(b) Verify the stable range of $K$ by using margin to determine PM for selected values of $K$.

(c) Use rlocus and rlocfind to determine the values of $K$ at the stability boundaries.

(d) Sketch the Nyquist plot of the system, and use it to verify the number of unstable roots for the unstable ranges of $K$.

(e) Using Routh's criterion, determine the ranges of $K$ for closed-loop stability of this system.

**Solution:**

(a) The Bode plot for $K = 1$ is:

![Bode Plot](image)

From the Bode plot, the closed-loop system is unstable for $K = 1$. But we can make the closed-system stable with positive GM by
increasing the gain $K$ up to the crossover frequency reaches at $\omega = 1.414 \text{ rad/sec} \ (K = 2)$, where the phase plot crosses the $-180^\circ$ line. Therefore:

$$1 < K < 2 \implies \text{The closed-loop system is stable.}$$

(b) For example, $PM = 6.66 \text{ deg}$ for $K = 1.5$.

(c) Root locus is:

$$j\omega\text{-crossing:}$$

$$1 + \frac{j\omega + 2}{(j\omega)^3 + (j\omega)^2 - 2} = 0$$
\[ \omega^2 - 2K + 2 = 0 \]
\[ \omega(\omega^2 - K) = 0 \]

\[ K = 2, \ \omega = \pm \sqrt{2}, \text{ or } K = 1, \ \omega = 0 \]

Therefore,

\[ 1 < K < 2 \implies \text{The closed-loop system is stable.} \]

(i) \( 0 < K < 1 \)

\[ N = 0, \ P = 1 \implies Z = 1 \]

One unstable closed-loop root.

(ii) \( 1 < K < 2 \)

\[ N = -1, \ P = 1 \implies Z = 0 \]

Stable.

(iii) \( 2 < K \)

\[ N = 1, \ P = 1 \implies Z = 2 \]

Two unstable closed-loop roots.

(e) The closed-loop transfer function of this system is:

\[
\frac{y(s)}{r(s)} = \frac{k \frac{1}{s - 1}}{1 + k \frac{1}{s - 1} \times \frac{s + 2}{(s + 1)^2 + 1}}
\]
\[
= \frac{K(s^2 + 2s + 2)}{s^3 + s^2 + Ks + 2K - 2}
\]

So the characteristic equation is:

\[ \implies s^3 + s^2 + Ks + 2K - 2 = 0 \]
Using the Routh’s criterion,
\[ \begin{align*}
  s^3 : & \quad 1 & & K \\
  s^2 : & \quad 1 & & 2K - 2 \\
  s^1 : & \quad 2 - K & & 0 \\
  s^0 : & \quad 2k = 2
\end{align*} \]

For stability,
\[ 2 - K > 0 \]
\[ 2K - 2 > 0 \]
\[ \implies 2 > K > 1 \]

0 < K < 1 Unstable
1 < K < 2 Stable
2 < K Unstable

27. Suppose that in Fig. 6.94,

\[ G(s) = \frac{-3.2(s + 1)}{s(s + 2)(s^2 + 0.2s + 16)}. \]

Use MATLAB’s margin to calculate the PM and GM for G(s) and comment on whether you think this system will have well damped closed-loop roots.

Solution:
MATLAB’s margin plot for the given system is:

![Bode plot](image-url)
From the MATLAB margin routine, $PM = 92.8^\circ$. Based on this result, Fig. 6.36 suggests that the damping will be $= 1$; that is, the roots will be real. However, closer inspection shows that a very small increase in gain would result in an instability from the resonance leading one to believe that the damping of these roots is very small. Use of MATLAB's damp routine on the closed loop system confirms this where we see that there are two real poles ($\zeta = 1$) and two very lightly damped poles with $\zeta = 0.0027$. This is a good example where one needs to be careful to not use Matlab without thinking.