoff adversely. The reason for this is twofold: Because an increase in $G^j_c$ reduces both country $i$’s probability $\phi^i$ of winning the war and the overall size of the common resource pool $\bar{X}$.

Proposition 2 below describes how the just described indirect effects of arming decisions on payoffs (together with Proposition 1) combine with the direct effects of changes in the shadow of the future and the distribution of factor ownership to influence payoffs.

**Proposition 2 (Payoffs)** The net effect of changes in the shadow of the future and in the distribution of factor ownership on payoffs depend on the nature of the initial equilibrium, especially, on whether countries’ arming decisions exhaust their respective resource endowments. Specifically,

(a) if $R^i \in (0, R^L_c)$, then
   - $dU^i_c/dR^i > 0$, $d^2U^i_c/d(R^i)^2 < 0$ and $\lim_{R^i \to 0} U^i_c = 0$;
   - $dU^i_c/d\delta < 0$;

(b) if $R^i \in [R^L_c, R^H_c]$, then $U^i_c = \beta \frac{\bar{R}}{4}$ and
\[ \begin{align*}
\text{o } dU_i^c / dR_i^c &= 0; \\
\text{o } dU_i^c / d\delta &= 0; \\
\text{(c) if } R_i^c \in [R_H^c, \hat{R}], \text{ then} \\
\text{o } dU_i^c / dR_i^c &> 0, \quad d^2U_i^c / d(R_i^c)^2 > 0 \text{ and } \lim_{R_i^c \to \hat{R}} U_i^c = \beta \hat{R}; \\
\text{o } dU_i^c / d\delta &> 0.
\end{align*} \]

Fig. 2(b) describes the dependence of country \( i \)'s payoff under conflict on the international distribution of factor ownership for alternative values in the discount factor \( \delta \).\(^{22}\) Note that \( U_i^c \) rises at a decreasing (increasing) rate in \( R_i^c \) for the alternative intervals in factor endowments detailed in the various parts of Proposition 2. Also note that \( U_i^c \) falls (rises) with increases in \( \delta \) again as noted in Proposition 2.

We may explain the intuition behind the above argument by studying the effects of resource redistributions, examined in parts (a) – (c) of Proposition 2 (see (8)) as follows. When country \( i \)'s resource constraint is active (part (a)), increases in \( R_i^c \) (with \( \hat{R} \) remaining fixed) affect \( U_i^c \) by relaxing the resource constraint on arming (which implies \( \partial U_i^c / \partial G_i > 0 \) since \( G_i = R_i^c \) in this case) and by inducing the unconstrained opponent \( j \) (\( \neq i \)) to adopt a more aggressive stance in its security policy due to strategic complementarity. This latter effect on welfare is negative. In the Appendix we show that the positive welfare effect due to increases in \( G_i \) dominates the adverse effect due to increases in \( G_j \). Moreover, at the margin, the net effect becomes weaker as \( R_i^c \) rises (i.e., \( U_i^c \) rises at a decreasing rate) essentially because the opponent’s aggressiveness rises at a decreasing rate along its best-response function. Naturally, \( \lim_{R_i^c \to 0} U_i^c = 0 \) while \( \lim_{R_i^c \to \hat{R}} U_i^c = \hat{R} \) as one country becomes the world in these cases.

Part (b) follows from Proposition 1(b), which points out that both countries’ guns are invariant to changes in factor ownership and the definition of \( U_i^c \) in (8). Part (c) can also be explained using logic similar to the one in part (a). The main difference is that, by the envelope theorem, the indirect effect on welfare due to changes in \( G_i \) vanishes (because there is slack in country \( i \)'s resource constraint). Moreover, the (constrained) opponent \( j \) (\( \neq i \)) behaves less aggressively as its resource endowment falls. Since, in this case, \( G_j \) falls faster than \( G_i \), country \( i \)'s welfare rises at an increasing rate.

It is also important to note that the impact of \( \delta \) on a country’s average payoff \( U_i^c \) crucially depends on whether country \( i \)'s resource constraint on its arming decision is active (part (a)) or inactive (parts (b) and (c)). For given arms, increases in \( \delta \) raise the value of a country’s average discounted payoff, so the direct effect of \( \delta \) on \( U_i^c \) is positive, regardless of whether this country’s arming is actively constrained or unconstrained by its resource

\(^{22}\)The figure does not depict the non-negativity constraint on guns to maintain proper scale. Nonetheless, it is worth keeping in mind that, if this constraint were active for, say, country 1, \( B_1^c \) would coincide with the relevant range of the horizontal axis.
endowment. Turning to the strategic effects, as noted in part (c) of Proposition 2, a longer shadow of the future \((\delta \uparrow)\) fuels arming incentives of a country whose resource constraint on arming is inactive. Thus, if country \(j\) is unconstrained, then \(dG^i_c/d\delta > 0\) causing (the constrained) country \(i\)'s payoff \(U^i_c\) to fall. In the Appendix we show that the indirect effect of \(\delta\) on \(U^i_c\) dominates the direct effect thus implying \(dU^i_c/d\delta < 0\) (for small changes in \(\delta\)). The unconstrained country \(j\)'s payoff is increasing in \(\delta\) because the strategic effect vanishes due to the fact that \(G^i_c = R^i\).

### 3.2 Settlement

To assess the validity of the classical liberal argument, we now allow countries to attempt to settle their resource dispute peacefully. One key feature of such settlements is that the have to be renegotiated in every period because long-term commitments are not feasible. A second defining feature is that they do not necessarily eliminate countries’ need to arm because arming is a source of leverage. Naturally, there is a cost to arming: By reducing the size of the resources left for the production of commodities, arming reduces the size of the bargaining set (Skaperdas and Syropoulos, 2002; Anbarci et al., 2002; Garfinkel and Skaperdas, 2000, Garfinkel and Syropoulos, 2015).

Let \(\lambda^i\) be an arbitrary division of \(\bar{X}\), so that \(Z^i = \lambda^i \bar{X}\), and define \(\omega^j \equiv \mu^j \lambda^i\), where \(\mu^j \equiv [1 + (\sigma \pi^j_T)^{-1}]^{-\frac{1}{\sigma}}\) is our measure of the gains from trade and \(\pi^j_T = \pi^j_T(\lambda^i)\) is the world market-clearing price that solves (4).\(^{24}\) Utilizing these facts in (5), we may rewrite per-period payoffs under settlement as \(w^j_T = \mu^j Z^i = \omega^j \bar{X}\) or, more compactly, as

\[
v^i \equiv v^i(\lambda^i, \bar{G}; \tau, \sigma) = \omega^j \bar{X}, \quad i \neq j = 1, 2. \tag{12}\]

In short, \(v^j\) depends on the division \(\lambda^i\) (through \(\omega^j\) which depends on \(\lambda^i\) but not on guns) and on guns \(\bar{G}\) (through \(\bar{X}\)).\(^{25}\)

How does arming under peace affects a country’s average payoff \(V^j\)? The answer hinges on the negotiation process that determines \(\lambda^j\). This share is endogenously determined as a

\(^{23}\)Though the success of peaceful settlements depends on the existence and size of a surplus, the payoff enhancing effects of conflict in the future may be more salient.

\(^{24}\)The dependence of \(\pi^j_T\) on \(\lambda^i\) (fully described in Lemma (A.2) in the Appendix) follows from (4) where \(\pi^j_T = \frac{\gamma^j_1}{1 - \sigma} = \frac{\gamma^j_1}{\lambda^j_T}\) and \(\gamma^j_1 = \gamma^j(p')\) where \(p' = \tau \pi^j_T\). (Note that \(\pi^j_T\) does not depend directly on guns.) The properties of \(\omega^j\) are studied in Lemma (A.4) in the Appendix. In particular, part (a) of Lemma (A.4) indicates that \(\lim_{\lambda^i \rightarrow 0} \omega^i = 1\) and \(\lim_{\lambda^i \rightarrow 0} \omega^j = 0\). Thus, a country does not stand to not gain from trade if it obtains either all of \(\bar{X}\) or none of it.

\(^{25}\)Among other things, three points deserve emphasis here when guns \(\bar{G}\) (and thus \(\bar{X}\)) are fixed. For \(\sigma \in (1, \infty)\) and \(\tau \in [1, \infty)\), first, the payoff \(w^j_T\) (i) satisfies \(w^j_T > w^j_A\) for all \(\lambda^i \in (0, 1)\), (ii) is maximized at an \(\lambda^i \in (\frac{1}{\sigma}, 1)\), and (iii) \(\lim_{\lambda^i \rightarrow 0} w^j_T = \lim_{\lambda^i \rightarrow 1} w^j_T = w^j_A\). Second, country \(i\)'s relative gains from trade \(\mu^i (= w^j_T/w^j_A)\) fall with increases in its share \(\lambda^i\) of \(\bar{X}\) and its absolute gains from trade, \(w^j_T - w^j_A\), are maximized at some \(\lambda^i_0 < \lambda^i_1\). Third, the above points suggest that there exist Pareto inferior allocations of \(\bar{X}\).
function of countries’ (irreversible and non-contractible) arming choices and the bargaining protocol they adhere to. Henceforth we assume that negotiators choose $\lambda^i$ so as to split the surplus.\footnote{See Garfinkel and Syropoulos (2017b) who compare division rules based on splitting-the-surplus, Nash bargaining and equal sacrifice.}

Consider period $t = 2$ and suppose that conflict did not arise in period $t = 1$. (Recall that under conflict in period $t = 1$ the defeated player vanishes; therefore, settlement is feasible in period $t = 2$ only if it is preceded by settlement in period $t = 1$.) Also suppose that, for any given $(G^i, G^j)$, a positive per period surplus $S \equiv v^i + v^j - u^i - u^j$ exists due to existence of gains from trade and no destruction. The "split-the-surplus" division defines $\lambda^i$ in that period implicitly as the solution to $v^i - u^i - (v^j - u^j) = 0$ which could be rewritten, after dividing by $\bar{X} (> 0)$, as

$$
\Psi^i = \Psi(\lambda^i, G^i, G^j; \sigma, \tau, \beta) \equiv \omega^i - \omega^j - \beta(\phi^i - \phi^j) = 0, \ i \neq j = 1, 2. \tag{13}
$$

Temporality assume $\lambda^i$ is a unique solution to (13). (Lemma 1 below addresses this issue.)

The problem facing country $i$’s policymaker in period $t = 2$ can thus be described as follows:

$$
\max_{G^i} v^i \left( G^i, G^j, \lambda^i(G^i, G^j; \cdot); \cdot \right), \ \text{s.t.} \ G^i \in [0, R^i]. \tag{14}
$$

Denote with $G^i_s$ and $v^i_s$ ("s" for "settlement") country $i$’s (= 1, 2) equilibrium arming and its associated payoff, respectively. Now, focusing on period $t = 1$ decisions, under settlement country $i$ will choose $G^i$ to maximize $V^i = \frac{1}{1 + \delta} \left[ v^i \left( G^i, G^j, \lambda^i(G^i, G^j; \cdot); \cdot \right) + \delta \max \left( v^i_s, u^i_c \right) \right]$, s.t. $G^i \in [0, R^i]$ and $u^i_c = U^i_c|_{s=0}$. But, as we will see, the presence of gains from trade and/or the absence of destruction under peace imply $v^i_s = \max (v^i_s, u^i_c)$. Thus, due to stationarity, the solution to the 2-period problem under settlement is identical to the one described in (14) in period $t = 2$. Since this implies $V^i_s = v^i_s$ for all $i = 1, 2$, it suffices to examine the equilibrium of the stage game in (14).

Going back to (13), it is clear that $\lambda^i$ hinges on the nature of payoffs $\omega^i$ and $\omega^j$ and the CSF. But $\omega^i$ itself depends on $\lambda^i$. Keeping in mind that $d\lambda^i = -d\lambda^i$, differentiation of $\omega^i$ gives

$$
\omega^i_{\lambda^i} = \mu^i \left( 1 - \frac{\gamma^i}{\lambda^j} / \Delta \right), \ i \neq j = 1, 2. \tag{15}
$$

The first term inside the parentheses (multiplied by $\mu^i$) is the direct effect of shifting a fraction of the common pool into country $i$. This effect is positive because, at constant prices, it expands country $i$’s income.\footnote{It’s crucial to keep in mind that here, and elsewhere in the paper, changes in $\lambda^i$ are accompanied by changes in $\lambda^j$.} The second term (also multiplied by $\mu^i$) is negative
because the resource reallocation affects country \( i \)'s terms of trade adversely. As explained in Lemma (A.2) in the Appendix, at constant prices, the expansion (contraction) of country \( i \)'s (j’s) resource base reduces the relative supply of \( i \)'s (j’s) importable (exportable), forcing the world market clearing price \( \pi^i_T \) to rise; hence the deterioration in \( i \)'s terms of trade. We describe the salient features of \( \omega^i \) in Lemma (A.4) of the Appendix. In particular, we prove that \( \omega^i \) is strictly concave in \( \lambda^i \) and that it attains a maximum at \( \lambda^i_T \in (\frac{1}{2}, 1) \).^{28}

Equipped with an understanding of the traits of \( \omega^i \), we go on to characterize the dependence of \( \Psi^i \) on \( \lambda^i \) in Lemma (A.8) of the Appendix. In that lemma we prove, among other things, that \( \Psi^i \) is strictly quasiconcave (resp., quasiconvex) in \( \lambda^i \) for \( \lambda^i \in [\frac{1}{2}, 1] \) (resp., \( \lambda^i \in [0, \frac{1}{2}] \), and that it attains a unique maximum (resp., minimum) at \( \lambda^i_{\text{max}} \) (resp., \( \lambda^i_{\text{min}} \)). In particular, \( \lambda^i_{\text{max}} \in (\frac{1}{2}, 1) \) for \( \sigma - \tau < 1 \), whereas \( \lambda^i_{\text{max}} = 1 \) for \( \sigma - \tau \geq 1 \). We thus to arrive at

\textbf{Lemma 1} Under the split-the-surplus sharing rule, there exist unique divisions \( \lambda^i \) \( \Psi^i \big|_{\phi^i=0} = 0 \) and \( \lambda^i = \{\lambda^i \mid \Psi^i \big|_{\phi^i=0} = 0 \} \) of the contested pool \( \lambda^i \) and \( \Psi^i \) with \( \lambda^i_{\text{max}} = 1 \) only if \( \sigma - \tau \geq 1 \) and \( \beta = 1 \).

Furthermore, for any \( G^i \in [0, R^i] \) and \( G^j \in [0, R^j] \), there exists a unique division \( \lambda^i \) \( \Psi^i \big|_{\phi^i=0} = 0 \) with the following traits:

\( a \) \( \lambda^i \in [\frac{1}{2}, \lambda^i_{\text{max}}] \), with \( \lambda^i = \lambda^i_{\text{max}} = 1 \) only if \( \sigma - \tau \geq 1 \) and \( \beta = 1 \).

\( b \) \( \lambda^i \in [\lambda^i, \lambda^i_{\text{max}}] \) and \( \lambda^i \big( G^i, G^j \big) = \lambda^i \big( G^i, G^j \big) \) for \( i \neq j = 1, 2 \)

\( c \) \( \lambda^i_{G^i} = -\frac{\Psi^i_{G^i}}{\Psi^i_{\lambda^i}} = \frac{2\beta \phi^i_{G^i}}{\omega^i_{\lambda^i} + \omega^i_{\lambda^j}} > 0 \) and \( \lambda^j_{G^j} = -\frac{\Psi^j_{G^j}}{\Psi^j_{\lambda^j}} = \frac{2\beta \phi^j_{G^j}}{\omega^j_{\lambda^i} + \omega^j_{\lambda^j}} < 0 \)

\( d \) \( G^i \geq G^j \implies \lambda^i \geq \lambda^j \).

Lemma 1 points out that, depending on the value of \( \lambda^i \), the \( \omega^i - \omega^j \) component of \( \Psi^i \) may rise above \( \beta \). But Lemma 1 also points out that the existence of a range of \( \lambda^i \) values under which \( \omega^i - \omega^j \) (and thus \( \Psi^i \)) is increasing in \( \lambda^i \). Part \( a \) shows that \( \Psi^i_{\lambda^i} > 0 \) for all \( \lambda^i \in [0, 1] \) if the degree of heterogeneity in the traded intermediates relative to the size of trade costs is sufficiently high (i.e., if \( \sigma - \tau \geq 1 \)). However, if \( \sigma - \tau < 1 \), then \( \omega^i - \omega^j > 1 \) (and thus \( \Psi^i > 0 \)) for a range of uneven \( \lambda^i \) divisions, which are inconsistent with (13) and thus screened out. Parts \( a \) and \( b \) emphasize that this boils down to limiting the set of admissible divisions, so that \( [\lambda^i, \lambda^i_{\text{max}}] \subset [0, 1] \) when \( \sigma - \tau < 1 \). In other words, when the gains from trade are sufficiently high, the split-the-surplus protocol awards a positive fraction of the common pool to all agents, even when one of them does not arm!^{29} This contrasts

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^28 Among other things, in Lemma (A.4) we also note that \( \omega^i \) and \( \omega^j \) are symmetric functions and describe the dependence of \( \lambda^i \) on the elasticity of substitution \( \sigma \) and trade cost \( \tau \).

^29 We explore the dependence of \( \lambda^i_{\text{max}} \) in \( \sigma, \tau \) and \( \beta \) in Lemma (A.7) in the Appendix. As will become clear later, this relationship helps identify situations under which no arming under settlement may arise as a perfectly coalition-proof equilibrium.
sharply with conflict where the winner appropriates all of $\tilde{X}$ under (6) with certainty when its rival does not arm. Furthermore, the smaller the value of $\tilde{X}^i$ the smaller the potential for any country’s arming decision to influence the division of the resource to its favor. While this does not constitute a complete description of countries’ incentives to arm under settlement it, nonetheless, reveals one of the ways that settlement (or, more specifically, the "split-the-surplus" solution) shapes the international distribution of $\tilde{X}$. The key point is that the larger the gains from trade, the more settlement tilts the balance towards more even allocations of the contested resource which in turn amplifies the world gains from trade.\footnote{The world gains from trade, per unit of $\tilde{X}$ ($>0$), can be described by $\Omega \equiv S/\tilde{X} = \omega^i + \omega^j - \beta$. How does $\Omega$ depend on the division $\lambda^i$? One can show that $\Omega$ is strictly concave in $\lambda^i$, attains a maximum at $\lambda_{\text{max}}^i = \frac{1}{2}$, and $\lim_{\lambda_{\text{max}}^i \to 0} \Omega = \lim_{\lambda_{\text{max}}^i \to 1} \Omega = 1 - \beta$. The properties of $\Omega$ are fully described in Lemma (A.7). There we also show that $\Omega$ rises with increases in the degree of input heterogeneity ($\sigma^i$), trade liberalization ($\tau^i$), and the severity of conflict ($\beta^i$). The former two parametric changes expand $\Omega$ because they enhance the gains from trade through $\omega^i$ and $\omega^j$. A fall $\beta$ raises $\Omega$ because it causes the rate of destruction under conflict to fall.}

Parts (c) and (d) provide detailed descriptions of the dependence of $\lambda^i (\cdot)$ on agents’ arming choices which help determine countries’ arming decisions. We now apply these insights to explore equilibria with positive arming. Differentiation of (12) gives

$$v_G^i = \omega^i \lambda_{G^i} \tilde{X} - \omega^i = \frac{\omega^i \lambda_{G^i}}{\omega^i + \omega^j} 2 \beta \phi_G^i \tilde{X} - \omega^i,$$

where $\lambda^i \in [\lambda^i, \tilde{X}]$ and $\lambda_{G^i} = 2 \beta \phi_G^i / (\omega^i + \omega^j) > 0$. The first term in the RHS of (16) captures country $i$’s marginal benefit ($MB^i$) to arming. The first term ($\omega^i \lambda_{G^i}$) captures the joint effect of an extra unit of the intermediate input the country can produce (by acquiring more of the common pool) on its income and terms of trade. The product of the second and third terms ($\lambda_{G^i} \tilde{X}$) capture the increase in its intermediate input due to the production of one more gun. Clearly, a country will have no interest in producing an extra gun if $\omega^i \lambda_{G^i} < 0$ (i.e., if the deterioration in its terms of trade overwhelmed the beneficial effect due to the increase in its income) as that would render its $MB^i$ negative. The last term in the RHS of (16) is country $i$’s marginal/opportunity cost ($MC^i$) to arming. This cost is the payoff reduction country $i$ suffers due the reduction in the size of common pool $\tilde{X}$. Since $MC^i \equiv \omega^i \equiv \mu^i \lambda^i$, the opportunity cost to arming varies in proportion to the country’s relative gains from trade and its share of the common pool.

Further inspection of $MB^i$ in (16) reveals that it is increasing in the size of $\tilde{X}$ and thus decreasing in $G$. Moreover, provided $G^j > 0$, $\lim_{G^j \to 0} MB^i = \infty$ because $\lim_{G^j \to 0} \phi_G^i = \infty$. Thus every country will have an interest in producing a positive quantity of guns in order to influence the division of the prize. How far will it wish to expand its production of
guns? The answer hinges on the sensitivity of \( MB^i \) and \( MC^i \) to changes in guns \( G^i \) and the international distribution of factor ownership.

Keeping in mind that, depending on relative size, a country’s initial resource endowment \( R^i \) may constrain its production of guns, it is useful to differentiate between "unconstrained" and "constrained" arming choices. We do this by placing a tilde "\( \tilde{\} \)" over unconstrained variables and functions under settlement (e.g., \( \tilde{v}^i \) is country \( i \)'s unconstrained per period payoff).

Lemma 2 below unveils several useful properties of \( \tilde{v}^i \).

**Lemma 2** Country \( i \)'s unconstrained per period payoff function \( \tilde{v}^i \) has the following properties for \( G^i \geq G^j > 0 \) (i \( \neq j \), 1, 2):

- (a) \( \partial^2 \tilde{v}^i/\partial (G^i)^2 < 0 \)
- (b) \( \partial^2 \tilde{v}^i/\partial G^i \partial G^j > 0 \)
- (c) \( \partial \tilde{B}^i_s/\partial G^j |_{\tilde{B}^i_s = G^j} < 1 \).

Part (a) points out that \( \tilde{v}^i \) is strictly concave in \( G^i \). This trait of \( \tilde{v}^i \) helps establish existence of equilibria. Part (b) states that \( G^i \) is a strategic complement for \( G^j \). As we will see shortly, strategic complementarity together with the fact that the slope of an unconstrained country’s best-response function detailed in part (c) is less than 1, ensures uniqueness of unconstrained equilibrium.

To see how the above ideas matter for the nature of equilibrium, let us arbitrarily assume that the resource constraint on guns for every country \( i = 1, 2 \) is inactive. Further, let us also assume (again arbitrarily) that \( \lambda^i = \lambda^j = \frac{1}{2} \), which implies \( \pi^i_\tau = 1, \gamma^i_j = \gamma^i_i \leq \frac{1}{2} \) (with equality if \( \tau = 1 \)), \( \mu^i = \mu \equiv [1 + \tau^{1-\sigma}]^{-\frac{1}{\sigma}}, \omega^i = \mu/2 \) and \( \omega^i_{\lambda^i} = \omega^i_{\lambda^j} \) for \( i \neq j = 1, 2 \).

Utilizing these ideas in (16), simplifying the resulting expression and setting it equal to 0 yields

\[
\tilde{v}^i_{G^i} = \beta \left( X \phi^i_{G^i} - \frac{m}{2} \right) = 0, \text{ where } m \equiv \mu/\beta \geq 1 \text{ for } i = 1, 2.
\]

It can now be seen that the symmetric structure of \( \phi^i \) implies \( G^i = G^j = \bar{G}/2 \) where, as a consequence, \( \phi^i_{G^i} = 1/(2G) \). Substituting this into the above equation, utilizing the fact that \( X = \bar{R} - \bar{G} \) and solving for the symmetric equilibrium gives \( \bar{G}^i_s = \frac{R^i/2}{1 + m} \) for \( i = 1, 2 \), which is independent of the shadow of the future \( \delta \). However, \( \bar{G}^i_s \) is decreasing in the gains from trade \( \mu \) and in the rate of destruction (jointly captured by \( m = \mu/\beta \)).

Let us now reflect on our first assumption (that every country’s resource constraint is inactive) while maintaining the assumption that \( \lambda^i = \lambda^j = \frac{1}{2} \). Because \( m \geq 1 \), we will
have \( G^i_s \leq \bar{R}/4 \); thus, if the allocation of factor ownership were symmetric (i.e., \( R^i = \bar{R}/2, i = 1, 2 \)) then neither country’s arming decision would be constrained by its initial resource endowment. By continuity, the presence of slack in every country \( i \)’s resource constraint under symmetry suggests that \( G^i_s \) will qualify as a solution even for some uneven distributions of factor ownership. Indeed, each country \( i \)’s resource constraint would not constrain its arming decision if

\[
R^i_L < R^i < R^i_H, \quad i = 1, 2,
\]

where

\[
R^i_L \equiv \left(1 - \frac{m}{1+m}\right) \bar{R}/2 \quad \text{and} \quad R^i_H \equiv \left(1 + \frac{m}{1+m}\right) \bar{R}/2.
\]

This brings us full circle back to our second assumption that \( \lambda^i = \frac{1}{2} \) for \( i = 1, 2 \). In Proposition 2 below we argue that, indeed, \( \lambda^i = \frac{1}{2} \) is what we would actually observe in an unconstrained equilibrium under settlement if \( R^i \in (R^i_L, R^i_H) \).

Is it possible for settlement to yield \( G^1_s = G^2_s \)? Proposition 2 suggests that this can happen for some parameter values.

**Proposition 3** (Arming) Under settlement, there exists a unique equilibrium with positive investments arming \( (G^1_s, G^2_s) \) such that \( G^i_s \leq \bar{R} \), with equality for at most one country. For \( i \neq j = 1, 2 \), \( m \equiv \mu/\beta \), and \( R^i + R^j = \bar{R} \), the equilibrium guns and shares under positive arming can be described as follows:

(a) (i) If \( R^i \in (0, R^i_L) \), then \( G^i_s = R^i \), \( G^j_s = \bar{B}^i_j (R^i, \cdot) \) and \( \lambda^i_s = (\bar{\lambda}^i, \frac{1}{2}) \).

(ii) If \( R^i \in [R^i_L, R^i_H] \), then \( G^i_s = R^i_L (m) \) and \( \lambda^i_s = \frac{1}{2} \).

(iii) If \( R^i \in (R^i_H, \bar{R}) \), then \( G^i_s = \bar{B}^i_j (R^j, \cdot) \), \( G^j_s = \bar{R} \) and \( \lambda^i_s = (\frac{1}{2}, \bar{\lambda}^i) \).

(b) \( d (R^i_H - R^i_L) / dm > 0 \).

In the absence of destruction \( (\beta = 1) \), there exists \((\sigma, \tau)\) pairs with positive gains from trade such that \( G^1_s = G^2_s = 0 \).

Settlement gives rise to a unique equilibrium in arming with positive quantities of guns by both contenders. If the international structure of asset ownership is sufficiently even, the resource constraint on arming in period \( t = 1 \) turns out to be inactive for both countries and their arms are equalized. On the other hand, if asset ownership is sufficiently uneven, the less affluent economy specializes completely in the production of arms in period \( t = 1 \) whereas its more affluent adversary diversifies its production and arms relatively more.

The finding that adversaries that are initially endowed with uneven factor endowments may end up producing identical quantities of arms in part (b) may appear surprising. Nevertheless, the logic behind it is fairly simple. Provided both countries’ FOCs under settlement
hold with equality (so that every country’s resource constraint on arming is inactive), the identity of countries turns out to be inconsequential because their respective marginal benefits and marginal costs are symmetric in terms of the shares of the resource they obtain. Thus, in this specification of the model, uneven factor ownership matters only when a country’s resource constraint limits its production of guns.\footnote{Notice that the resource constraint can be binding for at most one country. Also notice that the shadow of the future (\(\delta\)) does not affect arming.}

The arming equilibrium under settlement can be visualized with the help of the green curves in Fig. 3. (Ignore the other curves for now.\footnote{Henceforth, the color magenta to identify variables related to conflict, green to identify variables related to settlement and blue to identify variables related to unilateral deviations from settlement.}) As in the case of conflict, the initial distribution of factor ownership is captured by points on the straight line connecting points \(\bar{R}\) on the two axes. Allocations along this line that render \(G_1 = R^1\) (resp., \(R^2\)) sufficiently close to 0 imply \(G_1 = \tilde{B}_s^2 (R^1)\) (resp., \(G_2 = R^2\) and \(G_1 = \tilde{B}_s^1 (R^2)\). Allocations in segment \(D_1 D_2\) induce an equilibrium at \(D\). Starting in the neighborhood of \(R^1 = 0\), endowment reallocations that expand \(R^1\) (reduce \(R^2\)): (i) induce the equilibrium to move along \(\tilde{B}_s^2 (R^1)\) for \(R^1 \in (0, R^1_s)\) until point \(D\) is reached; (ii) do not affect the equilibrium at \(D\) for reallocations in segment \(D_1 D_2\) (or, equivalently, for \(R^1 \in [R^1_s, R^1_H]\); and (iii) reallocations along segment \(D_1 \bar{R}\) toward \(\bar{R}\) on the horizontal axis cause the equilibrium to move along \(\tilde{B}_s^1(R^2)\) in the direction of origin \(O\). Thus, while reallocations in \(D_1 \bar{R}\) toward the horizontal axis do not affect arming decisions, reallocations outside \(D_1 D_2\) toward \(\bar{R}\) on either axis induce both countries to produce less arms, thus reducing global \(G_s\). Importantly, due to the endogeneity of the gains from trade, both countries’ arming decisions are strategic complements all the way up to, including point \(D\). Fig. 4 (a) provides an alternative view of the dependence of a country’s arming on the distribution of asset ownership and the discount factor.

How the division of the contested resource responds to the endowment redistributions described above can be understood from our discussion of the determination of \(\lambda^{i}_{G_i}\) and \(\lambda^{j}_{G_j}\) in Lemma 2. What deserves some emphasis here is that, in designing its security policy, a country is aware (and mindful) of the impact of its arming on its gains from trade. For a relatively affluent economy whose arming decision is unconstrained by its initial resource endowment, embracing an equitable division of the common pool—through less intense arming—may be an appealing option because that promises to generate higher gains from trade. (This is especially true when \(\sigma\) or \(\tau\) are low.)

It is also worth thinking about Proposition 3 in the context of Lemma 1 where we demonstrated that, for any given (and feasible) guns, a social planner would choose to distribute \(\bar{X}\) equally between the two contenders. But that’s exactly what happens when both countries’ resource constraints on arming are inactive. The key difference is that, in the
noncooperative setting considered here, arming is endogenously determined and, as already emphasized, the quantity of guns produced is inversely to countries' gains from trade and the potential destructiveness of conflict. A second difference is that uneven distributions of asset ownership matter.

Let us now consider the possibility of an equilibrium in which \( G^1_s = G^2_s = 0 \), which Proposition 2 claims is a real possibility under settlement. This arming configuration will be consistent with equilibrium if no country can improve its payoff by producing an infinitesimal quantity of guns. To see this possibility, let us initially abstract from trade costs and destruction \((\gamma = 1, \beta = 1)\). We will argue that the condition for no arming to, indeed, be an equilibrium under settlement is that \( \sigma \in (1, 2) \). From part (a.ii) of Lemma (A.4), we know that both countries prefer an even allocation of the common pool to a division that arbitrarily allocates all of \( \bar{X} \) to one country, say, country 1, for all \( \sigma \in (1, 2) \). Focusing on \( \sigma \in (1, 2) \), now suppose that country 1 produces a positive but infinitesimally small quantity of guns, so that \( \phi^1 = 1 \). Since, by Lemma (1.ii), \( \lambda^1 = \bar{X}^1 < 1 \), the split-the-surplus solution delivers positive quantities of the contested pool to both countries, production and international exchange of intermediates, though limited, dominates in payoffs the allocation associated with \( \lambda^1 = 1 \). This tends to erode the relative appeal of \( \lambda^1 \). In fact, as \( \sigma \) falls toward \( 1 \), country 1 moves closer to its optimum allocation \( \lambda^1 \), which, of course, is preferable over \( \lambda^1 = 1/2 \). Eventually, when \( \sigma \) falls below \( 1 \), country 1’s payoff at \( \lambda^1 = 1/2 \) is rendered unappealing as compared to the division at \( \lambda^1 = \bar{X} < 1 \).

Let us now take a closer look at equilibrium payoffs under settlement. Focusing on the payoffs that would arise if peace and settlement arose in both periods, and as noted earlier, a country’s average discounted payoff \( V^i_s \) would coincide with its per period (stationary) payoff \( v^i_s \). The following proposition summarizes our salient findings in this context.

**Proposition 4 (Payoffs)** For any given \( \beta \in (0, 1] \), in equilibria with positive arming \((G^i_s > 0, i = 1, 2)\), a country’s average payoff under settlement \( V^i_s \) is independent of the shadow of future \( \delta \) and depends on factor ownership, the elasticity of substitution \( \sigma \) and trade costs \( \tau \) as follows:

(a) (i) If \( R^i \in (0, R^i_L) \), then \( \lim_{R^i \to R^i_L} dV^i_s/dR^i \leq 0 \) and \( \lim_{R^i \to 0} dV^i_s/dR^i > 0 \).

(ii) If \( R^i \in [R^i_L, R^i_H] \), then \( V^i_s = \frac{\beta m^2}{1+m} \bar{R}/2 \).

(iii) If \( R^i \in (R^i_H, \bar{R}) \), then \( V^i_s \) is increasing in \( R^i \).

(b) If \( \beta < 1 \) or if \( \sigma < \infty \) and \( \tau < \infty \), then \( V^i_s > \bar{R} \) (resp., \( V^i_s > 0 \)) for \( R^i \) close to \( \bar{R} \) (resp., 0).

(c) \( dV^i_s/d(-\xi) > 0 \) for \( \xi \in \{\sigma, \tau\} \).

If, as noted in Proposition 3, the equilibrium involves no arming (i.e., \( G^i_s = 0 \)), then \( V^i = \mu \bar{R}/2 \).
The green curves in Fig. 4(b) describe a country’s payoffs for alternative distributions of asset ownership and gains from trade. (Ignore the other curves for now.)

### 3.3 Settlement vs Conflict

In this section we examine how arming and payoffs differ across settlement and conflict. Arms and the division of the contested pool under these two regimes compare as follows:

**Proposition 5** (Arming and Resource Shares) Suppose \( \delta \to 0, \mu \to 1 \) and \( \beta \to 1 \). Then, in period \( t = 1 \), both countries’ will arm as much under conflict as they would under settlement (i.e., \( G^c_i = G^s_i \) for \( i = 1, 2 \)) and \([R^e_L, R^e_H] \to [R^s_L, R^s_H] \). However, if \( \delta > 0 \) or \( \mu > 1 \) or \( \beta < 1 \), then \( G^c_i \geq G^s_i \), with strict inequality for at least one country, and

1. the resource constraint on arms will bind for a larger set of factor allocations under conflict than under settlement (i.e., \([R^c_L, R^c_H] \subseteq [R^s_L, R^s_H] \));
2. if \( R^c_i \notin [R^c_L, R^c_H] \) and \( R^c_i \leq \bar{R}/2 \), then \( \lambda^i \geq \phi^i \) for \( i = 1, 2 \).

The proposition can be visualized with the help of Fig. 3, which illustrates how arming under conflict and settlement differ when \( \delta = 1 \). Fig. 4(a), which depicts a country’s arming decisions as a function of the distribution of factor ownership, also helps deepen one’s understanding of these points. A noteworthy implication of part (a) is that settlement leads to the equalization of guns for a wider range of endowment distributions than conflict. This suggests that \( \lambda^i \geq \phi^i \) for all \( R^i \in [R_L^e, \bar{R}/2] \), which implies that settlement ensures the less affluent side receives a larger share of the common pool as compared to conflict.

But, of course, arming is a recurrent event under settlement but not under conflict. Thus the comparison of payoffs under these two regimes is more subtle than might appear on first sight.

Since countries are allowed to communicate freely, they should be expected to adhere to that mode of conflict resolution that advances their mutual and self interests. While contracts on arming are impossible, division of the common pool is. Thus countries incentives to deviate from conflict as a group boils down to a comparison of payoffs under conflict and settlement.

Two points ought to be kept in mind when comparing average discounted payoffs. First, \( U^i_c \) is a function of \( \delta \), but \( V^i_s \) is not. Second, \( V^i_s \) is increasing in country \( i \)'s gains from trade (associated with a fall in \( \sigma \) or a fall in \( \tau \) which are subsumed in \( \mu \)) whereas \( U^i_c \) is independent of these gains. The properties of \( U^i_c \) were described in Proposition 2 (and Lemma A.4 in the Appendix). The properties of \( V^i_s \) and its dependence on the gains from trade were described in Proposition 4.

Suppose \( R^i \in [R^e_L, R^e_H] \), so that \( G^s_i = R^s_L(m) \) and \( V^i_s = \frac{\beta m^2}{1+m} \bar{R}/2 \) for \( i = 1, 2 \) (Propositions 3 and 4). Recall from Proposition 3 that \( d(R^e_H - R^e_L)/dm > 0 \) and from Proposition
4 that $V_s^i$ is increasing in $m$. Moreover, from Propositions 1 and 2 we have $G^i_c = R^i_J (\delta)$ and $U^i_c = \beta \frac{\delta}{4}$ for $R^i \in [R^i_L, R^i_H]$, where $\frac{d (R^i_J - R^i_L)}{d \delta} < 0$. (Thus, $U^i_c$ is invariant to changes in $\delta$ for all $R^i \in [R^i_L, R^i_H]$.) But, from the definitions of $R^c_J$ and $R^c_J$ for $J \in \{L, H\}$, one can see that \[ \lim_{m \to 0} R^c_J (m) = \lim_{\delta \to 0} R^c_J (\delta) = \bar{R}/4. \] Moreover, \[ \lim_{m \to 0} V_s^i = U^i_c = \beta \frac{\delta}{4} \] for any $R^i \in [R^i_L, R^i_H]$ in this case, while $[R^i_L, R^i_H] \cap [R^i_L, R^i_H]$ for any $\delta \in (0, 1]$ and $m > 1$.

Now suppose $\beta = 1$ and $\mu = 1$ initially (i.e., for now, there are no gains from trade). Proposition 2 implies that, in this case, $V_s^i = U^i_c = \beta \frac{\delta}{4}$ for all $R^i \in [R^i_L, R^i_H]$ (as noted above), whereas $V_s^i < U^i_c$ for $R^i \in (R^i_H, 1)$ ($i = 1, 2$). This suggests that the two countries would be indifferent between peace and war for $R^i \in [R^i_L, R^i_H]$ and that one one of them would choose war for $R^i \notin [R^i_L, R^i_H]$.

Next, consider the (benchmark) situation where, in addition to assuming $\beta = 1$ and $\mu = 1$, the players do not value future payoffs (i.e., $\delta = 0$). It should be clear that, in this case, conflict and settlement are indistinguishable regimes of conflict resolution, so $U^i_c = V_s^i$ for all $R^i \in (0, \bar{R})$ as indicated by the solid green curve in Fig. 4(b), which captures these payoffs.\(^{34}\) Continuing with this hypothetical situation, allow $\delta$ to rise. $V_s^i$ remains unaffected. Fig. 2(b) shows that $U^i_c$ will shift upwards with increases in $\delta$. For comparison purposes, $U^i_c$ ($i = 1, 2$) is illustrated in Fig. 4(b) for $\delta = 1$.

Next, let $\mu$ rise – perhaps because of a reduction in $\tau$ ($\sigma$) while $\sigma$ ($\tau$) is kept fixed in the background at some finite level. By Proposition 4, this would cause $V_s^i$ to shift upward, as indicated by the green curves in Fig. 4(b). As this happens, $V_s^i > U^i_c$ for sufficiently even and for sufficiently uneven international distributions of asset ownership. However, as $\mu$ rises beyond a certain threshold level (associated with points $C$ and $C'$ in Fig. 4(b)), settlement will dominate conflict for all possible $R^i \in R^i \in (0, \bar{R})$.

Proposition 6 below provides a detailed comparison of the payoffs associated with peace and war.

**Proposition 6** For any given $\delta \in (0, 1]$, there exists a threshold level of destruction $\beta_0 = \beta_0 (\delta) \in (0, 1)$ (where $\partial \beta_0 / \partial \delta < 0$) and a gains-from-trade threshold level

\[
\mu_0 = \mu_0 (\delta, \beta) = \begin{cases} 
1 & \text{if } \beta \leq \beta_0 \\
\tilde{\mu} (\delta, \beta) & \text{if } \beta > \beta_0 
\end{cases}
\]

(17)

where $\tilde{\mu} > 1$ (with $\tilde{\mu}_\delta > 0$ and $\tilde{\mu}_\beta > 0$), such that settlement dominates conflict (i.e., $V_s^i > U_c^i$ for $i = 1, 2$) under the following circumstances:

(a) if $\beta \in (0, \beta_0)$, then for any $R^i \in (0, \bar{R})$;

\(^{34}\)The case of $\delta = 0$ also informs us about how payoffs under conflict and settlement would compare in period $t = 2$. If $\mu > 1$ while $\beta = 1$, or if $\beta < 1$, $V_s^i > U_c^i$ because of gains from trade or the avoidance of destruction and because there is less arming (i.e., $G^i_c \leq G^i_c$ for $i = 1, 2$) under settlement. In other words, in the second period, settlement (almost) always dominates conflict.
(b) if $\beta \in (\beta_0, 1]$ and

(i) $\mu > \mu_0$, then for any $R^i \in (0, \bar{R})$

(ii) $\mu < \mu_0$, then only for sufficiently even and sufficiently uneven international allocations of asset ownership.

What deserves some emphasis here is that, when conflict is not very destructive and the gains from trade are moderate, settlement dominates conflict under two distinct types of situations, nicely illustrated in Fig. 4 (b): (i) when the international distribution of asset ownership is sufficiently even; and (ii) when this distribution is sufficiently uneven.

4 Unilateral Deviations

For settlement to arise as a perfectly coalition-proof equilibrium, settlement must be immune not just to coalitional deviations but also immune to unilateral deviations from it. Suppose the two sides expect settlement will arise down the road and as a consequence produce $(G^1_s, G^2_s)$ guns. Now consider the possible deviations that any country $i$ might undertake if it deviated from settlement in period $t = 1$. Two types of deviations are relevant here: First, for given $(G^i_s, G^j_s)$, a country may choose to declare "war" if that results in a larger payoff than would be the case under settlement. Second, if possible, a country may have an interest in changing its guns under the expectation that later on it will opt for war. Clearly, the first possibility is relevant when a country’s arming decision is limited by its resource endowment. The second possibility arises when the country’s resource constraint on its arming is not active. Because conflict will inevitably break out when a country violates settlement, under both situations, country $i$’s optimal arming under a unilateral deviation will be given by its best reply to $G^j_s (j \neq i = 1, 2)$ described in (10); that is, $B^i_c(G^j_s; \delta, R^i, \bar{R})$.

We can describe the salient features of this best reply with the help of Fig. 3 in the special case of $\delta = 1$. (The consideration of other values of $\delta$ is straightforward in this context.) Recall that the green curves in this figure describe the equilibrium under settlement for all possible allocations of asset ownership. The blue curve in Fig. 3 describes country 1’s payoff maximizing arming, when it deviates from settlement, for alternative distributions of asset ownership. The arrows describe the direction of change as $R^i$ rises along $(\bar{R}, \bar{R})$ in response to arbitrary redistributions of the asset. Inspection of this figure reveals that four intervals of asset allocations are relevant here in terms of the way country $i$’s arming $G^i_d ("d"$ for "deviation") is determined: (i) $(0, R^i_L)$; (ii) $(R^i_L, R^i_d)$; (iii) $(R^i_d, R^i_H)$; and (iv) $(R^i_H, \bar{R})$.

35It should be easy for one to see that settlement is not only immune to coalitional deviations in period $t = 2$ but also to unilateral deviations from it. The reason for this is that every country’s payoff under an optimal deviation when $\delta = 0$, coincides which its payoff under conflict. (For additional details, see the analysis that follows.)
In cases (i) and (ii), country $i$’s best reply is constrained by its initial resource endowment. In case (i), country $i$’s best reply coincides with its arming under settlement. It is under this case that it’s optimal deviation may entail a declaration of war even though no change in arming is recorded. In case (ii), country $i$’s optimal deviation entails producing a larger quantity of guns as compared to settlement until $R^i = R^i_L (< \bar{R}/2)$, where $G^i_d = \tilde{B}_c^i(G^i_s)$. For deviations beyond this allocation of resources, and into cases (iii) and (iv), country $i$’s best reply coincides with its best reply under conflict. In case (iii), country $i$ maintains its output of guns at $\tilde{B}_c^i(G^i_s)$, where its rival’s output $G^j_s$ remains at the unconstrained equilibrium under settlement. Importantly, for $R^i \in (R^i_L, R^i_H)$ both countries’ optimal deviations in arming entail $\tilde{B}_c^i(G^i_s) = \tilde{B}_c^j(G^j_s)$ with $G^i_s = G^j_s$. In case (iv), country $j$’s arming is constrained by its arming while country $i$ operates along its best-response function under conflict.

We illustrate country 1’s best replies described above with the blue curves in Fig. 5(a) in the special case of $\delta = 1$, $\beta = 1$ and $\mu = \mu_0$. For additional clarity and comparability, the figure also describes the quantities of guns produced under settlement (green curves) and conflict (pink curves). The thinner dashed-line curves describe the optimal quantities of guns by country 2.

Let $W_d^i \equiv U^i(G^i_d, G^j_s; \cdot)$ be the payoff to country $i$ under the optimal deviations described above, including the case where $G^i_d = G^j_s$ and the deviation involves a declaration of war with no adjustment in guns relative to settlement. The payoffs to both countries associated with the case described in the previous paragraph are depicted in Fig. 5(b). Notice, in particular, that in case (iii) (where $R^i \in (R^i_L, R^i_H)$), we have $W_d^i = V_s^i$ for $i = 1, 2$. (This is so because $\mu = \mu_0$ implies $V_s^i = U^i_{\mid R^i = R^i_H}$ (by the definition of $\mu_0$) and $W_d^i = U^i_{\mid R^i = R^i_H}$ for all $R^i \in [R^i_L, R^i_H]$.) This suggests that, while settlement dominates conflict for at least one country for allocations in $(R^i_L, R^i_H)$, the payoffs under settlement and an optimal deviation from it do not differ. As we will see in Proposition 7 below the key point is that the stability of settlement is threatened by unilateral deviations under such allocations. In case (iv) (where $R^i \in (R^i_H, \bar{R})$) we have $W_d^i = U^i_{\cdot}$ for $i = 1, 2$. Thus, in this case, the payoff to a country that deviates unilaterally coincides with its payoff under a coalitional deviation.

We may now describe the vulnerability of settlement to unilateral deviations as follows:

**Proposition 7** For any given $\delta \in (0, 1]$, settlement is immune to unilateral deviations (i.e., $V_s^i > W_d^i$ for $i = 1, 2$) under the following circumstances:

(a) if $\beta \in (0, \beta_0)$, then for any $R^i \in (0, \bar{R})$;

(b) if $\beta \in (\beta_0, 1]$ and

(i) $\mu > \mu_0$, then for any $R^i \in (0, \bar{R})$

(ii) $\mu < \mu_0$, then only for sufficiently uneven international allocations of asset ownership.
In the context of Fig. 5 (b), settlement is immune to unilateral deviations only for asset allocations in \((0, R_L)\) and in \((R_H^r, R)\). It should not be difficult for one to see that the main insight in part (b.ii) (i.e., that settlement is immune to unilateral deviations only for sufficiently asymmetric asset allocations) holds true for any \(\delta \in (0, 1]\). Importantly, under the conditions described in part (b), settlement is undermined by at least one country’s eagerness to deviate unilaterally from it when the international distribution of asset ownership is sufficiently even. Keep in mind that unilateral deviations generate a payoff for the deviating country that is at least as high as the payoff it would obtain if both countries deviated from settlement as a coalition.

5 The Stability of Armed Peace: How Trade Matters

Having identified the conditions under which countries will have an incentive to deviate from settlement as a coalition or unilaterally, we may now summarize the principal insights of our analysis as follows:

**Proposition 8 (Stability of Peace)** Peace is stable under

(a) **all** possible international allocations of asset ownership if the destructiveness of conflict is sufficiently high and/or the gains from trade are sufficiently high;

(b) sufficiently **uneven** international allocations of asset ownership if the destructiveness of conflict and the gains from trade are sufficiently low.

(c) The larger the degree of dissimilarity between traded commodities \((\sigma \downarrow)\) or the lower trade costs \((\tau \downarrow)\), the more likely that peace will prevail as a stable equilibrium.

(d) The longer the shadow of the future \((\delta \uparrow)\) or the lower the destructiveness of war, \((\beta \uparrow)\), the less likely that peace will arises as a stable equilibrium.

6 Concluding Remarks

In this paper, we study the emergence of open conflict (or war) vs. peaceful settlement (or armed peace) as an equilibrium outcome involving rational and forward looking policymakers. That is to say, the choice of conflict resolution is endogenously determined, depending on the destruction of open conflict, the strength of the shadow of the future and parameters that jointly determine the gains from trade. The analysis generates several interesting implications with regards to the effects of globalization on the volume of trade, arming and payoffs.

However, we have only touched the surface. Possible extensions include extending the model to an infinite horizon, allowing the loser of open conflict, allowing for the possibility
that the winner of the war also captures his rival’s technology or for the possibility of (colonial) trade with the defeated side, considering more than two countries and the possibility of third-party intervention, and finally endogenous destruction.

7 References


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A Appendix

Proof of Proposition 1: The proof follows from the discussion in the text.

Proof of Proposition 2: Parts (a) and (c). First note that, when both countries’ arming decisions are unconstrained by their initial resource endowments, the worldwide production of guns $G_c$ equals $2R_L^c = (1 + \frac{\delta}{2}) \frac{R}{2}$. Suppose now that $G^i_c = R^i$ (i.e., country $i$ is the "constrained" country) which arises when $R^i < R^c_L$ and implies $G^j_c > R^i$. Owing to strategic complementarity, in this case, we would have $G^i_c < 2R^c_L = (1 + \frac{\delta}{2}) \frac{R}{2}$. (This point will prove helpful shortly.)

Turning to the unconstrained country $j$, total differentiation of its FOC, $U^j_{G^i} = \frac{\beta}{1 + \delta} \left[ \phi^j_{G^i} (X + \delta R) - \phi^j \right] = 0$, gives

$$dU^j_{G^i} = U^j_{G^i} dG^i_c + U^j_{G^i} dG^j_c + U^j_{G^i} d\delta = 0,$$

(A.1)

where

$$U^j_{G^i} = \frac{\beta}{1 + \delta} \left[ \phi^j_{G^i} \phi^j / \phi^j_{G^i} - \phi^j_{G^i} \right]$$  (A.2a)

$$U^j_{G^i} = \phi^j_{G^i} \phi^j / \phi^j_{G^i} - 2\phi^j_{G^i}$$  (A.2b)

$$U^j_{G^i} = \frac{\beta}{1 + \delta} \phi^j_{G^i} R.'$$  (A.2c)

To obtain the equations in (A.2) we used $U^j_{G^i} = 0$. Utilizing the properties of the CSF together with the equations in (A.2) and the fact that $dG^i_c = dR^i$ in (A.1) yields

$$\frac{dG^j_c}{dR^i} = \frac{G^j_c - R^i}{2R^i} > 0$$  (A.3a)

$$\frac{dG^j_c}{d\delta} = \frac{R^i}{G^c} \frac{R}{2} > 0,$$  (A.3b)

where $\bar{G}^c = R^i + G^j_c$. Since payoffs depend on resource endowments through $\bar{R}$ (and not through their international allocation, i.e., $\partial U^j_{G^i} / \partial R^i = 0$), the effect of redistributing resources from country $j$ to country $i$ on average payoffs $U^j_{G^i}$ and $U^j_{G^i}$ can now be computed to be as follows:

$$\frac{dU^i_{G^i}}{dR^i} = U^i_{G^i} + U^i_{G^j} dG^j_c / dR^i

= \frac{\beta}{1 + \delta} \left\{ \left[ \phi^i_{G^i} (X + \delta R) - \phi^i \right] + \left[ \phi^i_{G^j} (X + \delta R) - \phi^i \right] \frac{dG^j_c}{dR^i} \right\}

= \frac{\beta}{1 + \delta} \left\{ \left[ \phi^j \phi^i_{G^i} / \phi^j_{G^i} - \phi^j \right] + \left[ \phi^j \phi^i_{G^j} / \phi^j_{G^j} - \phi^j \right] \frac{dG^j_c}{dR^i} \right\}$$

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\[
\frac{dU_i}{dR^i} = \frac{\beta}{1 + \delta} \left\{ \left[ \frac{G^i_c - R^i_c}{R^i} \right] - \left[ \frac{G^i_c - R^i_c}{2R^i} \right] \right\} \Rightarrow
\]

\[
1 + \frac{\beta}{1 + \delta} \left[ \frac{G^i_c - R^i_c}{2R^i} \right] > 0
\]  
\[(A.4)\]

\[
\frac{dU^j_c}{dR^j} = U^j_c + U^j_G i \frac{dG^j_c}{dR^j} + U^j_G j \frac{dG^j_c}{dR^j}
\]
\[
= \frac{\beta}{1 + \delta} \left\{ [0] + \left[ \phi^j_G (X + \delta R) - \phi^j \right] + [0] \frac{dG^j_c}{dR^j} \right\} \Rightarrow
\]

\[
\frac{dU^j_i}{dR^i} = - \frac{\beta}{1 + \delta} \left[ \frac{G^j_c}{R^i} \right] < 0.
\]  
\[(A.5)\]

To obtain the above expressions, we used the fact that \( U^j_G = 0 \), (A.3a), and the properties of the CSF. Differentiating (A.4) and (A.5) with respect to \( R^i \) and utilizing (A.3a) gives

\[
\frac{d^2U^i_c}{d(R^i)^2} = - \frac{\beta}{1 + \delta} \left[ \frac{G_c}{2(R^i)^2} \right] < 0
\]  
\[(A.6a)\]

\[
\frac{d^2U^j_i}{d(R^i)^2} = \frac{\beta}{1 + \delta} \left[ \frac{G^j_c}{2(R^i)^2} \right] > 0.
\]  
\[(A.6b)\]

Equation (A.6a) reveals that the constrained country’s average payoff \( U^i_c \) is concave in \( R^i \) (followed by equal changes in \( R^j \)). Since \( d^2U^j_i/d(R^i)^2 = d^2U^j_i/d(R^i)^2 > 0 \) by (A.6b), the unconstrained country’s equilibrium average payoff is convex in endowment. This proves the part of parts (a) and (b) that deals with endowment redistributions.

Let us now examine the payoff effects of \( \delta \).

\[
\frac{dU^i_j}{d\delta} = U^i_j + U^i_G i \frac{dG^i_c}{d\delta}
\]
\[
= \frac{\beta}{1 + \delta} \left[ \phi^i (X + \delta R) \right] + \beta \left[ \frac{\phi^i_G (X + \delta R) - \phi^i}{1 + \delta} \right] \frac{dG^i_c}{d\delta}
\]
\[
= \frac{\beta}{1 + \delta} \left[ \frac{R^i}{(1 + \delta)^2} \right] - \beta \left[ \frac{\phi^i + \phi^j}{1 + \delta} \right] \frac{dG^j_c}{d\delta}
\]
\[
= \frac{\beta}{(1 + \delta)^2} \left[ R^i - (1 + \delta) \frac{dG^j_c}{d\delta} \right].
\]

The first term on the fourth line is obtained from the fact that \( X = \bar{R} - \bar{G}_c \) and the definition of \( \phi^j \). The second term is obtained by utilizing the fact that \( \phi^j_G = -\phi^j_G \) in country \( j \)'s FOC (which requires \( \phi^j_G (X + \delta \bar{R}) = \phi^j \)). The last expression is obtained by noting that
\( \phi^i + \phi^j = 1 \) and simplifying. Utilizing (A.3b) in the expression above yields

\[
\frac{dU_i^c}{d\delta} = \frac{\beta \phi^i}{(1 + \delta)^2} \left[ G_i^c - (1 + \delta) \tilde{R} / 2 \right]
\]

(A.7)

which is negative, as noted at the beginning of the proof. Essentially, what this means is that the direct effect of \( \delta \) on \( U_i^c \), which is positive, is dominated by the indirect (and negative) effect of \( \delta \) on the unconstrained rival’s arming (which rises). Thus, under conflict, a constrained country’s average payoff necessarily falls if its rival is unconstrained. For the unconstrained country \( j \) the effect of increasing \( \delta \) on its average payoff \( U_j^c \) is positive because the direct effect is positive while the indirect effect is absent (because \( G_i^c = R^i \)).

**Part (c).** This part is a direct consequence of the fact that \( U_i^c = \beta \tilde{R}^i \) in this case. 

Let a "\( ^\wedge \)" over variables denote percent change (e.g., \( \tilde{x} \equiv dx/x \)). The definitions of expenditure shares \( \gamma^i_j = (p^i)^{1-\sigma}/[1 + (p^i)^{1-\sigma}] \) and domestic prices \( p^i = \tau \pi^i_T \) imply

\[
\tilde{\gamma}^i_j = -(\sigma - 1)\gamma^i_j(\tilde{\pi}^i_T + \tilde{\tau}) - \gamma^i_j \ln (p^i) \, d\sigma.
\]

Logarithmically differentiating (4) and simplifying the resulting expression gives

\[
\tilde{\pi}^i_T = \frac{1}{\Delta} \left\{ \tilde{Z}^i - \tilde{Z}^j + (\sigma - 1)(\gamma^i_j - \gamma^j_i)\tilde{\tau} - \left[ \gamma^i_j \ln (p^i) - \gamma^j_i \ln (p^j) \right] d\sigma \right\},
\]

(A.8)

where

\[
\Delta \equiv \varepsilon^i + \varepsilon^j - 1 > 0, \quad \varepsilon^i \equiv -\frac{\partial D^i_j/\partial p^i_j}{D^i_j/p^i_j} = 1 + (\sigma - 1) \left( 1 - \gamma^i_j \right), \quad i \neq j = 1, 2,
\]

is the Marshall-Lerner condition for stability of equilibrium. \( \Delta > 0 \) because \( \sigma > 1 \).\(^{36}\) Thus, increases in country \( i \)'s effective endowment \( Z^i \) affects its terms of trade adversely. Exactly the opposite is true for an increase in \( Z^j \). (See below for more on this.) Equation (A.8) reveals that the qualitative effect of an increase in trade cost \( \tau \) on country \( i \)'s terms of trade \( \pi^i_T \) depends on the two countries' expenditure shares of their respective importables which, in turn, depend on countries’ internal prices \( p^i \) and \( p^j \). Utilizing (A.8) and the fact that \( p^i = \tau \pi^i_T \), it is straightforward to show that \( 0 < \tilde{\pi}^i / \tilde{\tau} < 1 \) for \( i = 1, 2 \) (see (A.10b) below). Thus, trade costs inflate the relative price of a country’s importable but the pass-through is incomplete.

From (A.8) one can also see that the impact of the elasticity of substitution \( \sigma \) on \( \pi^i_T \) depends on the manner in which internal prices compare internationally. As we will see

\(^{36}\)Simplification of the above expression of \( \Delta \) allows us to rewrite it as \( \Delta = 1 + (\sigma - 1) \left( 1 - \gamma^i_j + 1 - \gamma^j_i \right) \) for \( i \neq j = 1, 2 \). In the special case of free trade (where \( \tau = 1 \)), \( \gamma^i_j + \gamma^j_i = 1 \) and thus \( \Delta = \sigma \).
shortly (Lemma (A.3)) the sign of this expression hinges on the comparison of the sizes of \( Z_i \) and \( Z_j \) or, equivalently, on the international allocation of \( \bar{X} \).

To sharpen our understanding of the manner in which the distribution of factor ownership matters for equilibrium prices, countries’ payoffs and their gains from trade, let us suppose \( Z_i = \lambda^i \bar{X} \) for \( i = 1, 2 \), where \( \lambda^i \geq 0 \) is an arbitrary division of the common pool \( \bar{X} \) (= \( R - G > 0 \)). To examine the importance of this division, let us also suppose \( R, G \) (and thus \( \bar{X} \)) remain fixed in the background. Lemma (A.1) below describes how \( \lambda^i \) affects \( \pi^i_T \) and \( \gamma^i_j \).

**Lemma A.1** For \( i \neq j = 1, 2 \), trade costs, the division \( \lambda^i \) of \( \bar{X} \) and expenditure shares are related as follows:

(a) \( \gamma^i_j + \gamma^j_i - 1 \geq 0 \) with equality if \( \tau = 1 \).

(b) If \( \lambda^i \geq \frac{1}{2} \) then \( \gamma^j_i \gamma^i_j - \gamma^i_j \gamma^i_j \leq 0 \).

**Proof:** Part (a). This part follows readily from the definition of expenditure shares which give

\[
\gamma^i_j + \gamma^j_i - 1 = \frac{1}{1 + (p^i)^{1-\sigma}} + \frac{1}{1 + (p^j)^{1-\sigma}} - 1
\]

\[
= \frac{(p^i p^j)^{\sigma-1} - 1}{[1 + (p^i)^{\sigma-1}] [1 + (p^j)^{\sigma-1}]}
\]

\[
= \frac{\tau^{2(\sigma-1)} - 1}{[1 + (p^i)^{\sigma-1}] [1 + (p^j)^{\sigma-1}]},
\]

for \( i \neq j = 1, 2 \). The part follows from the fact that \( \tau \geq 1 \).

Part (b). Utilizing the definitions of the expenditure shares gives

\[
\gamma^j_i \gamma^i_j - \gamma^i_j \gamma^i_j = \frac{(p^i)^{1-\sigma}}{[1 + (p^i)^{1-\sigma}]^2} - \frac{(p^j)^{1-\sigma}}{[1 + (p^j)^{1-\sigma}]^2}
\]

\[
= \frac{[\lambda^i p^i - (p^j)^{\sigma-1}] [(\pi^i_T)^{2(\sigma-1)} - 1]}{[1 + (p^i)^{\sigma-1}]^2 [1 + (p^j)^{\sigma-1}]^2}
\]

\[
= \frac{(p^j)^{\sigma-1} [(\pi^i_T)^{2(\sigma-1)} - 1]}{[1 + (p^i)^{\sigma-1}] [1 + (p^j)^{\sigma-1}]^2}.
\]

Noting that \( \tau \geq 1 \), the result follows by noting that \( \lambda^i \geq \frac{1}{2} \) implies \( \pi^i_T \geq 1 \). (See part (a) of Lemma (A.3) for details.)

**Lemma A.2** For any given \( \bar{X} \), its division \( \lambda^i \) affects country \( i \)'s terms of trade \( \pi^i_T \) and expenditure share \( \gamma^i_j \) as follows:

(a) \( \frac{\partial \pi^i_T}{\partial \lambda^i} > 0 \) and \( \frac{\partial^2 \pi^i_T}{\partial (\lambda^i)^2} \geq 0 \)

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(b) \( \lim_{\lambda \to 0} \pi_T^i = 0, \lim_{\lambda \to 1/2} \pi_T^i = 1 \) and \( \lim_{\lambda \to 1} \pi_T^i = \infty \).

(c) \( \lim_{\lambda \to 1/2} \gamma_j^i \leq \frac{1}{2}, \lim_{\lambda \to 1} \gamma_j^i = 0 \) and \( \lim_{\lambda \to 1} \gamma_j^i / \lambda^i = \infty \).

**Proof:** Part (a). Because the supply of country \( i \)'s intermediate input is \( Z^i = \lambda^i \hat{X} \) we have \( \hat{Z}^i - \hat{Z}^j = \lambda^i - \lambda^j = \left( \frac{1}{h} \right) ds^i \) in (A.8) for any given \( \lambda \); therefore

\[
\frac{\partial \pi_T^i}{\partial \lambda^i} = \frac{1}{\lambda^i \lambda^j \Delta} > 0,
\]

since \( \Delta > 0 \). This proves the first portion of part (a). The convexity of \( \pi_T^i \) in \( \lambda^i \) for \( \lambda^i \geq \lambda^j \) can be proven by noting that

\[
\frac{\partial^2 \pi_T^i / \partial (\lambda^i)^2}{\pi_T^i} = \frac{\lambda^i - \lambda^j}{(\lambda^i \lambda^j)^2 \Delta} + \frac{(\sigma - 1)}{\lambda^i \lambda^j \Delta^2} \left[ \pi_T^i \left( \partial \gamma_j^i / \partial \pi_T^i \right) + \pi_T^j \left( \partial \gamma_j^j / \partial \pi_T^j \right) \right] \frac{\partial \pi_T^i / \partial \lambda^i}{\pi_T^i}
\]

\[
= \frac{\lambda^i - \lambda^j}{(\lambda^i \lambda^j)^2 \Delta} + \frac{(\sigma - 1)}{\lambda^i \lambda^j \Delta^2} \left[ (\sigma - 1) \left( -\gamma_j^i \gamma_j^i + \gamma_j^j \gamma_j^j \right) \right] \frac{1}{\pi_T^i \lambda^i \lambda^j \Delta} \]

\[
= \frac{\lambda^i - \lambda^j}{(\lambda^i \lambda^j)^2 \Delta} + (\sigma - 1)^2 \left( \frac{\gamma_j^i \gamma_j^i - \gamma_j^j \gamma_j^j}{\pi_T^i \lambda^i \lambda^j \Delta^3} \right) > 0.
\]

The second part of the first equation follows from the definition of \( \Delta \). The second equation follows from the facts that \( p^i = \tau \pi_T^i, \pi_T^i = 1 / \pi_T^i \) and \( \pi_T^j \left( \partial \gamma_j^i / \partial \pi_T^j \right) = - (\sigma - 1) \gamma_j^i \gamma_j^i \) for \( i \neq j = 1, 2 \). The last inequality follows from the definitions of the expenditure shares and part (b) of Lemma (A.1) (which pointed out that \( \gamma_j^i \gamma_j^i - \gamma_j^j \gamma_j^j > 0 \) if \( \lambda^i > \lambda^j \)).

Part (b). The expenditure shares \( \gamma_j^i \) and \( \gamma_j^j \) can be rewritten as

\[
\gamma_j^i = \frac{1}{1 + \tau \sigma - 1 (\pi_T^i)^{\sigma - 1}} \quad \text{and} \quad \gamma_j^j = \frac{(\pi_T^i)^{\sigma - 1}}{(\pi_T^i)^{\sigma - 1} + \tau}.
\]

where \( \gamma_j^i = - (\sigma - 1) \left( 1 - \gamma_j^j \right) \pi_T^i \). Substituting these shares in \( \frac{\gamma_j^i}{\gamma_j^j} \pi_T^i = \frac{\lambda^i}{X} \) (which is a rearrangement of the world market clearing condition (4) in which \( Z^i = \lambda^i \hat{X} \) for \( i = 1, 2 \)) gives

\[
\left[ \frac{1 + \tau \sigma - 1 (\pi_T^i)^{\sigma - 1}}{\tau \sigma - 1 + (\pi_T^i)^{\sigma - 1}} \right] (\pi_T^i)^{\sigma} = \frac{\lambda^i}{1 - \lambda^i}.
\]

(A.9)

The remaining components of part (b) can now be proven by studying the behavior of \( \pi_T^i \) on the LHS of (A.9) as \( \lambda^i \) varies on the RHS. To do this we should keep in mind that, for any finite \( \tau \geq 1 \) and \( \sigma > 1 \), \( \tau \sigma - 1 \) in (A.9) is finitely positive. Now note that the RHS of (A.9) behaves as follows: (i) \( \lim_{\lambda \to 0} RHS = 0 \), (ii) \( \lim_{\lambda \to 1/2} RHS = 1 \), and (iii)
\[ \lim_{\lambda' \to 1} RHS = \infty. \] Clearly, the limits of the LHS much match the respective limits of the RHS in all three cases.

In case (i), the expression inside the square brackets of the LHS is finitely positive for all \( \pi^i_T \geq 0 \). Therefore, \( \lim_{\lambda' \to 0} LHS = 0 \) only if \( \lim_{\lambda' \to 0} \pi^i_T = 0 \). Similarly, in case (ii), \( \lim_{\lambda' \to 1/2} \pi^i_T = 1 \) because no other value of \( \pi^i_T \) ensures \( \lim_{\lambda' \to 1/2} LHS = 1 \). Lastly, in case (iii), \( \lim_{\lambda' \to -1} \pi^i_T = \infty \) because the expression inside the square brackets of the LHS in (A.9) is finitely positive for all \( \tau \geq 1 \) and \( \pi^i_T \geq 0 \) (including the case of \( \pi^i_T \to \infty \)).

**Part (c).** The first portion of part (c) follows readily from the definition of \( \gamma^j \) in part (b) and the fact that \( \sigma > 1 \). The second portion follows, first, by noting that \( \lim_{\lambda' \to 1} \pi^i_T = \infty \) (proven in part (b)), which implies \( \lim_{\lambda' \to -1} \left( \pi^i_T \right)^{\sigma - 1} = \infty \); and, second, by utilizing the definition of \( \gamma^j \) to obtain \( \lim_{\lambda' \to -1} \gamma^i_j = 0 \). The third component of part (c) follows by rewriting (4) as \( \frac{\gamma^j_i}{\lambda'} = \frac{\gamma^j_i}{\lambda^i} \pi^i_T \) and by noting that the limit of the RHS is

\[
\lim_{\lambda' \to -1} \left( \frac{\gamma^j_i}{\lambda'} \right) \times \lim_{\lambda' \to -1} \left( \pi^i_T \right) = \left[ \frac{\lim_{\lambda' \to -1} \left( \gamma^j_i \right)}{\lim_{\lambda' \to -1} \left( \lambda^i \right)} \right] \times \lim_{\lambda' \to -1} \left( \pi^i_T \right) = \left[ \frac{1}{1} \right] \times \infty. 
\]

This necessarily implies \( \lim_{\lambda' \to -1} LHS = \lim_{\lambda' \to -1} \left( \frac{\gamma^j_i}{\lambda^i} \right) = \infty \). Thus, the convergence of \( \gamma^i_j \) to 0 is slower than the convergence of \( \lambda^i \) to 0 as \( \lambda^i \to 0 \).

**Lemma A.3** For \( i \neq j = 1, 2 \), the division \( \lambda^i \) of \( \tilde{X} \) implies:

(a) If \( \lambda^i \gtrless \frac{1}{2} \) then \( \pi^i_T \gtrless 1 \) and \( p^j \gtrless p^j \).

(b) If \( \lambda^i \gtrless \frac{1}{2} \) then \( \gamma^i_j \gtrless \gamma^j_j \) and \( \gamma^j_j \gtrless \gamma^i_j \).

(c) If \( \lambda^i \lesssim \frac{1}{2} \) then \( d\pi^i_T/d\pi \lesssim 0 \) and \( d\pi^i_T/d\sigma \lesssim 0 \).

**Proof:** Parts (a) and (b) follow readily from parts (a) and (b) of Lemma (A.2) and the definitions of \( p^j \) and \( p^j \). Part (c) follows from part (a), Lemma (A.1) and the dependence of \( \pi^i_T \) or \( \tau \) and \( \sigma \) described in (A.8). This part suggests that larger trade costs and higher dissimilarity in traded commodities contain a home bias for the country with the largest effective endowment.

The effect of changes in countries’ effects endowments \( Z^i \) and \( Z^j \) on country \( i \)'s payoff \( w^i_T \) under trade is described by

\[
\tilde{w}^i_T = \tilde{Z}^i - \gamma^j_i \pi^i_T = (1 - \frac{\gamma^j_i}{\Delta})\tilde{Z}^i + \frac{\gamma^j_i}{\Delta}\tilde{Z}^j,
\]

where we used (A.8) and the fact that \( p^j \mu^i_{p^j}/\mu^i = -\gamma^j_j \). Since \( 0 < \frac{\gamma^j_i}{\Delta} < 1 \), \( w^i_T \) unambiguously rises with increases in country \( i \)'s effective endowment \( Z^i \), so immiserizing growth (due to an
adverse terms-of-trade effect) does not arise in this context. Similarly, an increase in country
\( j \)'s effective endowment \( Z_j \) expands \( w_i^j \) because of a favorable (to country \( i \)) terms-of-trade
effect.\(^{37}\)

If \( Z_i = \lambda_i \bar{X} \) and \( w_i^j = \omega_i^j \bar{X} \), how would the division \( \lambda_i \) of the common pool \( \bar{X} \) and the
quantity of guns \( \bar{G} \) affect \( w_i^j \)? Naturally, \( dw_i^j / d\bar{G} = -\omega_i^j \) and \( dw_i^j / ds_i = \bar{X} \omega_{\lambda i}^j \). As noted
in (15), one can show (from the definition of \( \omega_i^j \) and (A.8)) that

\[
\omega_{\lambda i}^j = \mu_i \left( 1 - \frac{\gamma_j^i / \lambda_i^j}{\Delta} \right). \tag{A.10a}
\]

Moreover, \( dw_i^j / d\xi = \bar{X} \omega_{\xi}^j \) for \( \xi \in \{ \tau, \sigma \} \), so the dependence of \( \omega_i^j \), not just on \( \lambda_i \) and \( \bar{G} \),
but also on trade costs and the elasticity of substitution are important. After some algebra,
we find

\[
\omega_i^j = \lambda_i^j \mu_i^j \frac{p_i^j}{\tau} = \omega_i^j \left( p_i^j \mu_i^j / \mu^i \right) \left( \tau p_i^j / p_i^j \right) \frac{1}{\tau} = \frac{\omega_i^j}{\tau} \left( -\gamma_j^i \right) \left[ 1 - \frac{1}{\Delta} (\sigma - 1)(\gamma_i^j - \gamma_j^i) \right]
\]

\[
= -\frac{\omega_i^j \gamma_j^i}{\tau \Delta} \left[ \Delta - (\sigma - 1)(\gamma_i^j - \gamma_j^i) \right] = -\frac{\omega_i^j \gamma_j^i}{\tau \Delta} \left[ 1 + 2 (\sigma - 1) \left( 1 - \gamma_i^j \right) \right] < 0,
\]

where the second line is obtained from the facts that \( p_i^j = \tau p_i^j \), \( p_i^j \mu_i^j / \mu^i = -\gamma_j^i \), the
definition of \( \Delta \) and equation (A.8). In short, the (direct) effect of an increase in trade costs
on a country’s payoff under trade, keeping \( \lambda_i \) and \( \bar{G} \) fixed, is negative.

The effect of \( \sigma \) on \( \omega_i^j \) is a bit more involved as in this case we have

\[
\omega_i^j = \lambda_i^j \left\{ \mu_i^j + \mu_i^j \frac{p_i^j}{\tau} \right\} = \omega_i^j \left\{ \mu_i^j / \mu^i + \left( p_i^j \mu_i^j / \mu^i \right) \left( p_i^j / p_i^j \right) \right\}
\]

\[
= \omega_i^j \left\{ -\frac{1}{(\sigma - 1)^2} \left[ (\sigma - 1) \gamma_i^j \ln (p_i^j) + \ln \left( 1 + (p_i^j)^{1-\sigma} \right) \right] + \frac{\gamma_j^i}{\Delta} \left[ \gamma_i^j \ln (p_i^j) - \gamma_j^i \ln (p_i^j) \right] \right\}.
\]

Utilizing the definition of \( \Delta \) and the properties of logarithms, the above equation can be
rewritten (after some additional algebra) as

\[
\omega_i^j = -\frac{\omega_i^j}{\Delta (\sigma - 1)^2} \left\{ \Delta \ln \left[ 1 + (p_i^j)^{1-\sigma} \right] - \gamma_j^i \ln \left[ (p_i^j)^{1-\sigma} \right] \right\} \tag{A.10c}
\]

\(^{37}\)Equi-proportionate increases in \( Z_i \) and \( Z_j \) would cause both countries’ welfare to rise proportionately
because they leave world prices intact.
\[-(\sigma - 1) \gamma^i_j \gamma^j_i \ln \left( (p^j)^{1-\sigma} \right) - (\sigma - 1) \gamma^i_j \gamma^j_i \ln \left( (p^i)^{1-\sigma} \right) \]
\[= -\frac{\omega^i}{\Delta (\sigma - 1)^2} \left\{ \Delta \ln \left[ 1 + (p^i)^{1-\sigma} \right] - \gamma^i_j \ln \left( (p^i)^{1-\sigma} \right) - (\sigma - 1) \gamma^i_j \gamma^j_i \ln \left( (p^i p^j)^{1-\sigma} \right) \right\} \]
\[= -\frac{\sigma}{\Delta (\sigma - 1)^2} \left\{ \Delta \ln \left[ 1 + (p^i)^{1-\sigma} \right] - \gamma^i_j \ln \left( (p^i)^{1-\sigma} \right) + 2 (\sigma - 1)^2 \gamma^i_j \gamma^j_i \ln (\tau) \right\} .\]

The move from the second equation to the third one is based on the fact that \( p^i p^j = (\tau p^i T) (\tau p^j T) = \tau^2 \). Inspection of the above equation reveals that \( \omega^i < 0 \): (i) because \( \Delta \ln \left[ 1 + (p^i)^{1-\sigma} \right] > \gamma^i_j \ln \left( (p^i)^{1-\sigma} \right) \) in the last equation since \( \Delta > \gamma^i_j \) and \( \ln \left[ 1 + (p^i)^{1-\sigma} \right] > \ln \left( (p^i)^{1-\sigma} \right) \); and (ii) because \( \tau \geq 1 \) implies \( \ln (\tau) \geq 0 \).

Let us now study in finer detail the dependence of \( \omega^i \) on the division \( \lambda^i \) of \( \bar{X} \). In particular, starting with \( \lambda^i = 0 \), let us ask how arbitrary reallocations of \( \bar{X} \) from country \( j \) to country \( i \) (\( i \neq j \)) affect \( \omega^i \) and \( \omega^j \). Going back to (A.10a), one can see that the direct effect of the resource transfer is to improve (worsen) the recipient’s (donor’s) purchasing power and thus its payoff. However, the transfer also causes the recipient (donor) country’s terms-of-trade to deteriorate (improve), so this indirect effect works against the direct effect.\(^{38}\)

The presence of this trade-off raises the following questions. Is there an optimal division, \( \lambda^i_T \), of the common pool that would maximize country \( i \)'s payoff \( \omega^i \)? If there is, what are its properties? For example, is \( \lambda^i_T < 1 \)? Furthermore, is it possible for resource transfers to immiserize both the recipient and the donor countries?\(^{39}\) These questions are of interest in their own right. But, as we will see later, they are of special interest in the context of the resource disputes we are studying because of their consequences for arming and the division of the common pool and, thus, for countries’ preferences over war and peace. Lemma (A.4) below summarizes several key insights in this context.

**Lemma A.4** (Payoffs) For any given guns, payoff \( \omega^i \) depends on the division \( \lambda^i \) of \( \bar{X} \), the elasticity of substitution \( \sigma \in (1, \infty) \) and trade costs \( \tau \in [1, \infty) \) as follows:

\[ \text{(a)} \quad \text{(i)} \lim_{\lambda^i \to 0} \omega^i = 0 \quad \text{and} \quad \lim_{\lambda^i \to 1} \omega^i = 1 \]

\(^{38}\)Once again, for now guns \( G \) and thus \( \bar{X} \) are kept fixed in the background. One can also think of such reallocations as a resource gift from country \( j \) (the donor) to country \( i \) (the recipient). Amano (1966) addressed this terms-of-trade issue in a variety of contexts. However, he did not study the welfare implications of resource transfers for both donor and recipient countries. Garfinkel and Syropoulos (2016) examined a variant of this issue in the context of a modified Ricardian model of trade and conflict.

\(^{39}\)In the standard trade literature that considers pure *income* transfers between two trading partners this possibility does not arise. In fact, stability of world trading equilibrium necessarily implies that the recipient enjoys a welfare improvement while the donor suffers a welfare loss. Prior work in this area (e.g., Brecher and Bhagwati, 1982; Bhagwati et al. 1983) also emphasized the idea that, indeed, transfers could worsen the recipient’s welfare in the presence of distortions. Grossman (1984) argued that, when goods are already traded freely, trade in factors can be welfare-reducing. However, his analysis was in the context of factor movements that require earnings in the host country to be transferred back to the country of origin. Moreover, he did not study the possible existence of immiserizing factor movements in the Pareto sense.
Another way to visualize these findings is to note that, for any 

\( \lambda_i \in (0, 1) \) and attains a maximum at 

\( \lambda^i_T = \arg \max_{\lambda^i} \omega^i \)

(b) \( \omega^i \) is strictly concave in \( \lambda^i \in (0, 1) \) and attains a maximum at \( \lambda^i_T = \frac{1}{2}, \) and \( \lim_{\lambda^i \to 1} \omega^i = 1 \) for \( \lambda^i_T = 1 \) for \( \xi \in \{\sigma, \tau\} \)

(c) \( \lambda^i_T \in (\frac{1}{2}, 1), ds^i_T/d\xi > 0, \lim_{\lambda^i \to 1} \lambda^i_T = \frac{1}{2}, \) and \( \lim_{\lambda^i \to \infty} \lambda^i_T = 1 \) for \( \xi \in \{\sigma, \tau\} \)

(d) \( \omega^i_{\lambda^i} < 0 \) and \( \omega^j_{\lambda^j} < 0 \) for all \( \lambda^i \in (\lambda^i_T, 1) \) \( (i \neq j = 1, 2) \)

(e) \( \partial \omega^i / \partial \xi < 0 \) for \( \xi \in \{\tau, \sigma\} \).

**Proof:** Part (a). For any \( \lambda^i \in (0, 1) \)

\[
\mu^i \equiv \left[ 1 + \tau^{1-\sigma} \left( \pi^i_T \right)^{1-\sigma} \right]^{1/(\sigma-1)} > 1
\]  

(A.11)

because \( \sigma > 1 \). From part (b) of Lemma (A.2) we know that: \( \lim_{\lambda^i \to 1} \pi^i_T = \infty \), which implies \( \lim_{\lambda^i \to 1} \mu^i = 1 \) and \( \lim_{\lambda^i \to 0} \pi^i_T = 0 \), which implies \( \lim_{\lambda^i \to 0} \mu^i = \infty \). The definition of \( \omega^i \) \( (\equiv \mu^i \lambda^i) \) readily implies \( \lim_{\lambda^i \to 1} \omega^i = 1 \). To prove \( \lim_{\lambda^i \to 0} \omega^i = 0 \), rewrite \( \omega^i \) as

\[
\omega^i = \left( \lambda^i / \pi^i_T \right) \left[ \left( \pi^i_T \right)^{\sigma-1} + \tau^{1-\sigma} \right]^{1/(\sigma-1)}
\]

and note that the expression inside the square brackets is finitely positive as \( \lambda^i \to 0 \) because \( \lim_{\lambda^i \to 0} \pi^i_T = 0 \) and \( \tau \in [1, \infty) \). Let us rearrange the world market clearing condition

\[
\frac{\gamma^i}{T^i} \pi^i_T = \frac{\lambda^i}{\lambda^j} \frac{\lambda^j}{\gamma^j}
\]

into \( \lambda^i / \pi^i_T = \lambda^j / \pi^j \). From part (a) of Lemma (A.2) we have \( \lim_{\lambda^i \to 0} \gamma^i = 1 \) and \( \lim_{\lambda^i \to 0} \gamma^j = 0 \) while \( \lambda^j \to 1 \); thus, \( \lim_{\lambda^i \to 0} \left( \lambda^i / \pi^i_T \right) = 0 \). This completes the proof to (i) of part (a).

To prove part (ii), note that \( \lim_{\lambda^i \to 1/2} \pi^i_T = 1 \) (part (b) of Lemma (A.2)) implies

\[
\lim_{\lambda^i \to 1/2} \omega^i = \frac{1}{2} \left[ 1 + \tau^{1-\sigma} \right]^{1/(\sigma-1)}.
\]

It is straightforward for one to verify that there exists a schedule \( \tilde{\sigma}(\tau) \) that is implicitly defined by setting the above expression equal to \( \lim_{\lambda^i \to 1} \omega^i = 1 \) and rearranging terms to obtain: \( f(\sigma, \tau) = 1 + \tau^{1-\sigma} - 2^{\sigma-1} = 0 \). One can verify the following: \( \tilde{\sigma}(1) = 2, \lim_{\tau \to \infty} \tilde{\sigma}(\tau) = 1 \) and (by the implicit function theorem)

\[
\tilde{\sigma}'(\tau) = -\frac{f_\tau}{f_\sigma} = -\frac{\sigma-1}{\tau \left( 2^{\sigma-1} \ln(2) + \ln(\tau) \right)} < 0.
\]

Another way to visualize these findings is to note that, for any \( \sigma \in (1, 2] \), there exists a range \( [1, \tilde{\tau}(\sigma)] \) of trade cost values such that \( \lim_{\lambda^i \to 1/2} \omega^i \geq \lim_{\lambda^i \to 1} \omega^i = 1 \) for all \( \tau \in [1, \tilde{\tau}(\sigma)] \), whereas \( \lim_{\lambda^i \to 1} \omega^i < \lim_{\lambda^i \to 1} \omega^i = 1 \) for all \( (\sigma, \tau) \in (1, 2] \times (\tilde{\tau}(\sigma), \infty) \cup (2, \infty) \times (1, \infty) \). In this case, \( \tilde{\tau}(\sigma) \) solves \( f(\sigma, \tau) = 0, \tilde{\tau}'(\sigma) < 0, \) and \( \tilde{\tau}(2) = 1 \).
Part (b). We know from part (c) of Lemma (A.2) that \(\lim_{\lambda \rightarrow 1} \gamma_j^i = 0\) while \(\lim_{\lambda \rightarrow 1} \gamma_i^j = 1\). Utilizing these observations in the definition of \(\Delta (\equiv 1 + (\sigma - 1)(1 - \gamma_j^i + 1 - \gamma_i^j))\) readily implies \(\lim_{\lambda \rightarrow 1} \Delta = \sigma > 1\). But from part (c) of Lemma (A.2) we also have \(\lim_{\lambda \rightarrow 1} \gamma_i^j/\lambda^i = \infty\). Thus, the expression for \(\omega_{\lambda^i}^j\), inside the parentheses in (A.10a) becomes negative as \(\lambda^i \rightarrow 1\) (i.e., \(\lim_{\lambda \rightarrow 1} \omega_i^j < 0\)). So, \(\omega^i \rightarrow 1\) from above as \(\lambda^i \rightarrow 1\). On the other hand, \(\omega^i\) is continuous in \(\lambda^i\) and from part (a) we know that \(\lim_{\lambda \rightarrow 0} \omega^i = 0\); therefore, \(\omega^i\) attains a maximum at some \(\lambda_T^i \in (1/2, 1)\) that solves \(\omega_i^j = 0\) in (A.10a).

We now prove that \(\omega_{\lambda^i}^j < 0\), which implies that \(\omega^i\) is concave in \(\lambda^i\) and thereby establishes the uniqueness of \(\lambda_T^i\). Differentiation of \(\omega_{\lambda^i}^j\) in (A.10a) gives

\[
\omega_{\lambda^i}^j = \mu^i \left\{ \frac{p^i \mu_i^j}{\mu^i} \left( \frac{p_j^i}{p^i} - \frac{p_i^j}{p^j} \right) \left( 1 - \frac{\gamma_j^i}{\lambda^i \Delta} \right) - \frac{\gamma_j^i}{(\lambda^i)^2 \Delta} \right\}
- \frac{p^i}{\lambda^i \Delta} \left( \frac{\partial \gamma_j^i}{\partial p^i} \right) \left( \frac{p_i^j}{p^i} - \frac{p_j^i}{p^j} \right)
+ \frac{\gamma_j^i}{\lambda^i \Delta^2} \left[ p^i \Delta p^i \left( \frac{p_i^j}{p^i} - \frac{p_j^i}{p^j} \right) + p^j \Delta p^j \left( \frac{p_j^i}{p^j} - \frac{p_i^j}{p^i} \right) \right].
\]

Utilizing the facts that \(p^i \mu_i^j/\mu^i = -\gamma_j^i\), \(p_j^i/p^i - p_i^j/p^j = 1/(\lambda^i \lambda^j \Delta)\), \(p^i(\partial \gamma_j^i/\partial p^i) = -(\sigma - 1) \gamma_j^i \gamma_i^j\), and \(p^i \Delta p^i = (\sigma - 1)^2 \gamma_j^i \gamma_i^j\) for \(i \neq j\) enables us to transform the above expression into:

\[
\omega_{\lambda^i}^j = \mu^i \left[ -\frac{\gamma_j^i}{\lambda^i \lambda^j \Delta} \left( 1 - \frac{\gamma_j^i}{\lambda^j \Delta} \right) - \frac{\gamma_j^i}{(\lambda^j)^2 \Delta} \right]
+ \frac{(\sigma - 1) \gamma_j^i \gamma_i^j}{\lambda^i \lambda^j \Delta} + \frac{\gamma_j^i (\sigma - 1)^2}{\lambda^i \lambda^j (\lambda^j)^2 \Delta^3} \left( \gamma_j^i \gamma_i^j - \gamma_j^i \gamma_i^j \right)
\]

\[
= -\frac{\gamma_j^i \omega_j^i}{(\lambda^i \lambda^j \Delta)^2} \left[ \gamma_j^i A + \gamma_i^j B \right],
\]

where

\[
A = 1 + \frac{(\sigma - 1)^2}{\Delta} \left( \gamma_i^j + \gamma_j^i - 1 \right) \quad \text{and} \quad B = \frac{\sigma(\sigma - 1)}{\Delta}.
\]

But \(A > 0\) (because \(\gamma_i^j + \gamma_j^i - 1 \geq 0\) from part (a) of Lemma (A.1)) and \(B > 0\) (because \(\sigma > 1\)). We must thus have \(\omega_{\lambda^i}^j < 0\) for \(i = 1, 2\).

Part (c): Having shown (in part (b)) that \(\lambda_T^i < 1\), we now prove \(\lambda_T^i > \frac{1}{2}\). Evaluating
\( \omega^i_{\lambda^i} \) at \( \lambda^i = 1/2 \) gives
\[
\omega^i_{\lambda^i} \big|_{\lambda^i=1/2} = 2\mu^i \left( \frac{1}{2} - \frac{\gamma^i_j}{\Delta} \right) > 0.
\]

The above expression is positive because \( \frac{1}{2} \geq \frac{\gamma^i_j}{\Delta} \) (part (c) of Lemma (A.2)) while \( 1 > \frac{1}{\Delta} \). Together these inequalities imply \( \frac{1}{2} \geq \frac{\gamma^i_j}{\Delta} \) at \( \lambda^i = 1/2 \); therefore, \( \lim_{\lambda^i \to 1/2} \omega^i_{\lambda^i} > 0 \). Moreover, \( \lim_{\xi \to \infty} \lambda^i_T = 1 \) for \( \xi \in \{\tau, \sigma\} \).

The dependence of \( \lambda^i_T \) on \( \xi \in \{\sigma, \tau\} \) can be studied by using the implicit function theorem, which implies \( d\lambda^i_T/d\xi = -\omega^i_{\lambda^i, \xi}/\omega^i_{\lambda^i, \lambda^i} \), where \( \omega^i_{\lambda^i, \lambda^i} < 0 \). It is straightforward for one to show (by differentiating \( \omega^i_{\lambda^i} \) with respect to \( \xi \) and evaluating the resulting expression at \( \lambda^i = \lambda^i_T \)) that \( \omega^i_{\lambda^i, \xi} > 0 \) for \( \xi \in \{\sigma, \tau\} \) which establishes the proof. The last two portions of part (c) follow readily by taking the appropriate limits of \( \lambda^i_T \).

**Part (d).** This part follows from part (c). The interesting part about this finding is that it immiserizes both countries.

**Part (e).** This part follows from equations (A.10b) and (A.10c).

Let us now reflect on countries’ payoffs under trade and autarky when (i) guns are set at some arbitrary level (so that \( \bar{X} \) remains fixed), and (ii) \( \bar{X} \) is divided (again arbitrarily) according to a fixed division \( \lambda^i \) under both trade regimes. Since \( w^i_T = \omega^i \bar{X} \) and \( w^i_A = \lambda^i \bar{X} \), the behavior of these payoffs can be understood by the behavior of \( \omega^i \) relative to that of \( \lambda^i \). Lemma (A.6) below summarizes the key ideas.

**Lemma A.5** For any given \( \bar{X} > 0 \), payoffs under trade \( (w^i_T) \) and under autarky \( (w^i_A) \) have the following properties:

(a) \( w^i_T > w^i_A \) for all \( \lambda^i \in (0, 1) \)

(b) \( \lim_{\lambda^i \to 0} w^i_T = \lim_{\lambda^i \to 0} w^i_A = 0 \)

Proof: All parts follow readily from Lemma (A.3). The important point for our purposes is twofold. First, when guns are exogenous (a situation that serves as a valuable benchmark), trade dominates autarky in payoffs for all possible allocations of the common pool except in the extreme cases where \( \lambda^i = 0 \) and \( \lambda^i = 1 \). Second, if the elasticity of

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40It’s important to keep in mind though that our upcoming comparison of conflict and settlement will be complicated by the following facts: (i) The size of the common pool \( \bar{X} \) under these regimes will differ because arming incentives across these regimes will differ. (ii) In addition to guns, the division of \( \bar{X} \) under these regimes differs and is endogenous.
substitution $\sigma$ is sufficiently close to 1 (so that the gains from trade are sufficiently high), a country enjoys a higher level of welfare under an even split of the common pool relative to a situation in which it controls the entire pool. As we will see later, this point can help understand the emergence of an equilibrium under settlement with less – and under some circumstances – no arming.

**Lemma A.6** For any feasible quantity of guns $G$ that yields a common pool of non-negligible size $X = R - G > 0$, the global gains from peace function $\Omega (\lambda^i; \sigma, \tau, \beta)$ is strictly concave in $\lambda^i$ and maximized at $\lambda_{\max}^i = \frac{1}{2}$. Moreover,

(a) $\lim_{\lambda^i \rightarrow 0} \Omega = \lim_{\lambda^i \rightarrow 1} \Omega = 1 - \beta$

(b) $\frac{\partial \Omega}{\partial (-\xi)} > 0$ for $\xi \in \{\sigma, \tau, \beta\}$.

**Proof:** The proof follows in a straightforward way from the properties of the individual components of $\Omega$, studied in Lemma (A.4). In particular, the strict concavity of $\Omega$ in $\lambda^i$ is due to the fact that it is the sum of the strictly concave functions $\omega^i$ and $\omega_j^j$. The reason a social planner would choose $\lambda_{\max}^i = \frac{1}{2}$ is threefold: (i) the production function $F(\cdot, \cdot)$ of the final good in each country (see (1)) is symmetric; (ii) the technologies of countries’ respective intermediate goods do not differ; and (iii) the rate of destruction $1 - \beta$ is fixed. Part (a) is fairly obvious: if all of $X$ is allocated to a single country, there are no gains from trade. Nonetheless, peace will still generate global gains through the avoidance of destruction. Part (b) follows from part (c) of Lemma (A.4) and the definition of $\Omega$ in (??).

**Lemma A.7** $\Psi^i$ in (13) is strictly quasiconcave (resp., quasiconvex) in $\lambda^i \in \left[\frac{1}{2}, 1\right]$ (resp., $\lambda^i \in [0, \frac{1}{2}]$); hence $\Psi^i$ admits a unique maximum $\lambda_{\max}^i = \max_{\lambda^i} \Psi^i$ (resp., unique minimum $\lambda_{\min}^i = \min_{\lambda^i} \Psi^i$), such that $\Psi^i_{\lambda^i} > 0$ for $\lambda^i \in (\lambda_{\min}^i, \lambda_{\max}^i)$. Moreover, $\lambda_{\min}^i = 1 - \lambda_{\max}^i$ and

(a) $\lambda_{\max}^i \in \left(\frac{1}{2}, 1\right)$ and $\Psi^i_{\lambda^i = \lambda_{\max}^i} > 0$ for $\sigma - \tau < 1$, whereas

(b) $\lambda_{\max}^i = 1$ and $\Psi^i_{\lambda^i = \lambda_{\max}^i} \geq 0$ (with equality if $\beta = 1$ and $\phi_j = 1$) for $\sigma - \tau \geq 1$.

**Proof:** To prove that $\Psi^i$ is strictly quasiconcave in $\lambda^i \in \left[\frac{1}{2}, 1\right]$ it is sufficient to show that there exists an $\lambda_{\max}^i \in \left[\frac{1}{2}, 1\right]$ such that $\Psi^i_{\lambda^i} > 0$ for $\lambda^i \in \left[\frac{1}{2}, \lambda_{\max}^i\right]$ and $\Psi^i_{\lambda^i} < 0$ for $\lambda^i \in (\lambda_{\max}^i, 1]$. First note from part (c) of Lemma (A.4) that $\omega^i_{\lambda^i} > 0$ and $\omega^j_{\lambda^i} > 0$ for $\lambda^i \in \left[\frac{1}{2}, \lambda^i_T\right)$, while $\omega^i_{\lambda^i} < 0$ as $\lambda^i \rightarrow 1$; therefore, $\Psi^i_{\lambda^i} > 0$ for $\lambda^i \in \left[\frac{1}{2}, \lambda^i_T + \varepsilon\right)$ for some $\varepsilon > 0$. We do not know the sign of $\Psi^i_{\lambda^i}$ for $\lambda^i \in [\lambda^i_T + \varepsilon, 1]$. There are two possibilities: Either $\Psi^i_{\lambda^i} > 0$ for all $\lambda^i \in \left[\frac{1}{2}, 1\right]$, in which case $\lambda_{\max}^i = 1$, or there exists at least one $\lambda_{\max}^i \in (\lambda^i_T, 1]$ such that $\Psi^i_{\lambda^i} = 0$. We start with the latter case and prove that $\lambda_{\max}^i$ is a unique maximizer of $\Psi^i$ because $\Psi^i_{\lambda^i = \lambda_{\max}^i} < 0$. 

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By its definition, \( \Psi_{\lambda^i \lambda^j} = \omega_{\lambda^i \lambda^j}^i - \omega_{\lambda^i \lambda^j}^j \) (\( i \neq j \)), where \( \omega_{\lambda^i \lambda^j}^i < 0 \) from \( \text{(A.12)} \) for \( i = 1, 2 \). Thus, to prove the above point it is sufficient to prove that \( \left( \frac{\omega_{\lambda^i \lambda^j}^i}{\omega_{\lambda^i \lambda^j}^j} \right) \bigg|_{\Psi_{\lambda^i} = 0} > 1 \). From the definition of \( \omega_{\lambda^i \lambda^j}^i \) in \( \text{(A.12)} \) we have

\[
\frac{\omega_{\lambda^i \lambda^j}^i}{\omega_{\lambda^i \lambda^j}^j} = \frac{\omega^i \gamma^i_j}{\omega^j \gamma^j_i} \left[ \gamma^i_i A + \gamma^i_j B \right] = \frac{\mu^i \gamma^i_j}{\mu^j \gamma^j_i} \left[ \gamma^i_i A + \gamma^i_j B \right].
\]

(A.14)

Furthermore, \( \omega_{\lambda^i}^i < 0 \) and \( \omega_{\lambda^j}^j > 0 \) at \( \bar{x}^i_{\text{max}} \). Applying \( \text{(A.10a)} \) onto \( \Psi_{\lambda^i} = \omega_{\lambda^i}^i + \omega_{\lambda^j}^j = 0 \) and rearranging terms gives

\[
\frac{\mu^i}{\mu^j} = \left( \frac{1 - \frac{\omega_{\lambda^i}^i}{\omega_{\lambda^j}^j}}{\Delta} \right) / \left( -1 + \frac{\omega_{\lambda^i}^i}{\omega_{\lambda^j}^j} \frac{\mu^i}{\mu^j} \right) > 0.
\]

Substituting the above expression in \( \text{(A.14)} \) allows us to obtain

\[
\left. \frac{\omega_{\lambda^i \lambda^j}^i}{\omega_{\lambda^i \lambda^j}^j} \right|_{\Psi_{\lambda^i} = 0} = \frac{\left( \lambda^i \Delta - \gamma^i_j \right) \gamma^j_i \left[ \gamma^i_i A + \gamma^i_j B \right]}{\left( -\lambda^j \Delta + \gamma^j_i \right) \gamma^i_i \left[ \gamma^j_j A + \gamma^j_i B \right]}.
\]

(A.14')

Noting that both the numerator and the denominator of the above expression are positive, we may subtract the latter from the former to obtain

\[
C = \left( \lambda^i \Delta - \gamma^i_j \right) \gamma^j_i \left[ \gamma^i_i A + \gamma^i_j B \right] - \left( -\lambda^j \Delta + \gamma^j_i \right) \gamma^i_i \left[ \gamma^j_j A + \gamma^j_i B \right].
\]

Thus, to prove that the expression in \( \text{(A.14')} \) is larger than 1 it suffices to prove that \( C > 0 \).

Utilizing the definitions of \( A \) and \( B \) from \( \text{(A.13)} \) and of \( \Delta \) in the above expression gives, after rearrangement and simplification,

\[
sign \left( C \right) = sign \left( C_1 + C_2 + C_3 \right)
\]

where

\[
C_1 \equiv \gamma^i_j \gamma^j_i \left( \lambda^i - \gamma^i_j \right) + \gamma^j_i \gamma^i_j \left( \lambda^j - \gamma^j_i \right) = \frac{\left( \left( p^i p^j \right)^{\sigma - 1} - 1 \right) \left[ \lambda^i \left( p^i \right)^{\sigma - 1} + \lambda^j \left( p^j \right)^{\sigma - 1} \right]}{\left[ 1 + \left( p^i \right)^{\sigma - 1} \right]^2 \left[ 1 + \left( p^j \right)^{\sigma - 1} \right]^2},
\]

\[
C_2 \equiv \left( \sigma - 1 \right) \left( \lambda^i \gamma^j_i \gamma^i_j + \lambda^i \gamma^j_i \gamma^i_i \right) > 0
\]

and

\[
C_3 \equiv \left( \sigma - 1 \right) \left( \gamma^i_i + \gamma^j_j \right) \left( \lambda^i \gamma^i_i \gamma^i_j + \lambda^j \gamma^j_i \gamma^j_i \right) > 0.
\]

But \( \left( p^i p^j \right)^{\sigma - 1} = \left( \pi^i_T \right)^{2(\sigma - 1)} \) > 1 in the definition of \( C_1 \) — from part (a) of Lemma \( \text{(A.3)} \)
and the fact that $\sigma > 1$ — so $C_1 > 0$. The sign of $C_3$ follows from part (a) of Lemma (A.1). Thus, $C > 0$ and $\Psi_{\lambda^i}^{(\lambda^i)} |_{\lambda^i = \lambda^i_{\text{max}}} < 0$.

We will now prove that $\text{sign} \left[ \lim_{\lambda^i \to 1} \Psi_{\lambda^i}^{(i)} \right] = \text{sign} \left[ \lim_{\lambda^i \to 1} \left( 1 + \omega_{\lambda^i}^{(i)} / \omega_{\lambda^i}^{(j)} \right) \right] = \text{sign} \left( \sigma - 1 - \tau \right)$. (The second equation holds true because $\lim_{\lambda^i \to 1} \omega_{\lambda^i}^{(j)} > 0$.) From (A.10) we have

$$1 + \frac{\omega_{\lambda^i}^{(i)}}{\omega_{\lambda^i}^{(j)}} = 1 + \frac{\mu^i \left( 1 - \frac{\gamma_{\lambda^i}^{(i)}}{\Delta} \right)}{\mu^j \left( 1 - \frac{\gamma_{\lambda^i}^{(j)}}{\Delta} \right)}, \text{ for } i \neq j = 1, 2. \quad (A.15)$$

From the definition of $\mu^i$ in (A.11), the facts that $p^i = \tau \pi_T^i$, $\pi_T^i = \tau / \pi_T^i$, and parts (b) and (c) of Lemma (A.2) we have $\lim_{\lambda^i \to 1} \mu^i = 1$, $\lim_{\lambda^i \to 1} \mu^j = \infty$, $\lim_{\lambda^i \to 1} \left( \gamma_{\lambda^i}^{(i)} / \lambda^i \right) = \infty$, $\lim_{\lambda^i \to 1} \left( \gamma_{\lambda^i}^{(j)} / \lambda^i \right) = 1$, and $\lim_{\lambda^i \to 1} \Delta = \sigma$. Now rewrite $\mu^j$ from (A.11) as

$$\mu^j = [1 + (p^j)^{\sigma - 1}]^{-\frac{1}{\sigma - 1}} = [p^j]^{-1} [1 + (p^j)^{\sigma - 1}]^{-\frac{1}{\sigma - 1}} = \frac{\pi_T^i}{\tau} \left[ 1 + (p^j)^{\sigma - 1} \right]^{-\frac{1}{\sigma - 1}}.$$ 

Substitution of $\pi_T^i = \frac{\lambda^i \gamma_{\lambda^i}^{(i)}}{\gamma_{\lambda^i}^{(i)}} = \frac{\gamma_{\lambda^i}^{(i)}}{\gamma_{\lambda^i}^{(j)}}$ from the world market clearing condition in the above expression and utilization of the resulting expression in (A.15) allows us to rewrite it as

$$1 + \frac{\mu^i \left( 1 - \frac{\gamma_{\lambda^i}^{(i)}}{\Delta} \right) \tau \frac{\gamma_{\lambda^i}^{(i)}}{\gamma_{\lambda^i}^{(j)}} \frac{\gamma_{\lambda^i}^{(i)}}{\lambda^i} \frac{\gamma_{\lambda^i}^{(j)}}{\lambda^i} \tau}{\left( 1 - \frac{\gamma_{\lambda^i}^{(j)}}{\Delta} \right) \left[ 1 + (\tau / \pi_T^i)^{\sigma - 1} \right]^{-\frac{1}{\sigma - 1}}} = 1 + \frac{\mu^i \left( \frac{\gamma_{\lambda^i}^{(i)}}{\gamma_{\lambda^i}^{(j)}} - \frac{\gamma_{\lambda^i}^{(j)}}{\lambda^i} \right) \tau}{\left( 1 - \frac{\gamma_{\lambda^i}^{(j)}}{\Delta} \right) \left[ 1 + (\tau / \pi_T^i)^{\sigma - 1} \right]^{-\frac{1}{\sigma - 1}}}.$$ 

It should now be possible for one to see that

$$\lim_{\lambda^i \to 1} \left( 1 + \omega_{\lambda^i}^{(i)} / \omega_{\lambda^i}^{(j)} \right) = 1 + \lim_{\lambda^i \to 1} \frac{\mu^i \left[ \frac{1}{\gamma_{\lambda^i}^{(i)}} - \frac{1}{\Delta} \right] \left( \frac{\gamma_{\lambda^i}^{(j)}}{\lambda^i} \right) \tau}{\left( 1 - \frac{\gamma_{\lambda^i}^{(j)}}{\Delta} \right) \left[ 1 + (\tau / \pi_T^i)^{\sigma - 1} \right]^{-\frac{1}{\sigma - 1}}}$$

$$= 1 + \frac{1 \times \left[ \frac{1}{\Delta} - \frac{1}{\tau} \right] \times 1 \times \tau}{\left( 1 - \frac{1}{\tau} \right) \left[ 1 + (\frac{\tau}{\Delta})^{\sigma - 1} \right]^{-\frac{1}{\sigma - 1}}}$$

$$= \frac{\sigma - 1 - \tau}{\sigma - 1}, \text{ since } \sigma - 1 > 0.$$ 

Hence, $\text{sign} \left[ \lim_{\lambda^i \to 1} \Psi_{\lambda^i}^{(i)} \right] = \text{sign} \left( \sigma - 1 - \tau \right)$.

Since $\Psi (\cdot)$ in (13) is a symmetric function with respect to $\lambda^i$ and guns (so that the properties of $\Psi^i$ also hold true for $\Psi^j$), $\Psi^j$ is strictly quasiconcave in $\lambda^j \in \left[ \frac{1}{2}, 1 \right]$; thus $\Psi^i$ is strictly quasiconvex in $\lambda^i \in \left[ 0, \frac{1}{2} \right]$ and attains a minimum at $\lambda^i_{\text{min}} = 1 - \lambda^i_{\text{max}}$. Moreover,
\[ \overline{x}_i^j = x_i^j \text{ for } i \neq j = 1, 2. \] We prove uniqueness below.

**Part (a).** Suppose \( \sigma - 1 - \tau < 0 \), so that the \( \omega^i - \omega^j \) component of \( \Psi^i \) approaches 1 from above as \( \lambda^i \to 1 \). Also note that \( \omega^i - \omega^j = 0 \) at \( \lambda^i = \frac{1}{2} \). It follows that \( \omega^i - \omega^j \) (and thus \( \Psi^i \)) will attain a maximum at some \( \overline{x}^i_{\text{max}} \in (\lambda^i_0^j, 1) \). This maximum has to be unique because \( \Psi^i|_{\lambda^i = \overline{x}^i_{\text{max}}} < 0 \) at all extrema in \( [\frac{1}{2}, 1] \), so \( (\omega^i - \omega^j)|_{\lambda^i = \overline{x}^i_{\text{max}}} > 1 \). The definition of \( \Psi^i \) now implies

\[
\Psi^i|_{\lambda^i = \overline{x}^i_{\text{max}}} = (\omega^i - \omega^j)|_{\lambda^i = \overline{x}^i_{\text{max}}} - \beta (\phi^i - \phi^j) > 1 - \beta (\phi^i - \phi^j) \geq 0,
\]

thus completing the proof to this part.

**Part (b).** Now suppose \( \sigma - 1 - \tau > 0 \), so that the \( \omega^i - \omega^j \) component of \( \Psi^i \) approaches 1 from below. To confirm the claim in this part, suppose that, in addition to \( \overline{x}^i_{\text{max}} = 1 \), there exists another (local) interior maximum such that \( \overline{x}^i_{\text{max}} < 1 \). Because \( \lim_{\lambda^i \to 1} \Psi^i_{\lambda^i} > 0 \) and \( \lim_{\lambda^i \to 1/2} \Psi^i_{\lambda^i} > 0 \) this type of maximum can exist only if, along with it, there also exists a local (interior) minimum at which \( \Psi^i_{\lambda^i} = 0 \). But this is impossible because we’ve shown that \( \Psi^i|_{\lambda^i = \overline{x}^i_{\text{max}}} < 0 \) at any interior extremum in \( [\frac{1}{2}, 1] \). Thus, \( \overline{x}^i_{\text{max}} = 1 \) is unique, \( (\omega^i - \omega^j)|_{\lambda^i = \overline{x}^i_{\text{max}}} = 1 \) and \( \Psi^i|_{\lambda^i = \overline{x}^i_{\text{max}}} = 1 - \beta (\phi^i - \phi^j) \geq 0 \) with equality only if \( \beta = 1 \) and \( \phi^i = 1 \).

**Proof of Lemma 1:** The unique division \( \overline{x} \) follows from the fact (proven in Lemma (A.8)) that the \( \omega^i - \omega^j \) component of \( \Psi^i \) is continuously increasing in \( \lambda^i \in [\frac{1}{2}, \overline{x}^i_{\text{max}}] \), taking values between 0 and a number \( \geq 1 \). Thus, if \( \phi^i = 1 \) (which, incidentally, arises if \( G^i > 0 \) and \( G^j = 0 \)), then, by the intermediate value theorem, there will exist a unique solution \( \overline{x}^i \) such that \( \Psi^i(\overline{x}^i) = 0 \). The reason \( \lambda^i = 1 - \overline{x} \) is the symmetric structure of \( \Psi(\cdot) \).

**Part (a).** The first portion of this part should be self-evident. The second portion follows from part (b) of Lemma (A.8).

The existence of a unique division \( \lambda^i(\cdot) = \{\lambda^i | \Psi^i(\lambda^i) = 0\} \), too, follows straightforwardly from the intermediate value theorem and Lemma (A.8).

**Part (b).** The first component is obvious. The second is due to the symmetric structure of \( \Psi(\cdot) \).

**Part (c).** The proof follows from the properties of the CSF in (6) and the fact that \( \Psi^i_{\lambda^i} = \omega^i_{\lambda^i} + \omega^j_{\lambda^i} > 0 \) for any feasible \( G^i \in [0, R^i) \) and \( G^j \in [0, R^j) \).

**Part (d).** Obvious.
**Proof of Lemma 2**: Part (a). To prove this part we need to prove that \( v_{iG} = \partial^2 \tilde{v}^i / \partial (G^i)^2 < 0 \). Differentiating (16) with respect to \( G^i \) and simplifying the resulting expression yields

\[
\tilde{v}_{iG}^i = \left[ \frac{\omega^i_{\lambda} \omega^j_{\lambda} + \omega^j_{\lambda} \omega^i_{\lambda}}{(\omega^j_{\lambda} + \omega^i_{\lambda})^2} (2\beta \tilde{X} \phi^i_{\lambda}) + \omega^i_{\lambda} \left( -2 + \tilde{X} \phi^i_{G^i} / \phi^j_{G^j} \right) \right] \lambda^i_{G^i}, \tag{A.13}
\]

where \( \omega^i_{\lambda} < 0 \) and \( \omega^j_{\lambda} < 0 \) from Lemma (A.4), while \( \omega^i_{\lambda} > 0 \) and \( \omega^j_{\lambda} > 0 \) from Lemma 1 \((i \neq j)\). (***). Since we also have \( \phi^i_{G^i} > 0 \), the first expression inside the square brackets will be negative. But the second expression inside these brackets is also negative since \( \phi^i_{G^i} < 0 \) for \( i = 1, 2 \). Lastly, from Lemma 1 \((b)\), we have \( \lambda^i_{G^i} > 0 \). It is thus clear that \( \tilde{v}_{iG}^i < 0 \) for \( i = 1, 2 \).

Part (b). Differentiating (16) with respect to \( G^j \) gives

\[
\tilde{v}_{iG}^j = \left[ \frac{\omega^i_{\lambda} \omega^j_{\lambda} + \omega^j_{\lambda} \omega^i_{\lambda}}{(\omega^j_{\lambda} + \omega^i_{\lambda})^2} (2\beta \tilde{X} \phi^i_{\lambda}) + \omega^j_{\lambda} \left( -2 + \tilde{X} \phi^i_{G^i} / \phi^j_{G^j} \right) \right] \lambda^j_{G^j}. \tag{A.14}
\]

From the proof to part \((a)\) the first term inside the square brackets is negative. Moreover, the properties of the CSF in (6) imply \( \phi^i_{G^i} < 0 \) and \( \phi^j_{G^j} > 0 \) for \( G^i \geq G^j \) \((i \neq j)\); so the second term is also negative. Lastly, \( \lambda^i_{G^i} < 0 \) for \( i \neq j \) from Lemma 2 \((a)\). For these reasons, we have \( \tilde{v}_{iG}^j > 0 \).

Part (c). The slope of country \( i \)'s unconstrained best-response function under settlement is \( \partial \tilde{B}^i_{s} / \partial G^j = -\tilde{v}_{iG}^j / \tilde{v}_{iG}^i > 0 \) for \( G^i \geq G^j \), where the positive sign follows from parts \((a)\) and \((b)\). Thus, \( G^i \) is a strategic complement for \( G^j \) for all \( G^i \geq G^j \). To complete the proof to this part, divide (A.13) by (A.14) and note from the definitions in the text that \( \lambda^i_{G^j} / \lambda^i_{G^i} = \phi^i_{G^j} / \phi^i_{G^i} < 0 \). Doing so gives

\[
\frac{\partial \tilde{B}^i_{s}}{\partial G^j} = - \left( \frac{\phi^i_{G^j}}{\phi^i_{G^i}} \right) \left[ \frac{-\omega^i_{\lambda} \omega^j_{\lambda} + \omega^j_{\lambda} \omega^i_{\lambda}}{(\omega^j_{\lambda} + \omega^i_{\lambda})^2} (2\beta \tilde{X} \phi^i_{\lambda}) + \omega^i_{\lambda} \left( 2 - \tilde{X} \phi^j_{G^j} / \phi^i_{G^i} \right) \right] \tag{A.15}
\]

Once again, the properties of the CSF imply that, at \( G^i = G^j \), we have \( \phi^i_{G^i} / \phi^i_{G^i} = -1 \) and \( \phi^j_{G^j} = 0 \). It should now be easy to see that, at \( G^i = G^j \), the above expression satisfies \( \partial \tilde{B}^i_{s} / \partial G^j \in (0, 1) \). \|

**Proof of Proposition 3**: We first argue that the two adversaries will produce equal quantities of guns if neither country’s arming is constrained by its initial resource endowment. We will prove this point by contradiction. Since \( \partial V^i / \partial G^i = 0 \) at an interior solution
for $i = 1, 2$ since in this case neither country’s resource constraint is active, from (16) we will have

$$\frac{MB^i}{MB^j} = \frac{MC^i}{MC^j} \Rightarrow \frac{\omega^i_G}{\omega^j_G} = \frac{\xi^i}{\xi^j} \Rightarrow \left( \frac{G^j}{G^i} \right) \left[ \frac{1 - \frac{\gamma^j}{\lambda^j \Delta}}{1 - \frac{\gamma^j}{\lambda^j \Delta}} \right] = \frac{\lambda^i}{\lambda^j}, \quad (A.16)$$

for $i \neq j$, where $MB^i$ and $MC^i$ denote country $i$’s marginal benefit and marginal cost to arming under settlement. Suppose $G^j/G^i < 1$ which (by Lemma 1 (d)) implies $\lambda^i/\lambda^j > 1$. Thus, to ensure equality, the value of the expression inside the square brackets will have to exceed 1. Because $\pi^i_T$ is increasing in $\lambda^i$ (part (a) of Lemma (A.2)) and, moreover, countries have symmetric technologies, we will have $\pi^i_T > 1$. In turn, this inequality implies (from the world market clearing condition (4)) $\frac{\lambda^i}{\lambda^j} > \frac{\lambda^j}{\lambda^i}$, which could be rewritten as

$$\frac{\gamma^j}{\lambda^j} > \frac{\gamma^i}{\lambda^i} \Rightarrow \frac{\gamma^j}{\lambda^j \Delta} > \frac{\gamma^i}{\lambda^i \Delta} \Rightarrow \frac{1 - \frac{\gamma^j}{\lambda^j \Delta}}{1 - \frac{\gamma^i}{\lambda^i \Delta}} < 1. \quad (A.17)$$

This contradicts our supposition that $G^j/G^i < 1$. Since this argument also applies to the case of $G^j/G^i > 1$, the supposition that $G^i \neq G^j$ at an interior solution is false. In short, $G^i = G^j$ when both countries’ arming decisions are unconstrained by their respective initial resource endowments.

Uniqueness of equilibrium follows readily from Lemma 2. Parts (a) – (c) are also direct consequences of Lemma 2. ||

**Proof of Proposition 4:** As explained in the independence of payoffs from $\delta$ under settlement is due to stationarity.

**Part (a).** The first point in part (a.i) is that, when country $i$’s arming is constrained by its resource endowment, then $V^i_s$ ↓ as $R^i \rightarrow R^i_L$ from the left (provided $\sigma < \infty$ and $\tau < \infty$). The second point in part (a.i) is that $V^i_s$ is increasing in $R^i$ for sufficiently low $R^i$. Thus, for $R^i \in (0, R^i_L)$, $V^i_s$ is non-monotonic in $R^i$ and attains a maximum at some $R^i \in (0, R^i_L)$. This suggests that a redistribution of resources from the poorer to the more affluent country prior to negotiation and bargaining will turn out to be payoff improving in the Pareto sense.

To prove this part, we start by noting that $R^i$ affect payoffs solely through guns (there is no direct effect). Thus

$$\frac{dV^i_s(R^i, \bar{B}^i_s(R^i))}{dR^i} = V^i_G + V^i_G \left[ \frac{d\bar{B}^i_s(R^i)}{dR^i} \right]$$

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\[
\begin{align*}
&= [X\omega^i, \lambda_{G^i}^i - \omega^i] + [X\omega^i, \lambda_{G^j}^i - \omega^j] \left[ \frac{d\tilde{B}_s^j(R^i)}{dR^i} \right] \\
&= [X\omega^i, \lambda_{G^i}^i - \omega^i] - [X\omega^i, \lambda_{G^i}^i + \omega^i] \left[ \frac{d\tilde{B}_s^j(R^i)}{dR^i} \right] \\
&= [X\omega^i, \lambda_{G^i}^i + \omega^i] \left[ \frac{X\omega^i, \lambda_{G^i}^i - \omega^i}{X\omega^i, \lambda_{G^i}^i + \omega^i} - \frac{d\tilde{B}_s^j(R^i)}{dR^i} \right],
\end{align*}
\]

for \(i \neq j, 2\). Since \(\tilde{X}\omega^i, \lambda_{G^i}^i + \omega^i > 0\) in \((A.18)\) we will have

\[
\text{sign} \left[ \frac{dV_s^i}{dR^i} \right] = \text{sign} \left[ \frac{\tilde{X}\omega^i, \lambda_{G^i}^i - \omega^i}{\tilde{X}\omega^i, \lambda_{G^i}^i + \omega^i} - \frac{d\tilde{B}_s^j(R^i)}{dR^i} \right].
\]

(A.19)

Now suppose \(R^i \to R^i_L\), which implies \(V_s^i \to 0\) and thus \((\tilde{X}\omega^i, \lambda_{G^i}^i - \omega^i) \to 0\) in \((A.19)\). But by Lemma 3 (c) we know that \(\lim_{R^i \to R^i_L} d\tilde{B}_s^j(R^i)/dR^i \in (0,1)\). Thus, the expression in the brackets in the RHS of \((A.19)\) is negative, as required. The case of \(R^i \to 0\) follows by noting that the first term in the brackets in the RHS of \((A.19)\) is less than 1 while the second term attains a value larger than 1 for \(R^i\) sufficiently close to 0.

Part \((a.ii)\) follows readily from the relevant definitions. We now turn to part \((iii)\). For simplicity, focus on \(R^i \in (0, R^i_L)\) and consider the impact of \(R^i\) on (the unconstrained) country \(j\)'s payoff. Since, in this case, \(G^i = R^i\) and \(V_s^j = 0\), we will have \(dV_s^i/dR^i = V_s^j < 0\); thus \(dV_s^i/dR^j > 0\).

Part \((b)\). Suppose \(R^i = \tilde{R}\), so that country \(j\) is inconsequential. Since \(\omega^i = 1\) in this case and there is no need for arming we will have \(v^i = \tilde{R}\). Now suppose \(R^j = \epsilon > 0\) is infinitesimally small, so that \(G^j_s = R^j\) (because \(R^j \in (0, R^j_L)\)) and \(R^i = \tilde{R} - \epsilon\) is arbitrarily close to \(\tilde{R}\). Thus, under settlement, country \(i\) has the option to arm in a way that captures the entire common pool \(\tilde{X}\) which would be arbitrarily close to \(\tilde{R}\). But this is not optimal for country \(i\). In fact, country \(i\) will have an interest in allocating a small but positive fraction of its resource into arming in a way that takes advantage of the fact that trading with country \(j\) generates gains form trade. Thus, country \(i\)' FOC (16) essentially implies a division \(\lambda^i < 1\). This suggests the \(\lim_{R^i \to \tilde{R}} V_s^i \to \tilde{R}\). By the same token, because country \(j\) will enjoy gains from trade it will attain a payoff that exceeds 0. If \(\sigma \to \infty\) there would be no gains from trade and the above argument does not hold.

Part \((c)\). The impact of \(\xi \in \{\sigma, \tau\}\) on \(V_s^i\) depends on the international distribution of asset ownership. (To be completed later.)
$R_L^c = (1 - \frac{1-\delta}{2})\overline{R}/2$ and $R_H^c = (1 + \frac{1-\delta}{2})\overline{R}/2$

Figure 1: Arming Equilibria under Conflict
**Figure 2**: Arming and Payoffs under Conflict for Alternative Distributions of Asset Ownership
\[ \delta = 1 \]

\[ R^c_L = \left(1 - \frac{1-\delta}{2}\right)\bar{R}/2 \quad \text{and} \quad R^c_H = \left(1 + \frac{1-\delta}{2}\right)\bar{R}/2 \]

\[ R^e_L = (1 - \frac{m}{1+m})\bar{R}/2 \quad \text{and} \quad R^e_H = (1 + \frac{m}{1+m})\bar{R}/2 \]

**Figure 3:** Arming Equilibria under Conflict, Settlement and Unilateral Deviations
Figure 4: Arming and Payoffs under Settlement for Alternative Distributions of Asset Ownership
Figure 5: Arming and Payoffs under Conflict, Settlement, and Unilateral Deviations for Alternative Distributions of Asset Ownership.